PARANAL: An Efficient Tool for Parametric Analysis of Criticality Safety

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Overview

- Introduction
- Features of PARANAL
- Methodology
- Examples
- Summary and Conclusions
Introduction

- Effective multiplication factor ($k_{\text{eff}}$) of a fissionable material system is a function of a large number of system variables $\mathbf{x} = [x_1, x_2, \ldots, x_n]$

$$k_{\text{eff}} = k(\mathbf{x})$$

where $x_i =$ mass, density, enrichment, moderation, geometry, reflection, etc.

- For nuclear criticality safety controls, a domain, $\mathbf{x}_s$, must be found such that

$$k(\mathbf{x}) \leq \text{USL (upper subcriticality limit)}$$

for all $\mathbf{x} \in \mathbf{x}_s$

- A safe control limit on variable $x_i$ can be expressed as

$$x_{\text{lim}} = \max(x_i') \text{ or } \min(x_i')$$

for $\mathbf{x}' \in$ domain ($\mathbf{x}_s$) boundary
Introduction (con’t)

• Parametric analysis is an excellent way to find the safe limit on a control variable (parameter). However, to find a solution that satisfies above requirements, analysts are faced with

  — tedious and time-consuming task of running multiple simulations
  — accurate identification of safe domain and safe limits

• Few tools are available that can automatically solve for entire ranges of specified variables and identify accurate limits. Most existing methods are based on a single-parameter regression fitting of $k_{\text{eff}}$, which may result in less accuracy due to limited and discrete $k_{\text{eff}}$ data and sometimes human errors.

• In order to achieve more accurate safety limits at a significantly low cost of analysts’ time, PARANAL has been developed with

  — numerical interpolation over entire ranges of specified variables
  — automation and visualization
Features of PARANAL

• PARANAL — Parametric Analyzer for Criticality Safety
  - Creating continuous $k_{\text{eff}}$ functions that interpolate discrete $k_{\text{eff}}$ data obtained from parametric simulations of a fissionable system
  - Determining safe $k_{\text{eff}}$ domains for specified parameters for a given USL
  - Searching the safety limit of a control parameter over the entire specified domain
  - Generating graphical and numerical results

• Numerical Interpolation Methods

  Lower order 2-D polynomial interpolation to fit the $k_{\text{eff}}$ function in pieces, include:
  - Bilinear
  - Bicubic
  - Bicubic Spline

• A Matlab-Based Tool
## Methodology

### PARANAL Interpolation Methods

<table>
<thead>
<tr>
<th>Features</th>
<th>Interpolation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bilinear</td>
</tr>
<tr>
<td>Closest neighborhood points</td>
<td>2×2 (4 points)</td>
</tr>
<tr>
<td>Interpolating Function</td>
<td>Piecewise Bilinear</td>
</tr>
<tr>
<td>Continuity</td>
<td>Function</td>
</tr>
<tr>
<td>Smoothness</td>
<td>Low</td>
</tr>
<tr>
<td>Accuracy</td>
<td>Low</td>
</tr>
<tr>
<td>Efficiency</td>
<td>High</td>
</tr>
</tbody>
</table>
Methodology (con’t)

2-D Interpolation Example

Bilinear

Bicubic Spline

Error

Error
Methodology (con’t)

• **k\text{eff} Contour Plot**
  – A graphical technique for representing a 3-D k\text{eff} function by plotting constant k\text{eff} slices on a 2-D parameter (x, y) format
  – The k\text{eff} contour plot is formed by:
    
    - Horizontal axis: parameter x
    - Vertical axis: parameter y
    - Lines: iso-k\text{eff} values

• **Safe Parameter Domain**
  – Safe parameter domain is a k\text{eff} contour region where k\text{eff}(x,y) < k_{\text{lim}} (safe limit of k\text{eff}).

• **Safe Parameter Limit**
  – The safe limit of a parameter x, y is given by
    
    $$x_{\text{lim}} \text{ or } y_{\text{lim}} = \text{MIN (x or y) or MAX (x or y)}$$
    
    under the condition of $k_{\text{eff}}(x,y) = k_{\text{lim}}$
Results – Example 1

Subcritical Mass Limit for 10 wt% Enriched Homogeneous UO₂-H₂O Mixture

- UO₂ Density: 10.96 g/cm²
- Geometry: Spherical
- Reflector: 1-foot thick H₂O
- Parameters (range):
  - UO₂ mass (8-16 kgs)
  - H₂O content (50-80 wt%)
- Subcritical $k_{eff}$ limit: 0.97
### Results – Example 1 (con’t)

#### Table 1. $k_{\text{eff}}^*$ Results of Spherical UO$_2$-H$_2$O System

<table>
<thead>
<tr>
<th>$k_{\text{eff}}^*$ (H$_2$O Moderation, wt%)</th>
<th>UO$_2$ Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>8</td>
</tr>
<tr>
<td>55</td>
<td>0.8687</td>
</tr>
<tr>
<td>60</td>
<td>0.8863</td>
</tr>
<tr>
<td>65</td>
<td>0.9002</td>
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<tr>
<td>70</td>
<td>0.9028</td>
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<tr>
<td>75</td>
<td>0.8987</td>
</tr>
<tr>
<td>80</td>
<td>0.8441</td>
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</table>

Note: $k_{\text{eff}}^* = k_{\text{eff}} + 3\sigma - \text{bias}$ ($\sigma$ = calculational standard deviation in $k_{\text{eff}}$)
Results - Example 1 (con’t)

Traditional 1-D Interpolation

[Graphs showing keff+3σ-bias vs. H2O (wt%) and keff+3σ-bias vs. UO2 Mass (kg)].
Results - Example 1 (con’t)

PARANAL 2-D Interpolation

Minimum safe UO₂ mass limit = **11.37 kgs** at optimal H₂O moderation = **64.85 wt%**

$k_{\text{eff}}$ verification: **0.9705±0.0011**
Results – Example 2

Subcritical Spacing Limit for an Infinite Array of Infinite-long Tanks Containing 8 wt% Enriched UO$_2$F$_2$ Solution

- UO$_2$F$_2$ Density: 6.37 g/cm$^2$
- Geometry: 8” in diameter
  Triangular pitch
- Reflector (bottom): 24” thick Concrete
- Interspersed H$_2$O: 0.00001 g/cm$^2$
- Parameters (range):
  Center-to-center spacing (100-300 cm)
  H$_2$O content (10-60 wt%)
- Subcritical k$_{\text{eff}}$ limit: 0.97
Results – Example 2 (con’t)

### Table 2. $k_{\text{eff}}^*$ Results of UO$_2$F$_2$ Tank Array

<table>
<thead>
<tr>
<th>H$_2$O Moderation (wt%)</th>
<th>Center-to-Center Spacing (cm)</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>10</td>
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<td>1.1931</td>
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<td>30</td>
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<tr>
<td>40</td>
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<td>1.3082</td>
<td>1.1439</td>
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<td>50</td>
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<tr>
<td>60</td>
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<td>1.1752</td>
<td>1.0238</td>
<td>0.9092</td>
<td>0.8305</td>
<td>0.7752</td>
</tr>
</tbody>
</table>
Results - Example 2 (con’t)

Traditional 1-D Interpolation

[Graphs showing keff+3σ-bias vs. H2O (wt%) and keff+3σ-bias vs. Center-to-Center Spacing (cm)]
Minimum safe Spacing = \textbf{229.2 cm} at optimal H$_2$O moderation = \textbf{32.22 wt\%}

$\kappa_{\text{eff}}$ verification: $\textbf{0.9690 \pm 0.0011}$
Summary and Conclusions

• PARANAL provides an exceptionally efficient tool for parametric studies of criticality safety analyses.

• PARANAL allows accurate determination of the safe domain and limits of criticality parameters with two-dimensional interpolation techniques.

• PARANAL can be extended to multiple (>2) parametric analyses using N-dimensional interpolation techniques, but the visualization of an N-dimensional $k_{eff}$ function and safe parameter domain will be difficult or impossible.

• Error estimates for interpolation may be taken into account in determining safe parameter limits, especially when extrapolation is needed.