Derivation of 2-D Thermoelastic Equations of Motion for a Finite Cylinder

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Outline

- Previous work by Wimett
- Current Work
- Results and Graphs
- Future Work

Why Do We Care?

- Thermoelastic equations can give accurate results when modeling material movement from forces due to temperature changes
  - Stresses, strains, displacements can be found
  - Applicable to criticality accident analysis
- Difficulties arise from modeling complicated geometries
  - “Easiest” is 1-D spherical geometry
  - “Second easiest” is simple 2-D cylindrical geometry
    - Godiva IV, SPR II can provide good experimental data to test against

Previous Work

- Wimett (1992) modeled the radial direction only, using approximations for the axial direction
  - Disk Approximation
  - Cylinder Approximation
  - Analytical solutions were found and compared to SPR II data
- Myers, et all (1995) developed original MRKJ, using a coupled neutronic-hydrodynamic method
- MRKJ was modified to use coupled neutronic-thermoelastic method
  - Spherical geometry only

Current Work

- Goal: Apply the neutronic-thermoelastic method to a 2-D cylindrical system
  - Derive the thermoelastic equations in 2-D cylindrical geometry
  - Test equations for simple problems
  - Implement new model into neutronic-thermoelastic code
  - Compare results to experimental data
- First two have been completed, currently working on part 3

Derivation

- Start from three sets of equations:
  - Stress-Strain Relations
  - Equations of Motion in Cylindrical Geometry
  - Strain-Displacement Relations in Cylindrical Geometry
- Concerned with r,z directions, so set derivatives with respect to theta equal to zero and theta displacements equal to zero
- Combine equations for final equation

Derivation: Stress-Strain Relations

$$\varepsilon_{rr} = \frac{1}{E}\left[\sigma_{rr} - \nu\left(\sigma_{\theta\theta} + \sigma_{zz}\right)\right] + \alpha\Delta T$$

$$\varepsilon_{\theta\theta} = \frac{1}{E}\left[\sigma_{\theta\theta} - \nu\left(\sigma_{rr} + \sigma_{zz}\right)\right] + \alpha\Delta T$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{rr} + \sigma_{\theta\theta}\right)\right] + \alpha\Delta T$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{z\theta} = 0$$

$$\varepsilon_{zr} = \frac{1}{2G}\sigma_{zr}$$

Derivation: Equations of Motion

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho \frac{\partial^2 w}{\partial t^2}$$

Derivation: Strain-Displacement Relations

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}$$

$$\varepsilon_{\theta\theta} = \frac{u}{r}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\varepsilon_{r\theta} = 0$$

$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)$$

$$\varepsilon_{z\theta} = 0$$

Derivation: Final Equations

$$\rho\frac{\partial^2 u}{\partial t^2}=\frac{E(\nu-1)}{(2\nu-1)(\nu+1)}\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]+\frac{E}{2(1+\nu)}\frac{\partial^2 u}{\partial z^2}-\frac{E}{2(2\nu-1)(\nu+1)}\frac{\partial^2 w}{\partial z\partial r}+\frac{E}{2\nu-1}\alpha\frac{\partial \Delta T}{\partial r}$$

$$\rho\frac{\partial^2 w}{\partial t^2}=\frac{E(\nu-1)}{(2\nu-1)(\nu+1)}\frac{\partial^2 w}{\partial z^2}+\frac{E}{2(1+\nu)}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right]-\frac{E}{2(2\nu-1)(\nu+1)}\left[\frac{\partial^2 u}{\partial z\partial r}+\frac{1}{r}\frac{\partial u}{\partial z}\right]+\frac{E}{2\nu-1}\alpha\frac{\partial \Delta T}{\partial z}$$

Derivation: Boundary and Initial Conditions

- Left Boundary Conditions:

$$u(r=0)=0$$
$$w(z=0)=0$$
$$\frac{\partial u(z=0)}{\partial z}=0$$
$$\frac{\partial w(r=0)}{\partial r}=0$$

$$\sigma_{rr}(r=R_i)=0$$
$$w(z=0)=0$$
$$\frac{\partial u(z=0)}{\partial z}=0$$
$$\sigma_{rz}(r=R_i)=0$$

- Outer Boundary Conditions (zero stress condition):

$$\sigma_{rr}(r=R)=0$$
$$\sigma_{zz}(z=H)=0$$
$$\sigma_{rz}(r=R)=0$$
$$\sigma_{rz}(z=H)=0$$

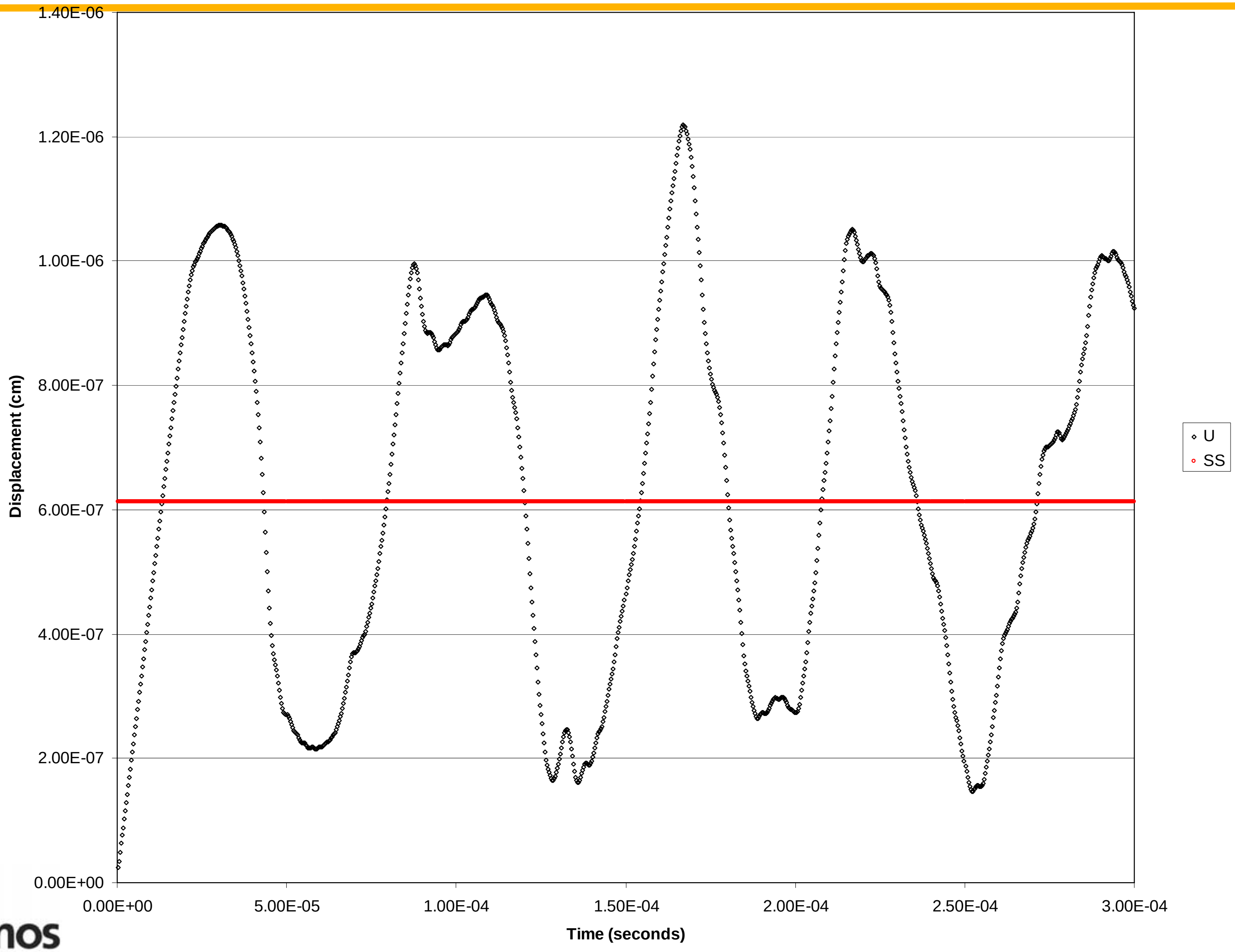
- Initial Conditions (at rest):

$$u(t=0)=0 \qquad \frac{\partial u(t=0)}{\partial t}=0$$
$$w(t=0)=0 \qquad \frac{\partial w(t=0)}{\partial t}=0$$

Results

- Small cylinder of Uranium
  - Diameter: 6”
  - Height: 6”
  - Constant Temperature Difference: 1 degree Celsius
- Boundary Conditions
  - Outer: Zero outer stresses
  - Inner: Zero displacements at origin
- Finite Difference Algorithm
  - Explicit Scheme
- Output
  - Displacements
  - Stresses, Strains

Displacement



Future Work

- Use this model in cylindrical MRKJ
  - Godiva IV, SPR
- Complications
  - No longer can use 1-D Partisn code without additional adjustments
  - Godiva IV has inner and outer core