Analytic One-Group S_2 Slab Problem with Isotropic Scattering and Fission Applied to Leakage and Neutron Multiplicity Sensitivity

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Introduction

- NC State University student and CNEC Fellow Alex Clark coded the sensitivity of the second moment of the neutron multiplicity counting distribution into SENSMG.
  - Mattingly worked out a multigroup deterministic method for calculating moments of the neutron multiplicity counting distribution using forward and adjoint solutions.[a]
  - O'Brien et al. worked out sensitivities.[b]
  - Clark et al. further developed the sensitivities and applied them to improve nuclear cross sections.[c]

- I used the rod problem for verification.

- In this talk I present the rod problem and two interesting results:
  - Derivative with respect to χ.
  - Derivative with respect to the slab width.

ANS

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Computer Code Abstract

Check for updates

SENSMG: First-Order Sensitivities of Neutron Reaction Rates, Reaction-Rate Ratios, Leakage, k_{eff}, and α Using PARTISN

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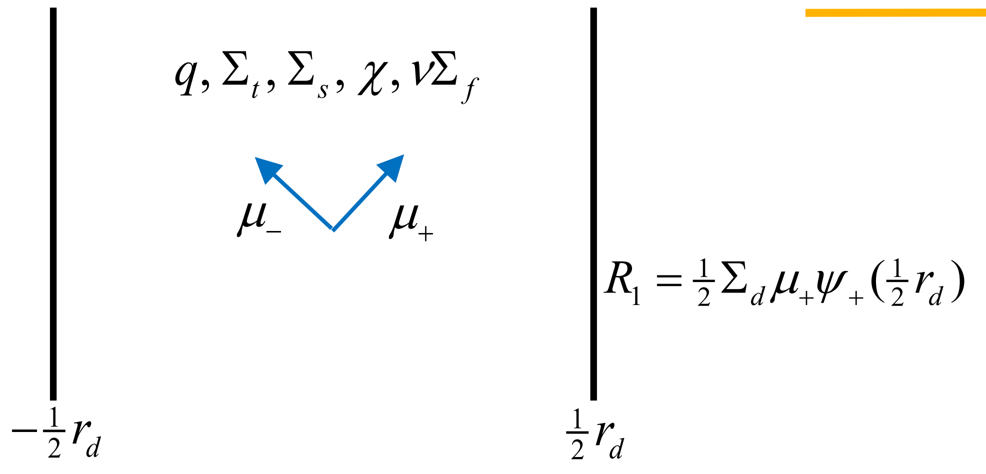
Abstract — *SENSMG is a tool for calculating the first-order sensitivities of reaction-rate ratios, k_{eff}, and α in critical problems and reaction-rate ratios, reaction rates, and leakage in fixed-source problems to multigroup cross*

If you are interested in difficulties regarding sensitivities w.r.t. χ, see my next talk!

[a] J. MATTINGLY, "Computation of Neutron Multiplicity Statistics Using Deterministic Transport," *IEEE Trans. Nucl. Sci.*, **59**, *2*, 314–322 (2012); https://doi.org/10.1109/TNS.2012.2185060.
[b] S. O'BRIEN, J. MATTINGLY, and D. ANISTRATOV, "Sensitivity Analysis of Neutron Multiplicity Counting Statistics Using First-Order Perturbation Theory and Application to a Subcritical Plutonium Metal Benchmark," *Nucl. Sci. Eng.*, **185**, *3*, 406–425 (2017); http://dx.doi.org/10.1080/00295639.2016.1272988.
[c] A. R. CLARK, J. MATTINGLY, and J. A. FAVORITE, "Application of Neutron Multiplicity Counting Experiments to Optimal Cross Section Adjustments," *Nuclear Science and Engineering*, submitted (2019).

An Analytic Transport Problem



- Homogeneous slab of width r_d.
- Constant isotropic neutron source rate density q.
- One neutron energy group.
  - The induced-fission spectrum in one group is 1, but we will carry it along in the equations.
- Scattering is isotropic.
- The quantity of interest R_1 is the leakage from the right side of the slab convolved with a response function.

- Two directions, right and left, with μ_+ the right-going direction cosine and μ_- the left-going.
  - Directions are constrained to satisfy $\mu_+ = -\mu_-$.
  - This problem is a regular S_2 discrete ordinates calculation.

- The equations for the forward right-going and left-going fluxes are

$$\mu_+ \frac{\partial \psi_+(r)}{\partial r} + \Sigma_t \psi_+(r) - \tfrac{1}{2}\Sigma_s(\psi_+(r) + \psi_-(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+(r) + \psi_-(r)) = q,$$

$$\mu_- \frac{\partial \psi_-(r)}{\partial r} + \Sigma_t \psi_-(r) - \tfrac{1}{2}\Sigma_s(\psi_+(r) + \psi_-(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+(r) + \psi_-(r)) = q,$$

with vacuum boundary conditions

$$\psi_+(-\tfrac{1}{2}r_d) = 0$$
$$\psi_-(\tfrac{1}{2}r_d) = 0.$$

The Adjoint Equations

- The equations for the adjoint right-going and left-going fluxes are

$$\mu_+ \frac{\partial \psi_+^*(r)}{\partial r} + \Sigma_t \psi_+^*(r) - \tfrac{1}{2}\Sigma_s(\psi_+^*(r) + \psi_-^*(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+^*(r) + \psi_-^*(r)) = 0,$$

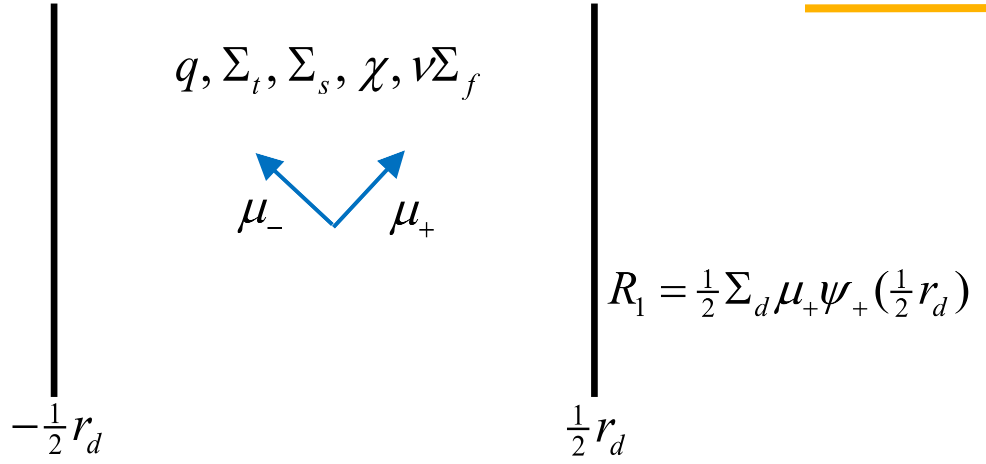
$$\mu_- \frac{\partial \psi_-^*(r)}{\partial r} + \Sigma_t \psi_-^*(r) - \tfrac{1}{2}\Sigma_s(\psi_+^*(r) + \psi_-^*(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+^*(r) + \psi_-^*(r)) = 0,$$

with a vacuum boundary condition on the left,

$$\psi_+(-\tfrac{1}{2}r_d) = 0,$$

and a source on the right,

$$\psi_-(\tfrac{1}{2}r_d) = \Sigma_d.$$



- No negative sign in front of the spatial derivative term because these are the *computational equations* (i.e. the equations that will actually be solved) obtained by replacing μ with $-\mu$ and recognizing that adjoint particles travel backwards.

- Thus, "left-going" and "right-going" here are in the computational sense, not the mathematical sense, in that right-going computational adjoint particles are really going to the right.

Solution of Coupled Partial Differential Equations
Step 1: Remove the Coupling

$$\mu_+ \frac{\partial \psi_+(r)}{\partial r} + \Sigma_t \psi_+(r) - \tfrac{1}{2}\Sigma_s(\psi_+(r)+\psi_-(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+(r)+\psi_-(r)) = q$$
$$\mu_- \frac{\partial \psi_-(r)}{\partial r} + \Sigma_t \psi_-(r) - \tfrac{1}{2}\Sigma_s(\psi_+(r)+\psi_-(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+(r)+\psi_-(r)) = q$$

- Take the derivative of the μ_+ equation with respect to r
- Write the μ_- equation as $\frac{\partial \psi_-(r)}{\partial r} = \ldots$
- Write the μ_+ equation as $\psi_-(r) = \ldots$
- Combine equations to yield

$$\frac{\partial^2 \psi_+(r)}{\partial r^2} - \frac{\Sigma_t}{\mu_+^2}\left(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f\right)\psi_+(r) = -\frac{\Sigma_t}{\mu_+^2} q$$

- Same procedure for $\psi_-(r)$, $\psi_+^*(r)$, and $\psi_-^*(r)$.

$$\frac{\partial^2 \psi_+^*(r)}{\partial r^2} - \frac{\Sigma_t}{\mu_+^2}\left(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f\right)\psi_+^*(r) = 0$$

Solution of Coupled Partial Differential Equations
Step 2: Solve Second-Order Ordinary Differential Equations

$$\frac{\partial^2 \psi_+(r)}{\partial r^2} - \frac{\Sigma_t}{\mu_+^2}\left(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f\right)\psi_+(r) = -\frac{\Sigma_t}{\mu_+^2} q$$

- The right-going forward flux is

$$\psi_+(r) = c_1 \cos(\lambda r) + c_2 \sin(\lambda r) + \psi_P$$

Particular solution

$$\psi_P = \frac{q}{\Sigma_t - \Sigma_s - \chi\nu\Sigma_f}$$

$$\lambda = \frac{1}{\mu_+}\sqrt{-\Sigma_t(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f)}$$

- The negative sign precedes the first Σ_t because, for this problem, the term in the parentheses is negative.

- The trigonometric solution accounts for the imaginary roots of the characteristic equation.

Solution of Coupled Partial Differential Equations
Step 3: Apply Boundary Conditions

- Evaluate $\mu_+ \dfrac{\partial \psi_+(r)}{\partial r} + \Sigma_t \psi_+(r) - \tfrac{1}{2}\Sigma_s(\psi_+(r)+\psi_-(r)) - \tfrac{1}{2}\chi\nu\Sigma_f(\psi_+(r)+\psi_-(r)) = q$

with

$\psi_+(r) = c_1\cos(\lambda r) + c_2\sin(\lambda r) + \psi_P$

at $r = \tfrac{1}{2}r_d$ (the right boundary), using $\psi_-(\tfrac{1}{2}r_d) = 0$.

- The result is

$$\begin{bmatrix} -\mu_+\lambda\sin(\tfrac{1}{2}\lambda r_d) + (\Sigma_t - \tfrac{1}{2}\Sigma_s - \tfrac{1}{2}\chi\nu\Sigma_f)\cos(\tfrac{1}{2}\lambda r_d) & \mu_+\lambda\cos(\tfrac{1}{2}\lambda r_d) + (\Sigma_t - \tfrac{1}{2}\Sigma_s - \tfrac{1}{2}\chi\nu\Sigma_f)\sin(\tfrac{1}{2}\lambda r_d) \\ \cos(\tfrac{1}{2}\lambda r_d) & -\sin(\tfrac{1}{2}\lambda r_d) \end{bmatrix}$$

$$\times \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -(\Sigma_t - \tfrac{1}{2}\Sigma_s - \tfrac{1}{2}\chi\nu\Sigma_f)\psi_P + q \\ -\psi_P \end{bmatrix}$$

- The solution is

$$c_1 = \frac{\psi_P}{D}\left(\Sigma_t\sin(\tfrac{1}{2}\lambda r_d) + \mu_+\lambda\cos(\tfrac{1}{2}\lambda r_d)\right)$$

$$c_2 = \frac{\psi_P}{D}\left[\mu_+\lambda\sin(\tfrac{1}{2}\lambda r_d) - (\Sigma_t - \Sigma_s - \chi\nu\Sigma_f)\cos(\tfrac{1}{2}\lambda r_d)\right]$$

$$D = -\mu_+\lambda\cos(\lambda r_d) - (\Sigma_t - \tfrac{1}{2}\Sigma_s - \tfrac{1}{2}\chi\nu\Sigma_f)\sin(\lambda r_d)$$

Solution

$$\psi_+(r) = c_1 \cos(\lambda r) + c_2 \sin(\lambda r) + \psi_P$$
$$\psi_-(r) = c_1 \cos(\lambda r) - c_2 \sin(\lambda r) + \psi_P$$
$$\psi_+^*(r) = c_3 \cos(\lambda r) + c_4 \sin(\lambda r)$$
$$\psi_-^*(r) = c_5 \cos(\lambda r) + c_6 \sin(\lambda r)$$

- The forward and adjoint scalar fluxes are

$$\begin{aligned}\phi(r) &= \tfrac{1}{2}\left(\psi_+(r) + \psi_-(r)\right) \\ &= c_1 \cos(\lambda r) + \psi_P\end{aligned}$$

$$\begin{aligned}\phi^*(r) &= \tfrac{1}{2}\left(\psi_+^*(r) + \psi_-^*(r)\right) \\ &= \tfrac{1}{2}\left((c_3 + c_5)\cos(\lambda r) + (c_4 + c_6)\sin(\lambda r)\right)\end{aligned}$$

- The detector response is

$$\begin{aligned}R_1 &= \tfrac{1}{2}\Sigma_d \mu_+ \psi_+(\tfrac{1}{2} r_d) \\ &= \tfrac{1}{2}\Sigma_d \mu_+ \left(c_1 \cos(\tfrac{1}{2}\lambda r_d) + c_2 \sin(\tfrac{1}{2}\lambda r_d) + \psi_P\right)\end{aligned}$$

The Second Moment

- The second moment R_2 of the count rate distribution is

$$R_2 = {}_2S + {}_2S_{s.f.},$$

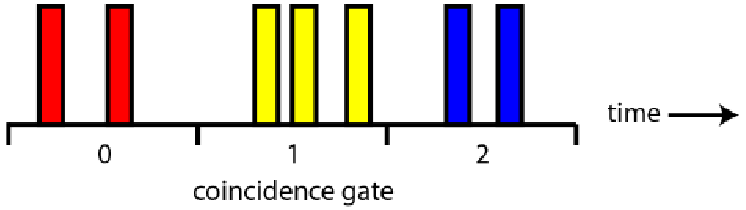
where

$${}_2S = \int_{-r_d/2}^{r_d/2} dr\,\overline{\nu(\nu-1)}\Sigma_f \left(\chi\phi^*(r)\right)^2 \phi(r) = \overline{\nu(\nu-1)}\Sigma_f \chi^2 \int_{-r_d/2}^{r_d/2} dr \left(\phi^*(r)\right)^2 \phi(r)$$

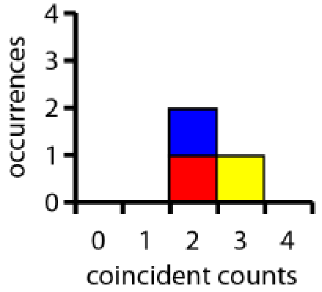
and

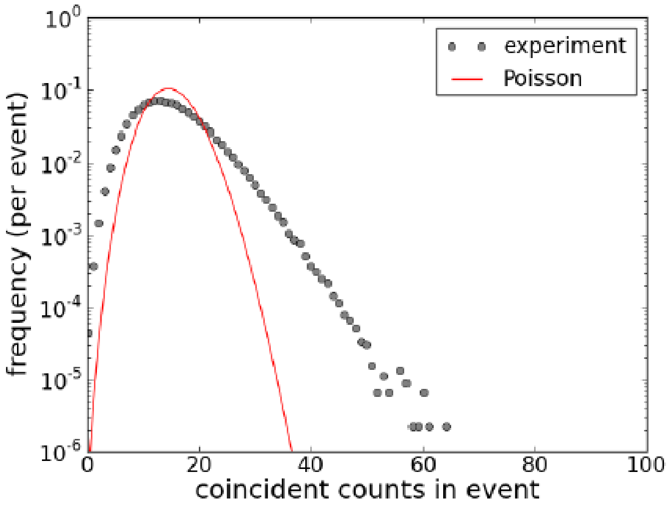
$${}_2S_{s.f.} = \int_{-r_d/2}^{r_d/2} dr \left(\frac{\overline{\nu(\nu-1)}}{\bar{\nu}}\right)_{s.f.} q\left(\chi_{s.f.}\phi^*(r)\right)^2 = \left(\frac{\overline{\nu(\nu-1)}}{\bar{\nu}}\right)_{s.f.} q\chi_{s.f.}^2 \int_{-r_d/2}^{r_d/2} dr \left(\phi^*(r)\right)^2$$

- χ is the material induced-fission spectrum and $\chi_{s.f.}$ is the material spontaneous-fission spectrum.



- R_2 is the second moment of this distribution





O'Brien et al., *Nucl. Sci. Eng.* **185** (2017).

The Second Moment (cont.)

- Using PARTISN, a vector χ, and the Nuclear Data Interface (NDI) at LANL, the induced-fission spectrum is defined for mixtures (in a one-group problem) as

$$\chi = \frac{\sum_{i=1}^{I} \chi_i \nu\sigma_{f,i} N_i f_i}{\sum_{i=1}^{I} \nu\sigma_{f,i} N_i f_i},$$

where f_i is the spectrum weighting function and I is the number of fissionable isotopes.

  + If the NDI is not used or if a matrix χ is used, $f_i = 1$.

- For the one-group problem, $\chi_{s.f.} = 1$.

- $\overline{\nu}$ and $\overline{\nu(\nu-1)}$ are the first and second factorial moments of the fission multiplicity distributions. These are isotopic nuclear data. The products $\overline{\nu(\nu-1)}\Sigma_f$ and $\left(\overline{\nu(\nu-1)}/\overline{\nu}\right)_{s.f.} q$ are defined for mixtures as

$$\overline{\nu(\nu-1)}\Sigma_f = \sum_{i=1}^{I} N_i \overline{\nu(\nu-1)}_i \sigma_{f,i}$$

and

$$\left(\frac{\overline{\nu(\nu-1)}}{\overline{\nu}}\right)_{s.f.} q = \sum_{i=1}^{I} N_i \left(\frac{\overline{\nu(\nu-1)}}{\overline{\nu}}\right)_{s.f.,i} q_i .$$

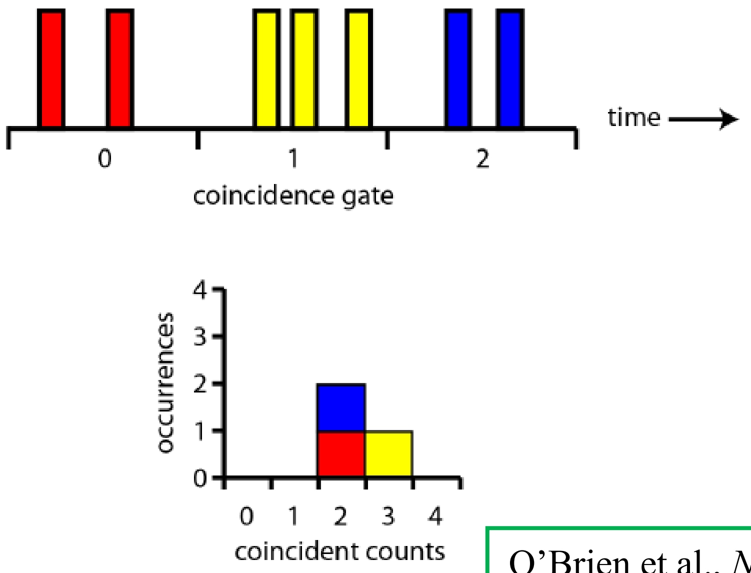
- Only isotopes with data for the moments will contribute to the material quantities $\overline{\nu(\nu-1)}\Sigma_f$ and $\left(\overline{\nu(\nu-1)}/\overline{\nu}\right)_{s.f.} q$.

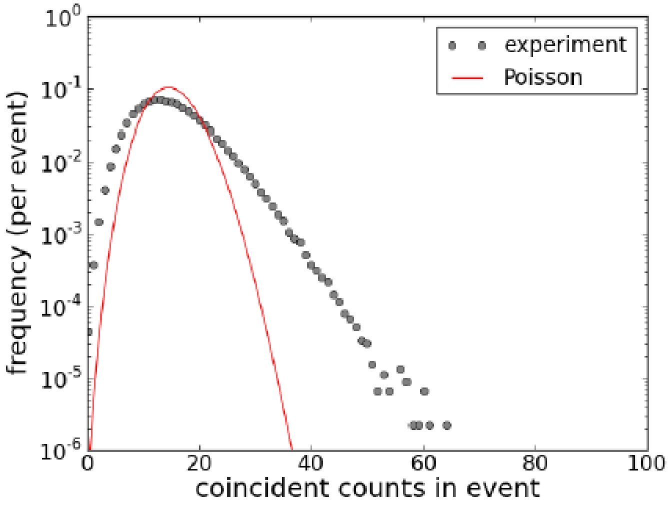
Feynman *Y*

- The Feynman *Y* asymptote is

$$Y = \frac{R_2}{R_1}.$$

- A measure of the variance in the neutron multiplicity counting distribution in excess of the variance in a Poisson distribution.





O'Brien et al., *Nucl. Sci. Eng.* **185** (2017).

- The width of the distribution in excess of the Poisson distribution is characteristic of multiplying material.

Flux Functionals

- The volume integral of the square of the adjoint scalar flux is

$$\int_{-r_d/2}^{r_d/2} dr\left(\phi^*(r)\right)^2 = \frac{1}{8\lambda}\left[(c_3+c_5)^2\left(\lambda r_d + \sin(\lambda r_d)\right) + (c_4+c_6)^2\left(\lambda r_d - \sin(\lambda r_d)\right)\right]$$

- The volume integral of the square of the adjoint scalar flux multiplied by the forward scalar flux is

$$\int_{-r_d/2}^{r_d/2} dr\left(\phi^*(r)\right)^2 \phi(r) = \frac{c_1}{6\lambda}\left[\tfrac{1}{4}(c_3+c_5)^2\left(9\sin(\tfrac{1}{2}\lambda r_d) + \sin(\tfrac{3}{2}\lambda r_d)\right) + (c_4+c_6)^2 \sin^3(\tfrac{1}{2}\lambda r_d)\right]$$

$$+\psi_P \int_{-r_d/2}^{r_d/2} dr\left(\phi^*(r)\right)^2$$

Derivatives with Respect to an Arbitrary Input Parameter (Material Property)

- $$\frac{\partial \psi_P}{\partial \alpha_x} = \frac{\psi_P}{q}\frac{\partial q}{\partial \alpha_x} - \frac{\psi_P}{\Sigma_t - \Sigma_s - \chi\nu\Sigma_f}\frac{\partial(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f)}{\partial \alpha_x}$$

- $$\frac{\partial \lambda}{\partial \alpha_x} = \frac{\lambda}{2}\left(\frac{1}{\Sigma_t}\frac{\partial \Sigma_t}{\partial \alpha_x} + \frac{1}{(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f)}\frac{\partial(\Sigma_t - \Sigma_s - \chi\nu\Sigma_f)}{\partial \alpha_x}\right), \quad \frac{\partial}{\partial \alpha_x}\left(\frac{1}{\lambda}\right) = -\frac{1}{\lambda^2}\frac{\partial \lambda}{\partial \alpha_x}$$

- $$\frac{\partial c_1}{\partial \alpha_x} = \left(\frac{1}{\psi_P}\frac{\partial \psi_P}{\partial \alpha_x} - \frac{1}{D}\frac{\partial D}{\partial \alpha_x}\right)c_1 + \frac{\psi_P}{D_{(12)}}\frac{\partial \Sigma_t}{\partial \alpha_x}\sin(\tfrac{1}{2}\lambda r_d) + \frac{\psi_P}{D}\left[\left(\frac{\Sigma_t r_d}{2} + \mu_+\right)\cos(\tfrac{1}{2}\lambda r_d) - \frac{\mu_+ \lambda r_d}{2}\sin(\tfrac{1}{2}\lambda r_d)\right]\frac{\partial \lambda}{\partial \alpha_x}$$

- etc.

- $$\frac{\partial R_1}{\partial \alpha_x} = \tfrac{1}{2}\Sigma_d \mu_+\left[\frac{\partial c_1}{\partial \alpha_x}\cos(\tfrac{1}{2}\lambda r_d) + \frac{\partial c_2}{\partial \alpha_x}\sin(\tfrac{1}{2}\lambda r_d) + \frac{\partial \psi_P}{\partial \alpha_x} + \frac{r_d}{2}\left(c_2\cos(\tfrac{1}{2}\lambda r_d) - c_1\sin(\tfrac{1}{2}\lambda r_d)\right)\frac{\partial \lambda}{\partial \alpha_x}\right]$$

- $$\frac{\partial Y}{\partial \alpha_x} = \frac{1}{R_1}\left(\frac{\partial_2 S}{\partial \alpha_x} + \frac{\partial_2 S_{s.f.}}{\partial \alpha_x} - Y\frac{\partial R_1}{\partial \alpha_x}\right)$$

Derivatives with Respect to the Slab Width

- $$\frac{\partial \psi_P}{\partial r_d} = 0\,, \quad \frac{\partial \lambda}{\partial r_d} = 0$$
- $$\frac{\partial c_1}{\partial r_d} = -\frac{c_1}{D}\frac{\partial D}{\partial r_d} + \frac{\psi_P \lambda}{2D}\left(\Sigma_t \cos(\tfrac{1}{2}\lambda r_d) - \mu_+ \lambda \sin(\tfrac{1}{2}\lambda r_d)\right)$$
- etc.
- $$\frac{\partial R_1}{\partial r_d} = \tfrac{1}{2}\Sigma_d \mu_+ \left[\frac{\partial c_1}{\partial r_d}\cos(\tfrac{1}{2}\lambda r_d) + \frac{\partial c_2}{\partial r_d}\sin(\tfrac{1}{2}\lambda r_d) + \frac{\lambda}{2}\left(c_2 \cos(\tfrac{1}{2}\lambda r_d) - c_1 \sin(\tfrac{1}{2}\lambda r_d)\right)\right]$$
- $$\frac{\partial Y}{\partial r_d} = \frac{1}{R_1}\left(\frac{\partial_2 S}{\partial r_d} + \frac{\partial_2 S_{s.f.}}{\partial r_d} - Y\frac{\partial R_1}{\partial r_d}\right)$$

Test Problem

- Material is plutonium with density 14 g/cm^3

Nuclide	Density [atoms/(b·cm)]	Weight Fraction
Pu-239	0.03385770516	0.96
Pu-240	0.001404851530	0.04

- Slab width = 4 cm
- Neutron source rates:
  - Total neutron source rate density is q = 585.3096779 neutrons/cm^3·s

Nuclide	Neutrons/s/(10^{24} atoms)
Pu-239	5.90346862E+00
Pu-240	4.16492268E+05

- 618-group MENDF71X collapsed to 1 energy group
- PARTISN (discrete-ordinates) parameters: 0.0005-cm mesh; P_0 scattering expansion
- First and second factorial moments of the multiplicity:

Event	$\overline{\nu}$	$\overline{\nu(\nu-1)}$
Thermal fission of ^{239}Pu	2.8794	6.7728
Spontaneous fission of ^{240}Pu	2.1563	3.8242

J. W. Boldeman and M. G. Hines, "Prompt Neutron Emission Probabilities Following Spontaneous and Thermal Neutron Fission," *Nucl. Sci. Eng.*, **91**, 114–116 (1985).

- Regular S_2 ordinates $\mu_{\pm} = \pm 1/\sqrt{3}$
- Response function $\Sigma_d = 0.009875877948$

Responses

Response	Analytic	SENSMG	Difference
R_1	1.57256464E+02	1.572564E+02	–0.00001%
R_2	7.54409818E+02	7.544096E+02	–0.00003%
Y	4.79732153E+00	4.797320E+00	–0.00002%

Sensitivity with Respect to the Pu-240 Induced-Fission Spectrum

- PARTISN constructs the material induced-fission spectrum χ using $\chi = \dfrac{\chi_{\mathrm{Pu}239}\nu\sigma_{f,\mathrm{Pu}239}N_{\mathrm{Pu}239}f_{\mathrm{Pu}239} + \chi_{\mathrm{Pu}240}\nu\sigma_{f,\mathrm{Pu}240}N_{\mathrm{Pu}240}f_{\mathrm{Pu}240}}{\nu\sigma_{f,\mathrm{Pu}239}N_{\mathrm{Pu}239}f_{\mathrm{Pu}239} + \nu\sigma_{f,\mathrm{Pu}240}N_{\mathrm{Pu}240}f_{\mathrm{Pu}240}}$.

- The unnormalized derivative of χ with respect to the Pu-240 fission spectrum is $\dfrac{\partial\chi}{\partial\chi_{\mathrm{Pu}240}} = \dfrac{\nu\sigma_{f,\mathrm{Pu}240}N_{\mathrm{Pu}240}f_{\mathrm{Pu}240}}{\nu\sigma_{f,\mathrm{Pu}239}N_{\mathrm{Pu}239}f_{\mathrm{Pu}239} + \nu\sigma_{f,\mathrm{Pu}240}N_{\mathrm{Pu}240}f_{\mathrm{Pu}240}}$.

- $$\frac{\partial_2 S_{s.f.}}{\partial\chi_{\mathrm{Pu240}}} = \left(\frac{\overline{\nu(\nu-1)}}{\overline{\nu}}\right)_{s.f.} q\chi_{s.f.}^2 \frac{\partial}{\partial\chi_{\mathrm{Pu240}}} \int_{-r_d/2}^{r_d/2} dr\left(\phi^*(r)\right)^2$$

- $$\frac{\partial_2 S}{\partial\chi_{\mathrm{Pu240}}} = \overline{\nu(\nu-1)}\Sigma_f\left(2\chi\frac{\partial\chi}{\partial\chi_{\mathrm{Pu240}}}\int_{-r_d/2}^{r_d/2} dr\left(\phi^*(r)\right)^2\phi(r) + \chi^2\frac{\partial}{\partial\chi_{\mathrm{Pu240}}}\int_{-r_d/2}^{r_d/2} dr\left(\phi^*(r)\right)^2\phi(r)\right)$$

Sensitivity[(a)]	Analytic	SENSMG	Difference
$S_{R_1,\chi_{\mathrm{Pu}240}}$	1.879239E-01	1.879239E-01	0.00000%
$S_{R_2,\chi_{\mathrm{Pu}240}}$	5.736636E-01	5.736636E-01	–0.00001%
$S_{Y,\chi_{\mathrm{Pu}240}}$	3.857397E-01	3.857397E-01	–0.00001%

(a) Unconstrained.

Derivative with Respect to the Slab Width

- An equation for the adjoint-based derivative of the Feynman Y to interface locations and the outer boundary has yet to be derived formally.

- SENSMG uses a straightforward extension of the equation for the derivative of the mean count rate R_1,[d,e]

$$\frac{\partial R_1}{\partial r_n} = \int_{S_n} dS \int_{4\pi} d\hat{\mathbf{\Omega}} \sum_{g=1}^{G} \left\{ \psi^{*g}(r,\hat{\mathbf{\Omega}}) \Delta Q_n + \psi^{*g}(r,\hat{\mathbf{\Omega}}) \left(\Delta F_n - \Delta A_n \right) \psi^{g}(r,\hat{\mathbf{\Omega}}) \right\},$$

where the Δ terms are differences across surface S_n.

Sensitivity	Analytic	SENSMG	Difference
$\partial R_1 / \partial r_d$	7.688378E+02	7.688377E+02	–0.00002%
$\partial R_2 / \partial r_d$	1.091584E+04	1.091583E+04	–0.00008%
$\partial Y / \partial r_d$	4.595981E+01	4.595979E+01	–0.00004%

[d] K. C. BLEDSOE, J. A. FAVORITE, and T. ALDEMIR, "Using the Levenberg-Marquardt Method for Solutions of Inverse Transport Problems in One- and Two-Dimensional Geometries," *Nuclear Technology*, **176**, *1*, 106–126 (2011); https://doi.org/10.13182/NT176-106.
[e] J. A. FAVORITE and E. GONZALEZ, "Revisiting Boundary Perturbation Theory for Inhomogeneous Transport Problems," *Nucl. Sci. Eng.*, **185**, *3*, 445–459 (2017); https://doi.org/10.1080/00295639.2016.1277108.

Summary and Conclusions

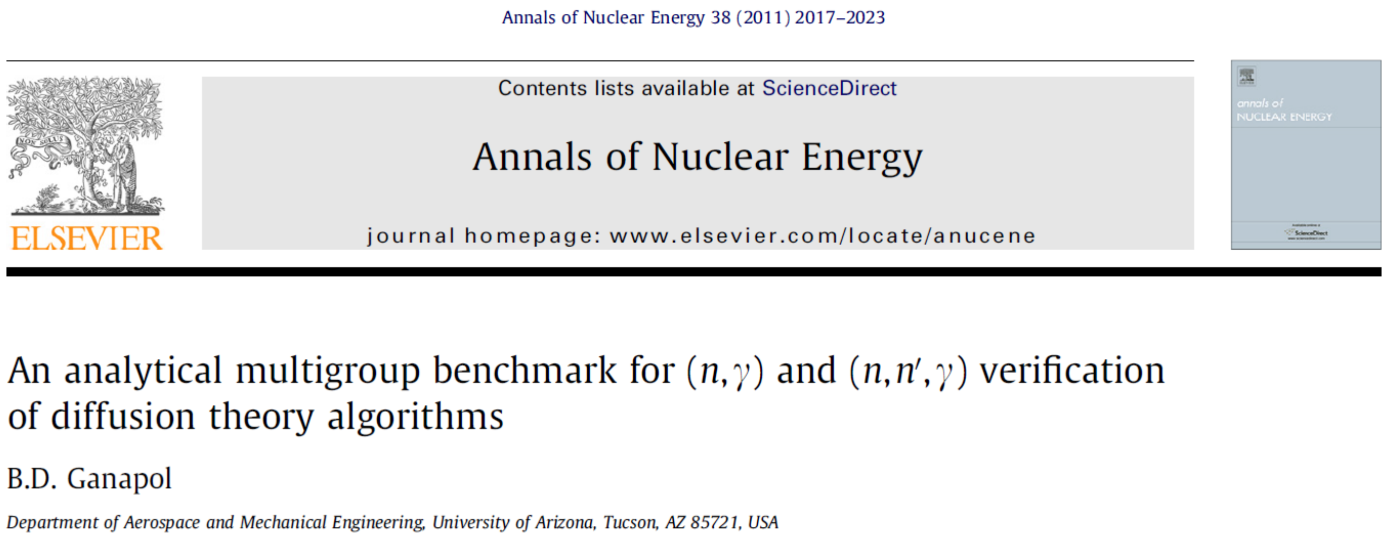
- The S_2 slab or "rod" problem has been applied to verify the adjoint-based derivatives of R_1 and R_2, the first and second moments of the neutron multiplicity counting distribution

- Keep this analytic problem in mind!

- Ganapol has published the solution of the rod problem in arbitrary groups ($G > 1$).[f]

  + I wanted to use Ganapol's solution, but I couldn't figure out how to take analytic derivatives.

  + An exercise for a student….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An analytical multigroup benchmark for (n,γ) and (n,n',γ) verification of diffusion theory algorithms

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- The rod problem helped us figure out what to do about adjoint-based derivatives with respect to χ.

- The rod problem verified our derivatives with respect to outer boundary.

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[f] B. D. GANAPOL, "An Analytical Multigroup Benchmark for (n,γ) and (n,n′,γ) Verification of Diffusion Theory Algorithms," *Ann. Nucl. Eng.*, **38**, 2017–2023 (2011); https://doi.org/10.1016/j.anucene.2011.04.013.