Statistical Noise for Criticality Safety Specialists

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Noise = Nuisance (not information)

- Statistics are said to allow anything that the practitioner wants to show
- Incorrect understanding helps
- Statistics theory and conclusions are taken out of their original contexts
- Long experience with criticality safety provides many examples
- Three examples are presented now

Outline

1. Correlation between calculated reactivities (could be measured)
2. Use of all available information sources to reduce uncertainties
3. Reliable statistical determination of zero observations

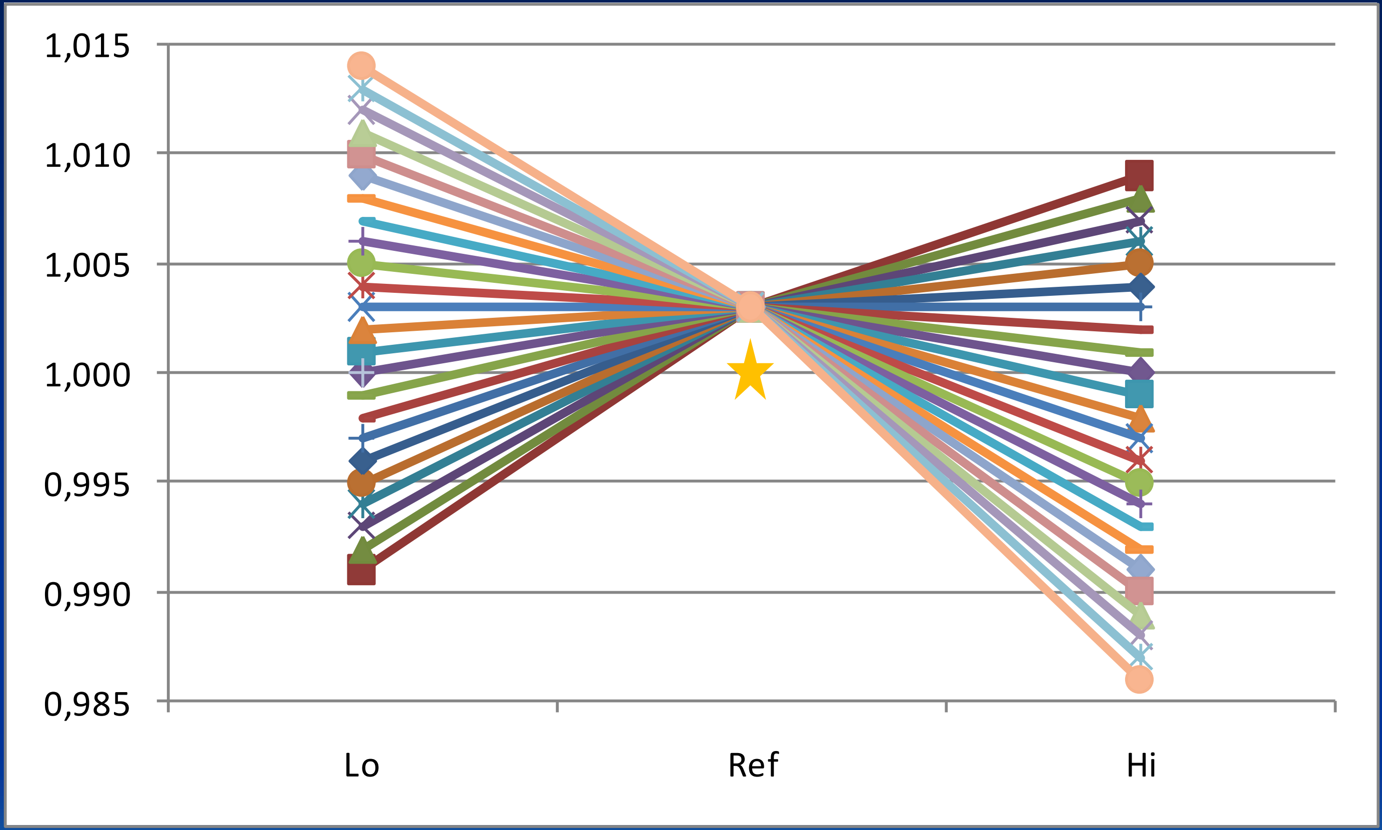
1. Reactivity correlation

- Correlation is not a complicated term
- Correlation is not limited to statistics
- The cause of a correlation should be understood to make it reliable
- Reactivities (calculated or measured) can be correlated
- Correlation of uncertainties is a safety problem (linear, not quadratic summation)

K_{eff} + reactivity additions

- Reference: $k_{eff,r} = 0.891 \pm 0.011$
- Perturbation 1: $k_{eff,1} = 0.892 \pm 0.012$
- Reactivity 1: $\Delta k_{eff,1} = 0.001 \pm 0.016$
- Perturbation 2: $k_{eff,2} = 0.893 \pm 0.012$
- Reactivity 2: $\Delta k_{eff,2} = 0.002 \pm 0.016$

Large number of reactivities



★ Correct Reference value

Two different correlation cases

- I: The sum of reactivities are added to the reference value to get the final value
- Total uncertainty: $\sqrt{\Sigma(\sigma_i^2)} = 0.025$? No! Two uncertainties are anti-correlated: $k_{eff} = \underline{0.891} + (0.892 - \underline{0.891}) + (0.893 - 0.891)$
- Correlation I: $\sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_r^2)} = 0.020$

Where $\sqrt{(\ldots)}$ is the square root of listed terms

Correlation case II

- II: The sum of reactivities are added to a new calculation value 0.894 ± 0.011 for final value
- Total uncertainty: $\sqrt{\Sigma(\sigma_i^2)} = 0.025$? No!
  Two uncertainties are correlated:
  $k_{eff} = 0.894+(0.892-\underline{0.891})+(0.893-\underline{0.891})$
- Correlation II: $\sqrt{(\sigma_n^2 + \sigma_1^2 + \sigma_2^2 + (2\sigma_r)^2)} = 0.030$

Simple equations without correlations

$$\rho_i = \frac{\Delta k_i}{k_i} = \frac{k_i - k_{ref,i}}{k_{ref,i}}$$

$$s_i^{\ 2} = \frac{u_i^2}{\delta x_i^2}\left[\left(k_{\delta xi} - k_{ref,i}\right)^2 \pm \left(s_{M,\delta xi}^2 + s_{M,ref,i}^2\right)\right]$$

$$\left(s_M\right)^2 = \sum_{i=1}^{I} \frac{u_i^2}{\delta x_i^2}\left[\left(s_{M,\delta xi}^2 + s_{M,ref,i}^2\right)\right]$$

Correlation of N uncertainties (sorted)

$$\left(s_M\right)^2 = \sum_{i=1}^{I} \frac{u_i^2}{\delta x_i^2} s_{M,\delta xi}^2 + \sum_{i=N+1}^{I} \frac{u_i^2}{\delta x_i^2} s_{M,ref,i}^2 + \left(\sum_{i=1}^{N} \frac{u_i}{\delta x_i} \right)^2 s_{M,r}^2$$

- A single correlated uncertainty $s_{M,r}$
- If more than one reference calculation, the last term is split to each group of correlations
- Make all $u_i/\delta x_i$ = 1.000 and all $s_{M,\delta xi}$, $s_{M,ref,i}$ and $s_{M,r}$ identical: s *Note: N≤I*

$$\left(s_M / s\right) = \sqrt{(2I - N) + N^2}$$

Examples

I	N	s_M/s
8	0	4.0
8	5	6.0
8	8	8.5
32	0	8.0
32	16	17.4
32	32	32.5

Part 1 - Conclusions

- In evaluations of critical experiments, and sometimes in safety evaluations, many uncertainties are covered
- A reasonable model for dealing with correlations reduces evaluation errors and supports efforts to minimize uncertainties
- Avoid reference value – Use upper and lower values with reference value to check linearity
- Large parameter variation reduces uncertainty

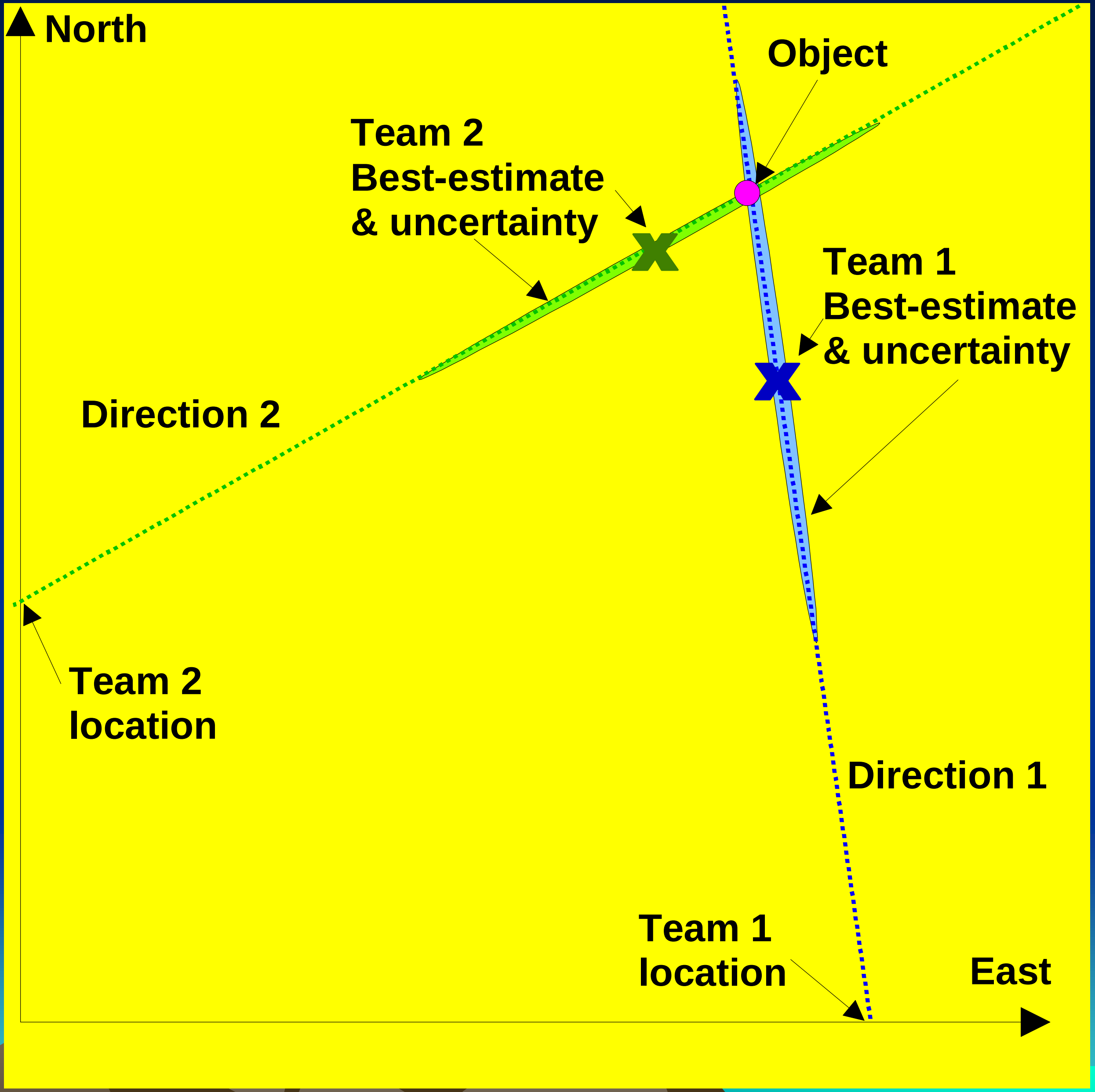
2. Statistical misunderstandings

- “A systematic effect can’t be reduced by additional observations”
- “Systematic effects increase the uncertainty”
- “Precision of uncertainty should not be higher than of the measured/calculated value”
- “Don’t compare with other evaluations to avoid being influenced by them”
- “Independent parameters -> independent reactivities”

Use all information and use it wisely

- Evaluated critical experiments are extremely valuable as benchmarks
- K_{eff}, or other criticality parameters should be complemented with other measurements
- Quantity of benchmarks has unfortunately been emphasized rather than quality
- Combining two different perspectives can be extremely effective

Different views help



Part 2 - Conclusions

- Combination of information can reduce uncertainties and reveal outliers
- The ICSBEP Uncertainty Guide, though very good, needs further work
- Statistical “truths” need further discussion
- Systematic and random effects, correlations and other effects are not necessarily complicated

3. Statistical determination of zero

- MCNP warns you if a tally is zero
- I got no votes in the last election – I warn you
- What if zero (no observations) is correct?
- Monte Carlo, just as other statistical sampling methods, rely on representative sampling
- Non-zero tallies are often caused by including invalid observations (votes). Discard them!

Spent fuel cask – Irradiated fuel

- The axial fuel composition varies strongly
- The central region often has low reactivity.
- Small region at fuel top dominates but bottom region almost as reactive
- A flat initial fission source generates many fissions near the bottom – Transient phase!
- Converged fission source -> No fissions
- Specialists claim zero fissions is a problem

Zero is correct!

- Start tally with converged fission source.
- Normal case less than ten million neutrons.
- Run many more neutrons.
- If up to one billion neutrons are needed to produce a fission in a region, the normal case fissions are zero in that region.
- Non-zero fissions is evidence of transient phase influence.

Zero is OK!

- An irradiated fuel assembly may be divided into 20 axial zones and several radial zones
- One suggested solution to avoid zero fissions is to modify the geometry and compositions
- Changed fuel assembly specifications require significant efforts and may not be safe in other scenarios
- Learn to recognize when the fissions in a region could correctly be zero.

Part 3 - Conclusions

- A tally is not always requested because the result is interesting or important
- It is important to understand the results of a calculation
- In burnup credit, the axial peak (end effect) is probably well understood
- Zero fissions in a region in a converged calculation is not understood but often correct

Summary

- Correlations need better treatment (between uncertainties, between benchmarks, etc.)
- Many statistical “truths” are misunderstood
- Zero observations (e.g. fission source in a region) can be determined statistically
- A certified criticality safety specialist should have 5 years experience at licensing authority to be trusted with safety responsibility

Reflection 15: Burnup credit makes mixed array issue more realistic

- Fresh fuel assumption – Few assembly types
- Burnup credit make all assemblies different
- Lower reactivity for one fresh fuel type does not necessarily be the case for irradiated fuel
- Different geometry (e.g. 14x14 or 16x16) of fuel assemblies is a mixed array safety issue
- OECD/NEA exercise on “Large arrays” not complete; Fuel assemblies in water is easier.