GE Hitachi
Nuclear Energy

PARANAL: An Efficient Tool for Parametric Analysis of Criticality Safety

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Data, Analysis, and Operations for Nuclear Criticality Safety

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Overview

- Introduction
- Features of PARANAL
- Methodology
- Examples
- Summary and Conclusions

Introduction

- Effective multiplication factor (k_{eff}) of a fissionable material system is a function of a large number of system variables $\mathbf{x}=[x_1, x_2, \ldots, x_n]$

$$\mathbf{k_{eff} = k(x)}$$

where x_i = mass, density, enrichment, moderation, geometry, reflection, etc.

- For nuclear criticality safety controls, a domain, $\mathbf{x_s}$, must be found such that

$$\mathbf{k(x) \leq USL\ (upper\ subcriticality\ limit)}$$

for all $\mathbf{x \in x_s}$

- A safe control limit on variable x_i can be expressed as

$$\mathbf{x_{lim} = max(}x_i'\mathbf{)\ or\ min(}x_i'\mathbf{)}$$

for $\mathbf{x' \in}$ domain ($\mathbf{x_s}$) boundary

Introduction (con't)

- Parametric analysis is an excellent way to find the safe limit on a control variable (parameter). However, to find a solution that satisfies above requirements, analysts are faced with
  - — *tedious and time-consuming task of running multiple simulations*
  - — *accurate identification of safe domain and safe limits*
- Few tools are available that can automatically solve for entire ranges of specified variables and identify accurate limits. Most existing methods are based on a single-parameter regression fitting of k_{eff}, which may result in less accuracy due to limited and discrete k_{eff} data and sometimes human errors.
- In order to achieve more accurate safety limits at a significantly low cost of analysts' time, PARANAL has been developed with
  - — *numerical interpolation over entire ranges of specified variables*
  - — *automation and visualization*

Features of PARANAL

- **PARANAL – Parametric Analyzer for Criticality Safety**
  - Creating continuous k_{eff} functions that interpolate discrete k_{eff} data obtained from parametric simulations of a fissionable system
  - Determining safe k_{eff} domains for specified parameters for a given USL
  - Searching the safety limit of a control parameter over the entire specified domain
  - Generating graphical and numerical results
- **Numerical Interpolation Methods**

  Lower order 2-D polynomial interpolation to fit the k_{eff} function in pieces, include:
  - Bilinear
  - Bicubic
  - Bicubic Spline
- **A Matlab-Based Tool**

Methodology

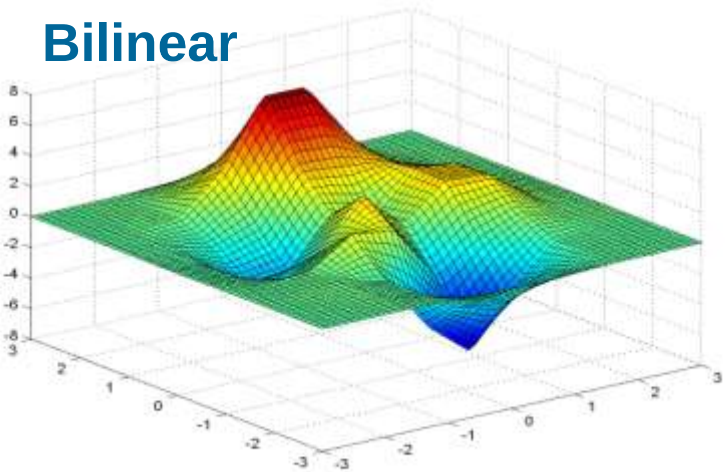
PARANAL Interpolation Methods

Features	Interpolation Method		
	Bilinear	Bicubic	Bicubic Spline
Closest neighborhood points	2x2 (4 points)	4x4 (16 points)	4x4 (16 points)
Interpolating Function	Piecewise Bilinear	Piecewise Bicubic Hermit Polynomial	Bicubic Spline
Continuity	Function	Function 1st Derivative	Function 1st Derivative 2nd Derivative
Smoothness	Low	Higher	Highest
Accuracy	Low	Higher	Highest
Efficiency	High	Low	Low

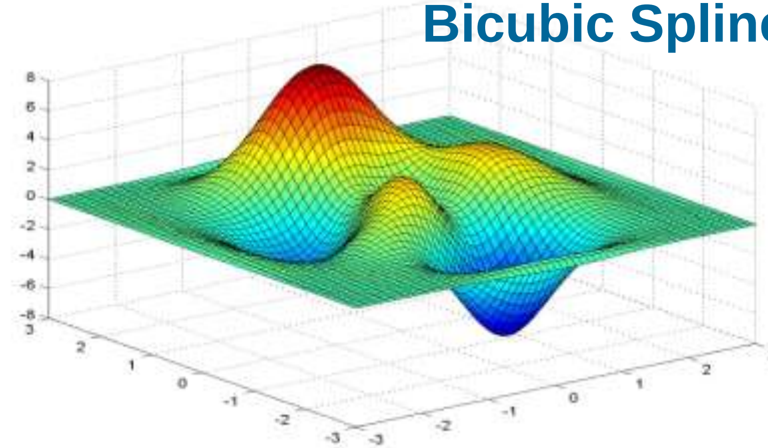
Methodology (con’t)

2-D Interpolation Example

Bilinear



Bicubic Spline



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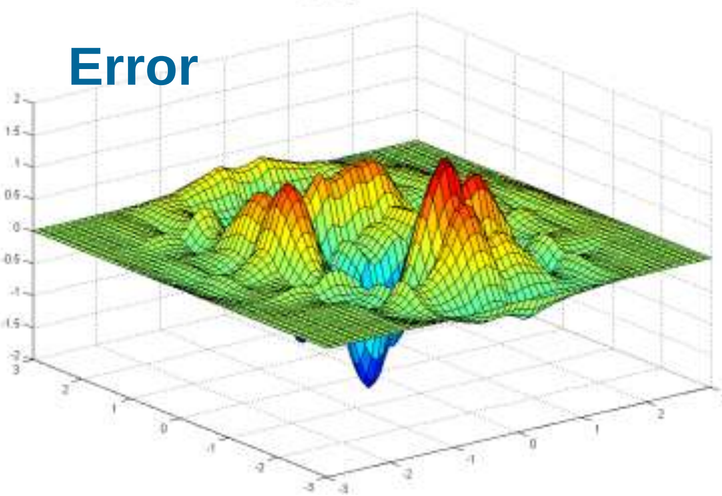

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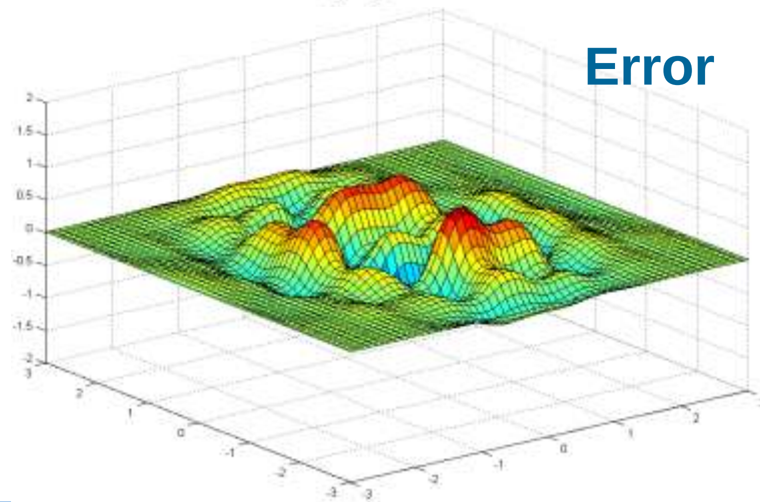
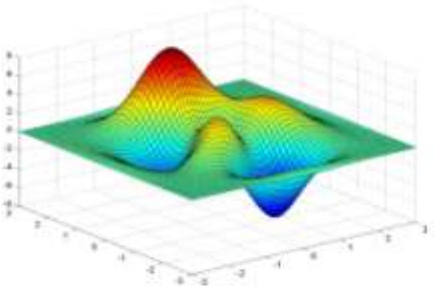
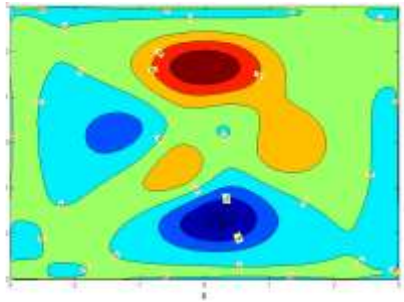


Methodology (con't)

- **k_{eff} Contour Plot**
  - A graphical technique for representing a 3-D k_{eff} function by plotting constant k_{eff} slices on a 2-D parameter (x, y) format
  - The k_{eff} contour plot is formed by:

Horizontal axis:	parameter x
Vertical axis:	parameter y
Lines:	iso-k_{eff} values





- **Safe Parameter Domain**
  - Safe parameter domain is a k_{eff} contour region where k_{eff} (x,y) < k_{lim} (safe limit of k_{eff}).

- **Safe Parameter Limit**

– The safe limit of a parameter x, y is given by

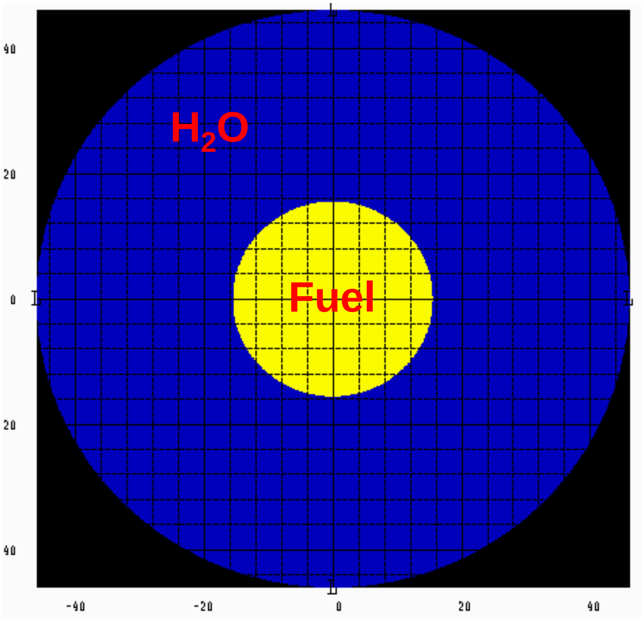
$$x_{lim} \text{ or } y_{lim} = \text{MIN (x or y) or MAX (x or y)}$$

under the condition of k_{eff} (x,y) = k_{lim}

Results – Example 1

Subcritical Mass Limit for 10 wt% Enriched Homogeneous UO_2-H_2O Mixture

- UO_2 Density: 10.96 g/cm^2
- Geometry: Spherical
- Reflector: 1-foot thick H_2O
- Parameters (range):
  - UO_2 mass (8-16 kgs)
  - H_2O content (50-80 wt%)
- Subcritical k_{eff} limit: 0.97



Results – Example 1 (con't)

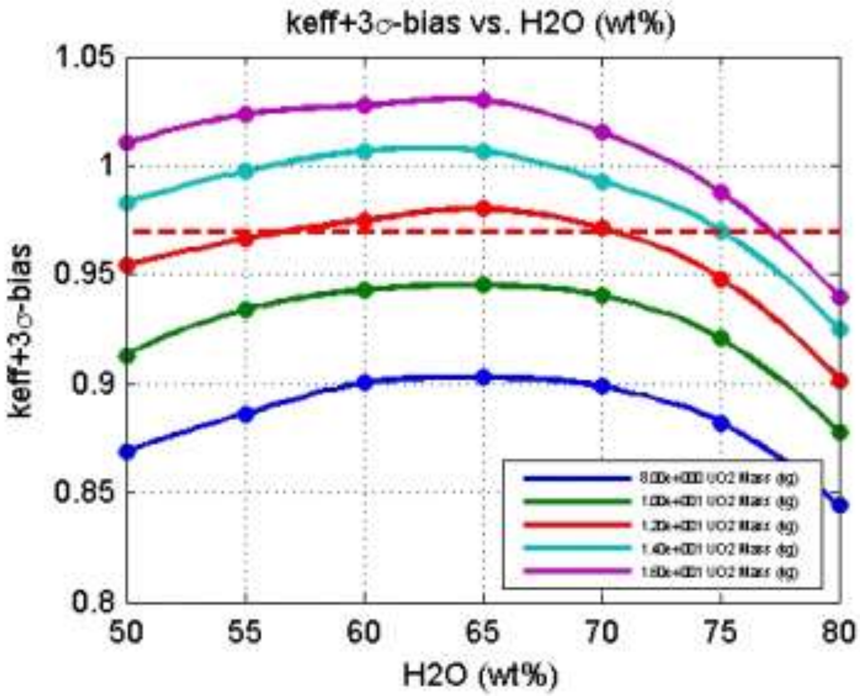
Table 1. k_{eff}* Results of Spherical UO_2-H_2O System

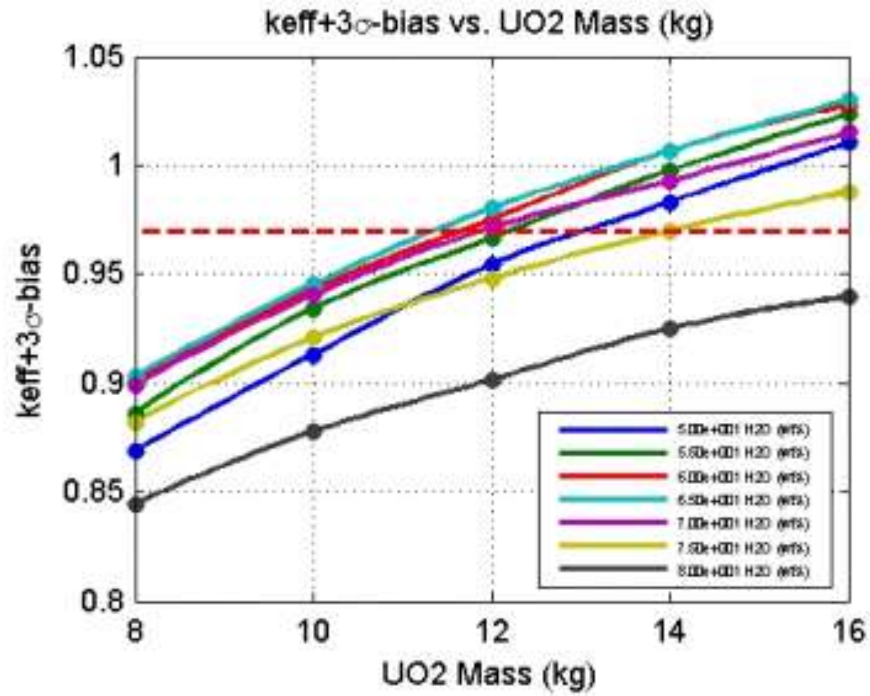
k_{eff}*		UO_2 Mass (kg)				
		8	10	12	14	16
H_2O Moderation (wt%)	50	0.8687	0.9129	0.9541	0.9830	1.0101
	55	0.8863	0.9339	0.9664	0.9973	1.0233
	60	0.9002	0.9428	0.9748	1.0066	1.0274
	65	0.9028	0.9451	0.9801	1.0066	1.0299
	70	0.8987	0.9402	0.9713	0.9929	1.0150
	75	0.8821	0.9206	0.9481	0.9701	0.9877
	80	0.8441	0.8776	0.9016	0.9250	0.9394

Note: $k_{eff}^* = k_{eff} + 3\sigma$ - bias (σ = calculational standard deviation in k_{eff})

Results - Example 1 (con't)

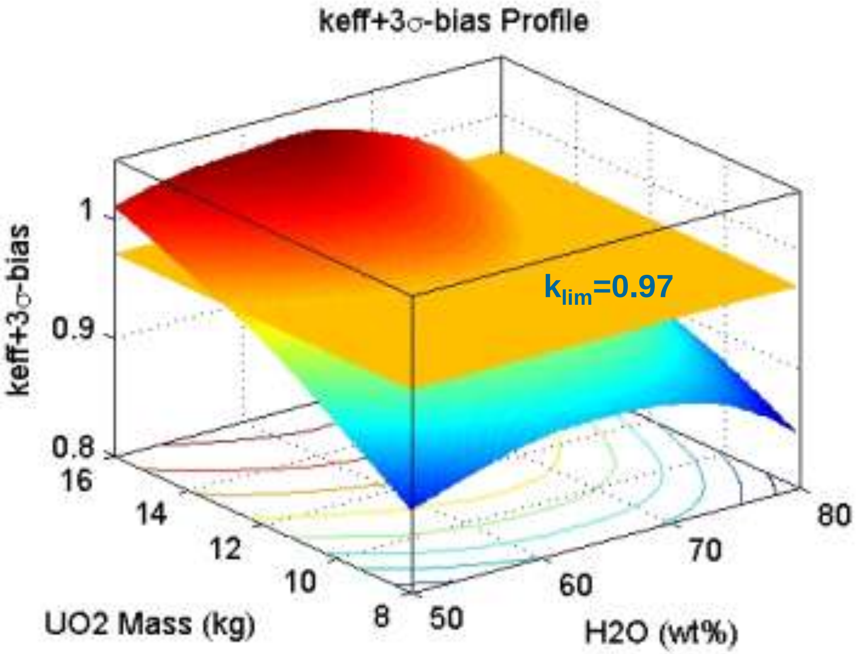
Traditional 1-D Interpolation

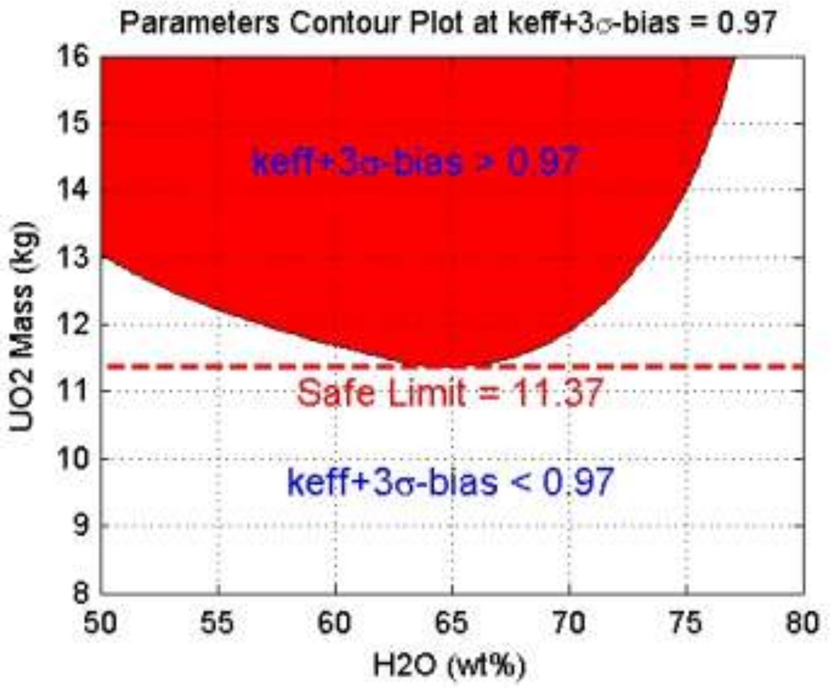




Results - Example 1 (con't)

PARANAL 2-D Interpolation





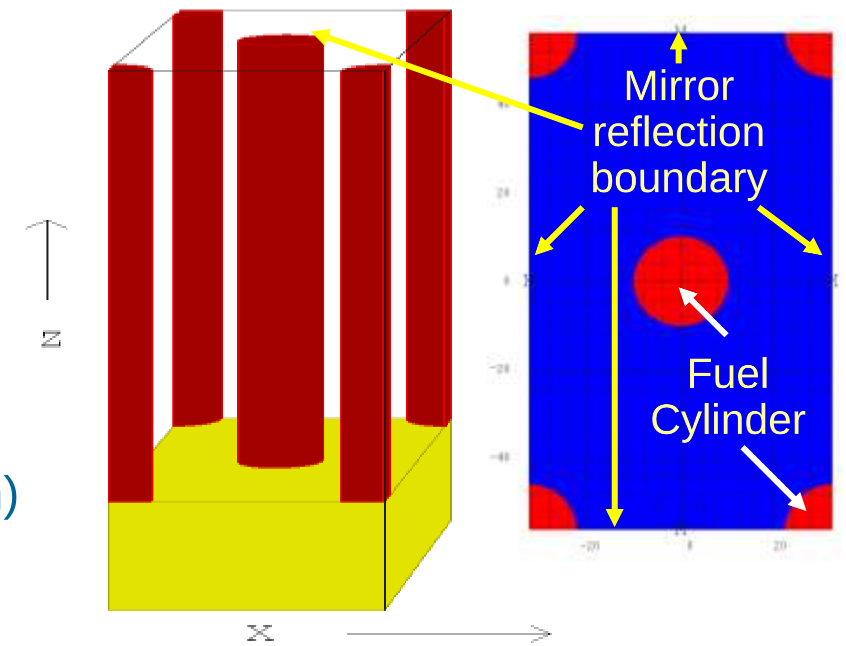
Minimum safe UO_2 mass limit = **11.37 kgs** at optimal H_2O moderation = **64.85 wt%**

k_{eff} verification: **0.9705±0.0011**

Results – Example 2

Subcritical Spacing Limit for an Infinite Array of Infinite-long Tanks Containing 8 wt% Enriched UO_2F_2 Solution

- UO_2F_2 Density: 6.37 g/cm^2
- Geometry: 8” in diameter
  Triangular pitch
- Reflector (bottom): 24” thick Concrete
- Interspersed H_2O: 0.00001 g/cm^2
- Parameters (range):
  Center-to-center spacing (100-300 cm)
  H_2O content (10-60 wt%)
- Subcritical k_{eff} limit: 0.97



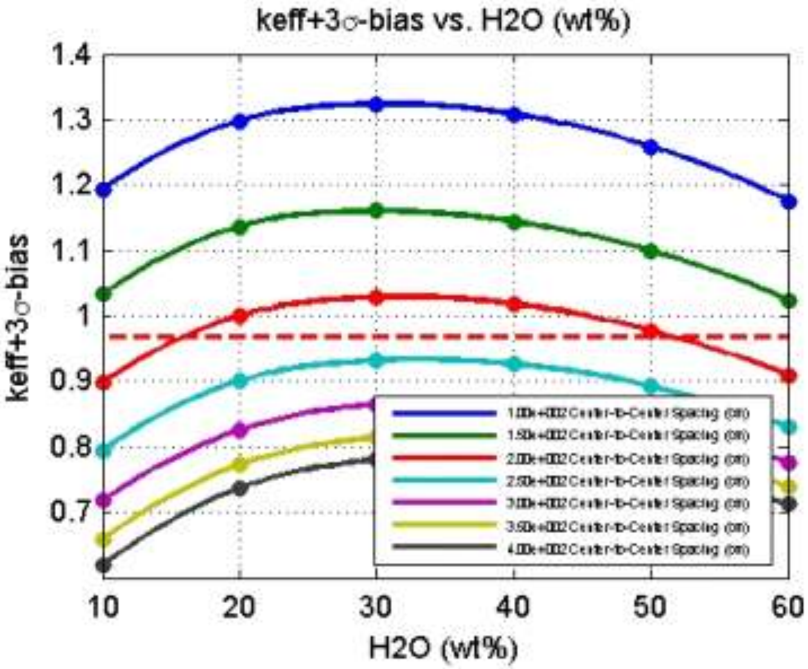
Results – Example 2 (con't)

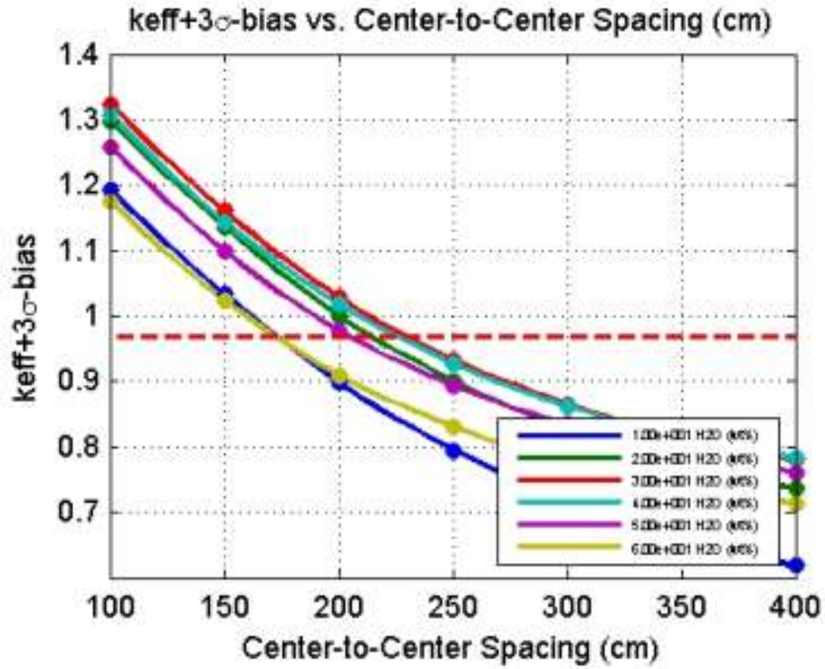
Table 2. k_{eff}* Results of UO_2F_2 Tank Array

k_{eff}*		Center-to-Center Spacing (cm)				
		100	150	200	250	300
H_2O Moderation (wt%)	10	1.1931	1.0330	0.8982	0.7952	0.7178
	20	1.2981	1.1368	1.0000	0.9005	0.8249
	30	1.3240	1.1603	1.0290	0.9321	0.8657
	40	1.3082	1.1439	1.0185	0.9264	0.8620
	50	1.2590	1.0995	0.9776	0.8932	0.8346
	60	1.1752	1.0238	0.9092	0.8305	0.7752

Results - Example 2 (con't)

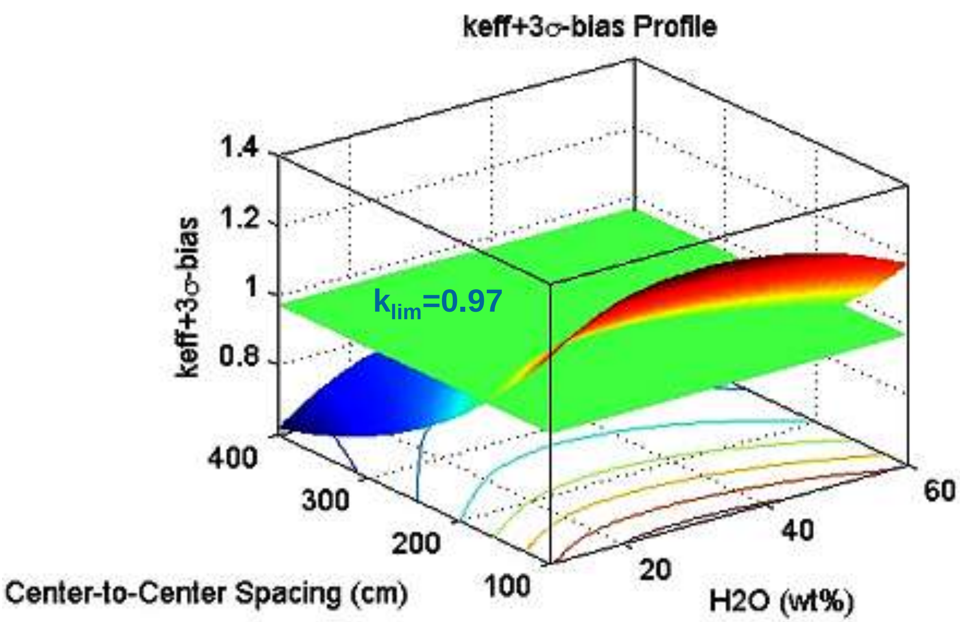
Traditional 1-D Interpolation

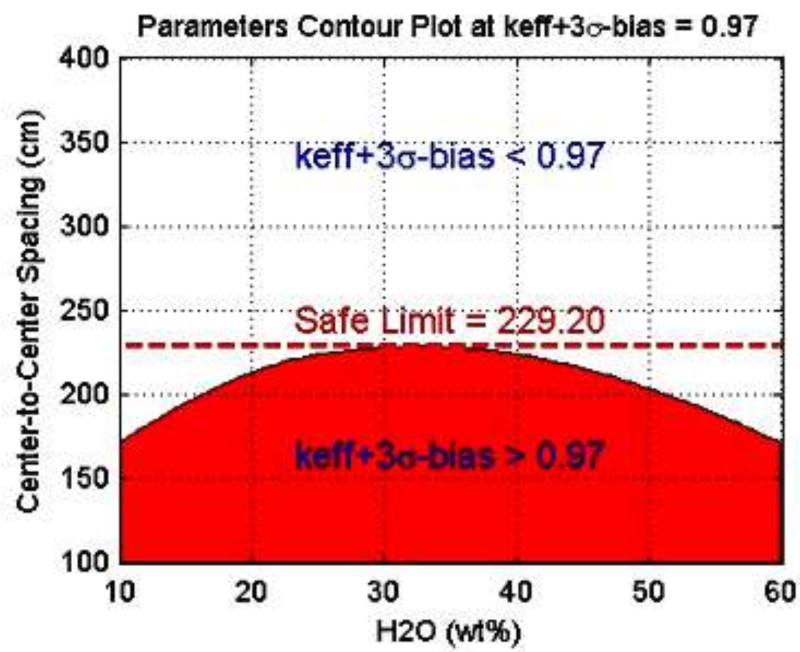




Results - Example 2 (con't)

PARANAL 2-D Interpolation





Minimum safe Spacing = **229.2 cm** at optimal H_2O moderation = **32.22 wt%**

k_{eff} verification: **0.9690±0.0011**

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Summary and Conclusions

- PARANAL provides an exceptionally efficient tool for parametric studies of criticality safety analyses.
- PARANAL allows accurate determination of the safe domain and limits of criticality parameters with two-dimensional interpolation techniques.
- PARANAL can be extended to multiple (>2) parametric analyses using N-dimensional interpolation techniques, but the visualization of an N-dimensional k_{eff} function and safe parameter domain will be difficult or impossible.
- Error estimates for interpolation may be taken into account in determining safe parameter limits, especially when extrapolation is needed.