Subcriticality Measurement by Neutron Source Multiplication Method with Detected-Neutron Multiplication Factor

Nagoya University ☺Tomohiro Endo,
Akio Yamamoto

Kyoto University Cheol Ho Pyeon,
Takahiro Yagi

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Neutron source multiplication method (NSM)

- Subcriticality measurement technique based on

$$CR = \varepsilon\left(S + S\times k + S\times k^2 + \cdots\right) = \frac{\varepsilon S}{1-k}$$

CR :count rate, k : neutron multiplication factor
S :neutron source strength, ε :detector efficiency

- Features
  - Simple technique by neutron source and detector
  - Reactivity difference $\Delta\rho$ can be measured from the reference state where k_{eff} is known beforehand

Modified NSM [1]

- Neutron count rate

calculated neutron flux at target subcritical state

$$CR = \frac{k_{\mathrm{eff}}}{1-k_{\mathrm{eff}}} \frac{\left\langle \Sigma_{\mathrm{d}} \psi_{\mathrm{calc}} \right\rangle}{\left\langle \varphi^{\dagger}_{\mathrm{calc}} \mathbf{F} \psi_{\mathrm{calc}} \right\rangle} \left\langle S \varphi^{\dagger}_{\mathrm{calc}} \right\rangle$$

fundamental adjoint eigenfunction

Correction to extract fundamental mode component of λ-eigenvalue equation

[1] Mizoo, N., 1977. JAERI-M 7135, (JAERI).

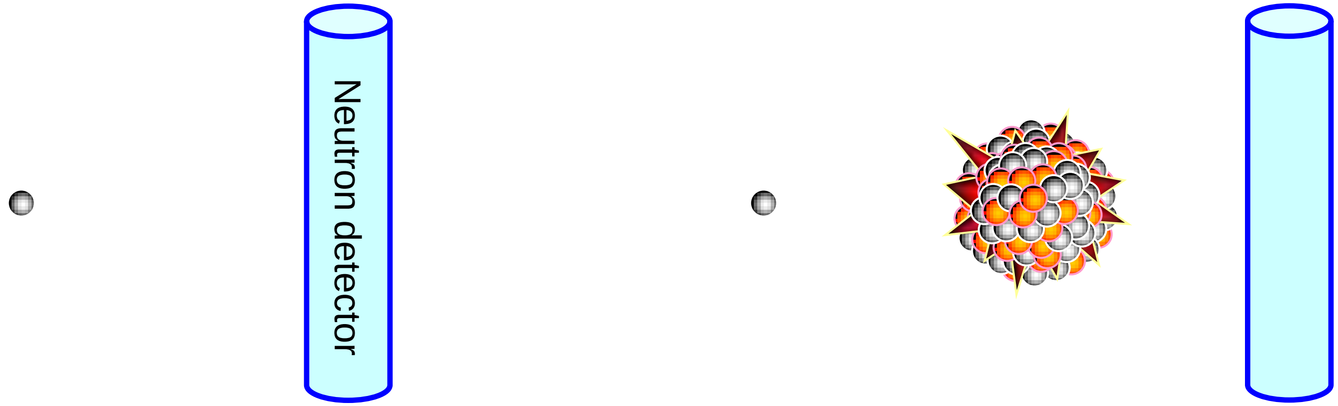
Purpose of this study

- Issues of Modified NSM
  - Numerical results has uncertainties
    - size, nuclide compositions, cross-sections
    - analytical models
  - Estimated correction factors have also uncertainties
- Purpose
  - To clarify appropriate detector positions to reduce the impact of correction factors in NSM experiment
    - Ideally, correction factors nearly equal to unity
- Reconsideration of definition of measured “neutron multiplication factor”

Decomposition of neutron flux

source-flux term | fission-flux term

$$\left\langle \Sigma_d \phi \right\rangle = \left\langle \Sigma_d \phi_s \right\rangle + \left\langle \Sigma_d \phi_f \right\rangle$$



detection of primary neutron due to source

detection of fission neutron

Detected-neutron multiplication factor[2]

$$k_{\mathrm{det}} \equiv \frac{\left\langle \Sigma_{\mathrm{d}}\, \phi_{\mathrm{f}} \right\rangle}{\left\langle \Sigma_{\mathrm{d}}\, \phi \right\rangle} = \frac{\left\langle I^{\dagger}\, \mathbf{F}\, \phi_{\mathrm{s}} \right\rangle}{\left\langle I^{\dagger}\, \mathbf{A}\, \phi_{\mathrm{s}} \right\rangle}$$

$\mathbf{A}$ annihilation operator

$\mathbf{F}$ production operator

$I^{\dagger}$ detector importance

- Ratio of "detected fission-neutron" to "detected all neutron"
  - Similar to the definition of "Subcritical mulitiplication factor: k_{sub}[3]"
  - Note: reaction rate of neutron detection is used in the difinition of k_{det}

[2] T. Endo, et al., *Ann. Nucl. Energy*, **38**, pp.2417-2427 (2011).
[3] K. Kobayashi and K. Nishihara, Nucl. Sci. Eng., **136**[2], pp.272-281 (2000).

Relationship between neutron count rate and k_{det}

$$\left\langle \Sigma_d \phi \right\rangle = \frac{\left\langle \Sigma_d \phi_s \right\rangle}{1 - k_{det}} = \left\langle \Sigma_d \phi_s \right\rangle \left(1 + k_{det} + k_{det}^2 + \cdots\right)$$

- Detected total neutron count rate can rigorously expressed by summation of Infinite geometric series
  - 1st term：detected primary neutron due to emission of external source
  - Common ratio： k_{det}

NSM expression using k_{det}

$$k_{\text{eff, target}} = \frac{1}{f_{\text{c,target}}} \left\{ 1 - f_{\text{s}} \left(1 - f_{\text{c,ref}} k_{\text{eff, ref}} \right) \frac{\langle \Sigma_{\text{d}} \phi_{\text{ref}} \rangle}{\langle \Sigma_{\text{d}} \phi_{\text{target}} \rangle} \right\}$$

- Correction factor for source-flux

$$f_{\text{s}} \equiv \langle \Sigma_{\text{d}} \phi_{\text{s,target}} \rangle / \langle \Sigma_{\text{d}} \phi_{\text{s,ref}} \rangle$$

- Conversion factor from k_{det} to k_{eff}

$$f_{\text{c}} \equiv k_{\text{det}} / k_{\text{eff}}$$

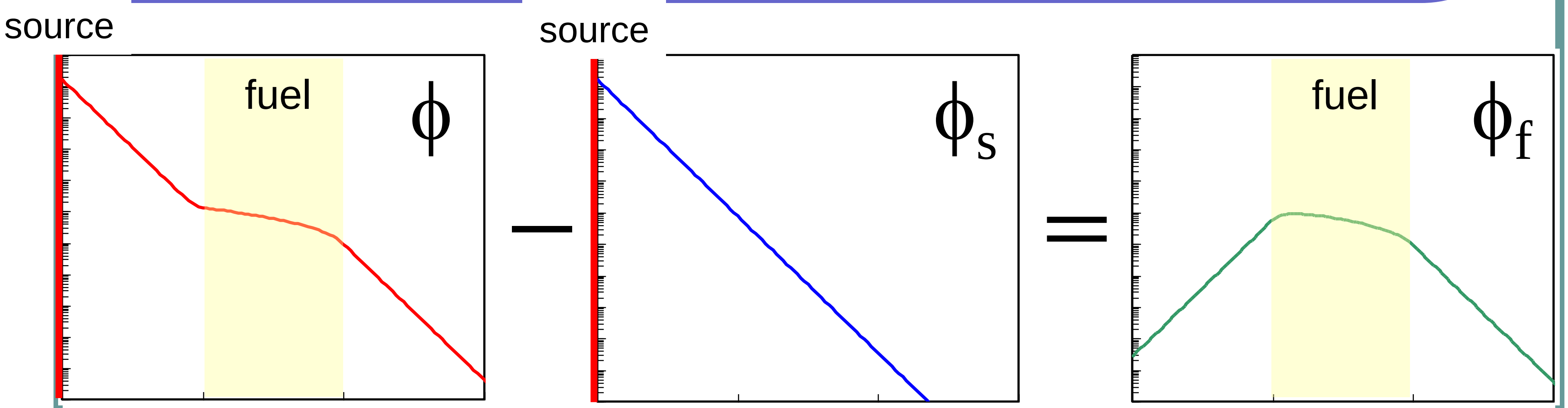
- Note: f_s & f_c can be evaluated by only forward calculations, without adjoint ones

Appropriate detector position in NSM experiment

$$k_{\text{eff, target}} = \frac{1}{f_{\text{c, target}}} \left\{ 1 - f_{\text{s}} \left(1 - f_{\text{c,ref}}\, k_{\text{eff, ref}} \right) \frac{\langle \Sigma_{\text{d}}\, \phi_{\text{ref}} \rangle}{\langle \Sigma_{\text{d}}\, \phi_{\text{target}} \rangle} \right\}$$

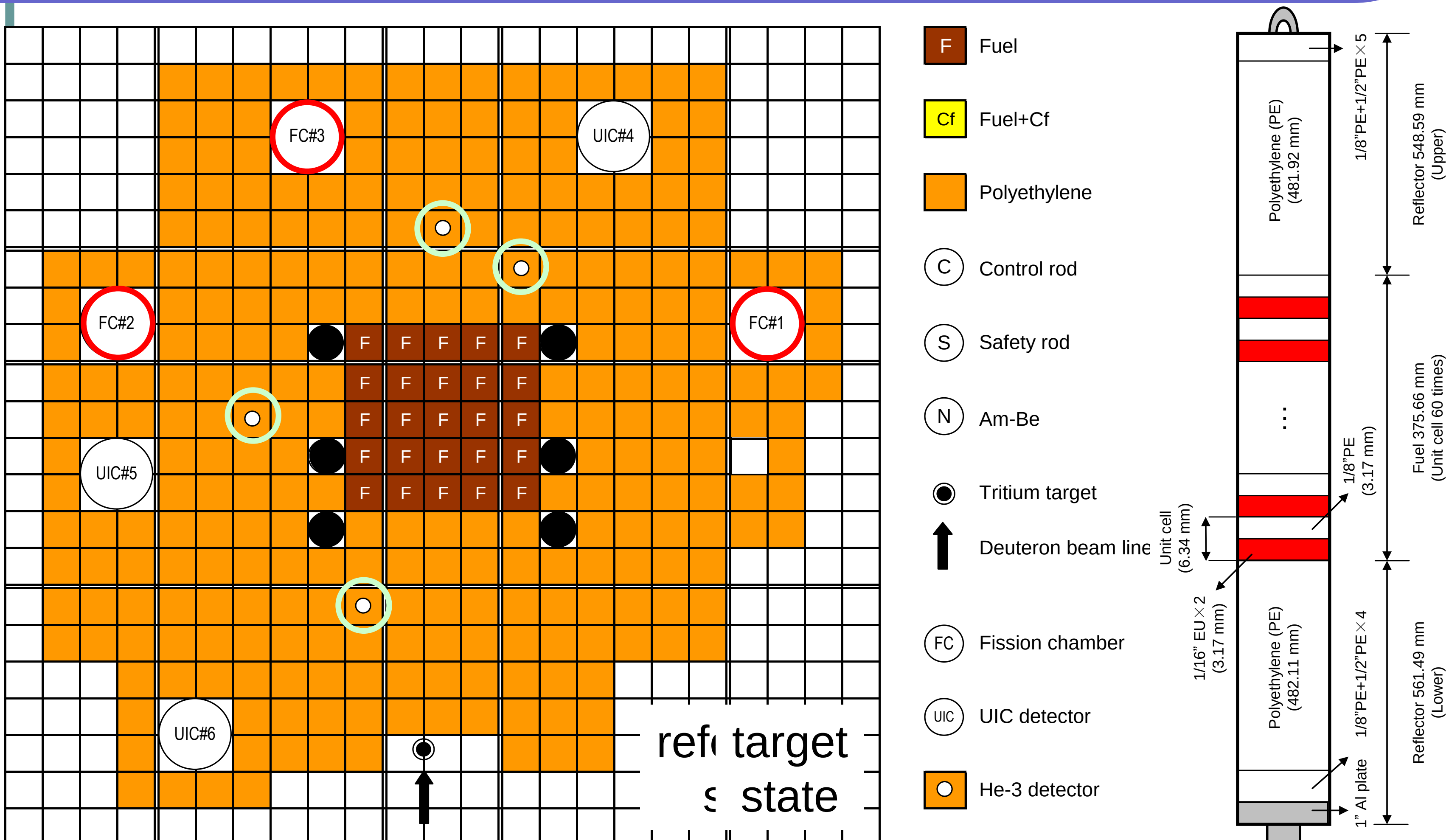
- Detector position where $k_{det}=k_{eff}$
  - Conversion factor f_c is equal to unity
  - This search can be achieved by only forward calculations at reference state
- Detector position where $f_s=1$ is better
  - Note: need for forward calculations for target state

How to numerically solve k_{det}

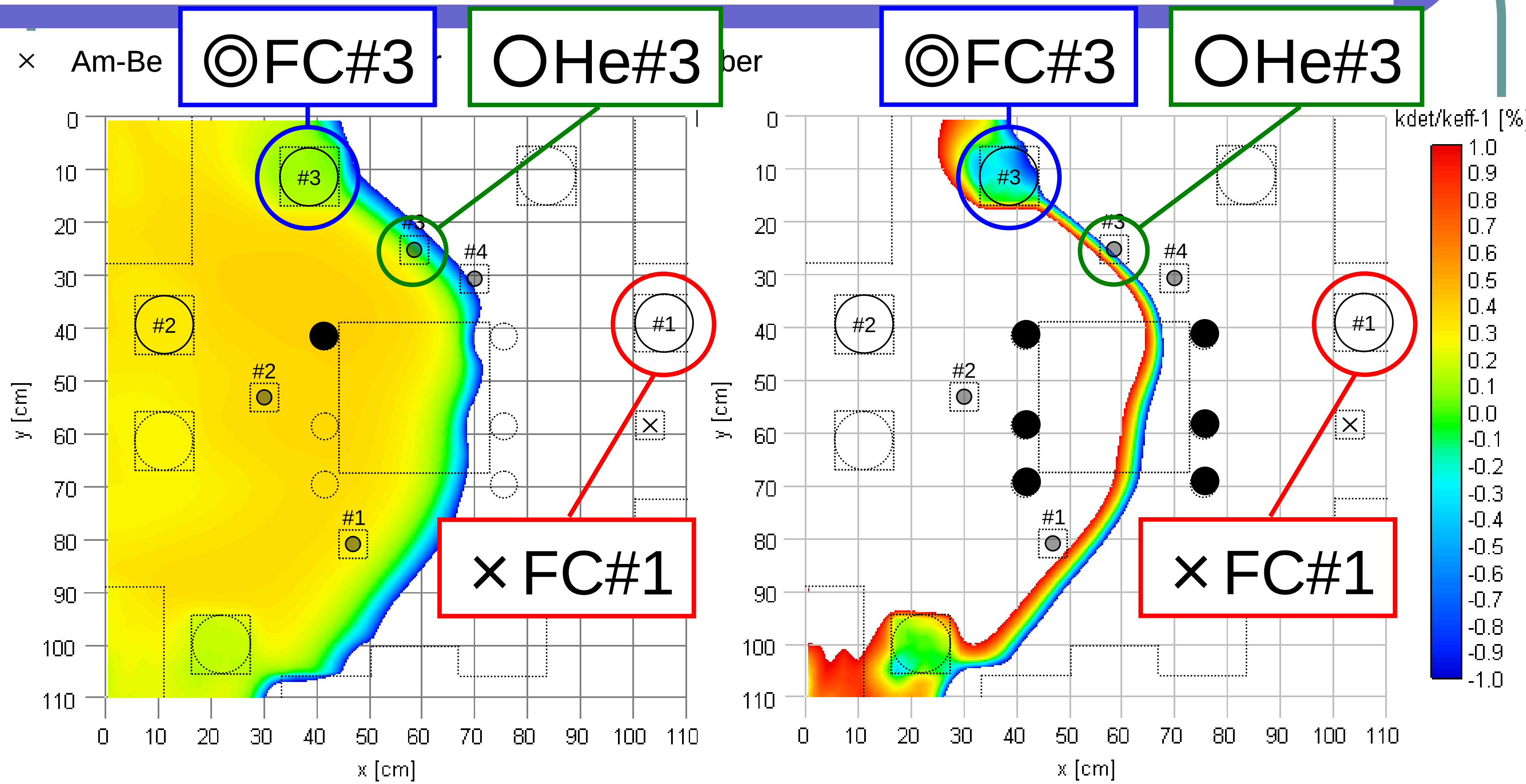


1. Forward external source problem with fission
2. Forward external source problem without fisison ($\nu\Sigma_f$=0 or ν=0)
3. ϕ_f is evaluated by $\phi - \phi_s$
4. $k_{det} = \langle \Sigma_d \phi_f \rangle / \langle \Sigma_d \phi \rangle$

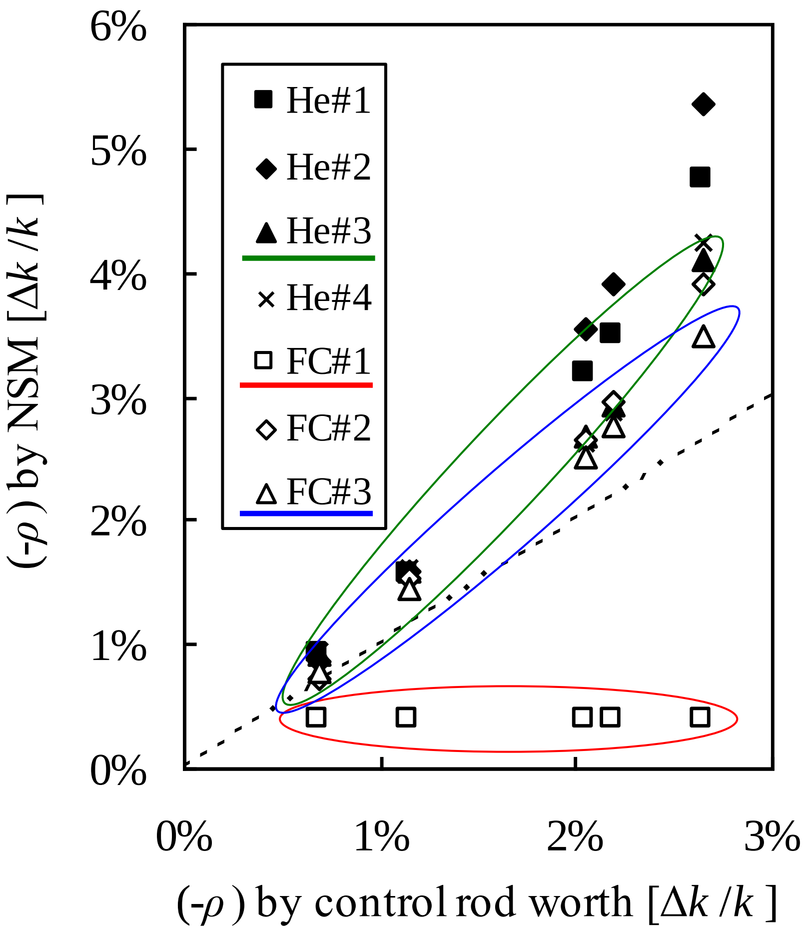
NSM experiment at Kyoto University Critical Assembly(KUCA)



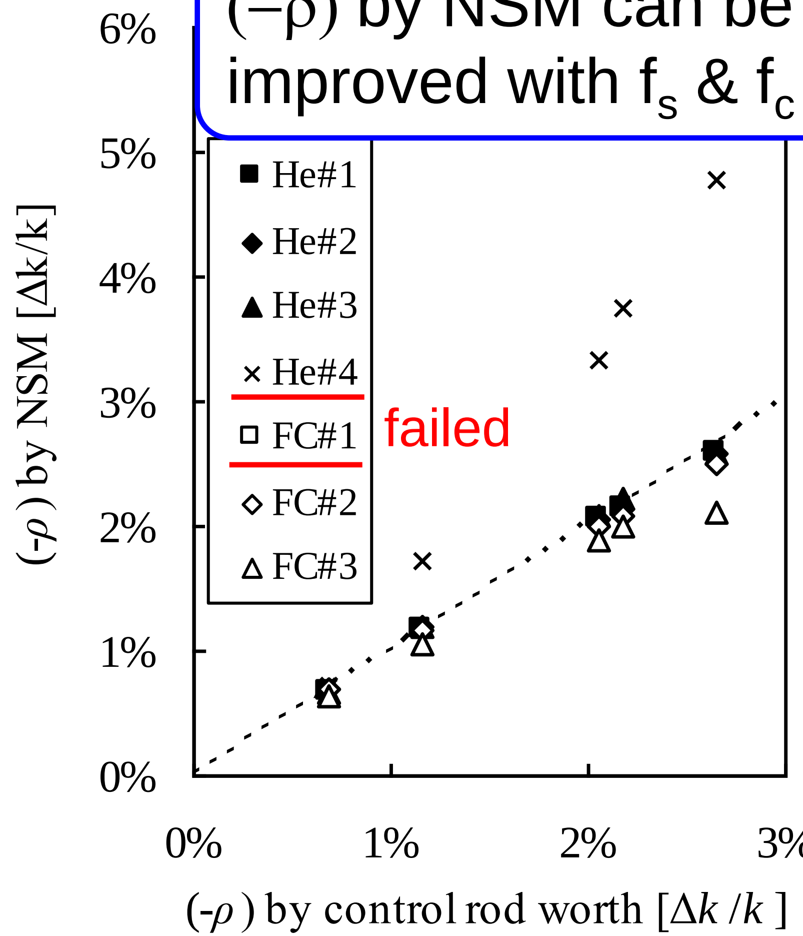
k_{det}=k_{eff} map for Am-Be source by SRAC/DANTSYS,16-group Sn transport calc.



Result of NSM for Am-Be source



without corerction

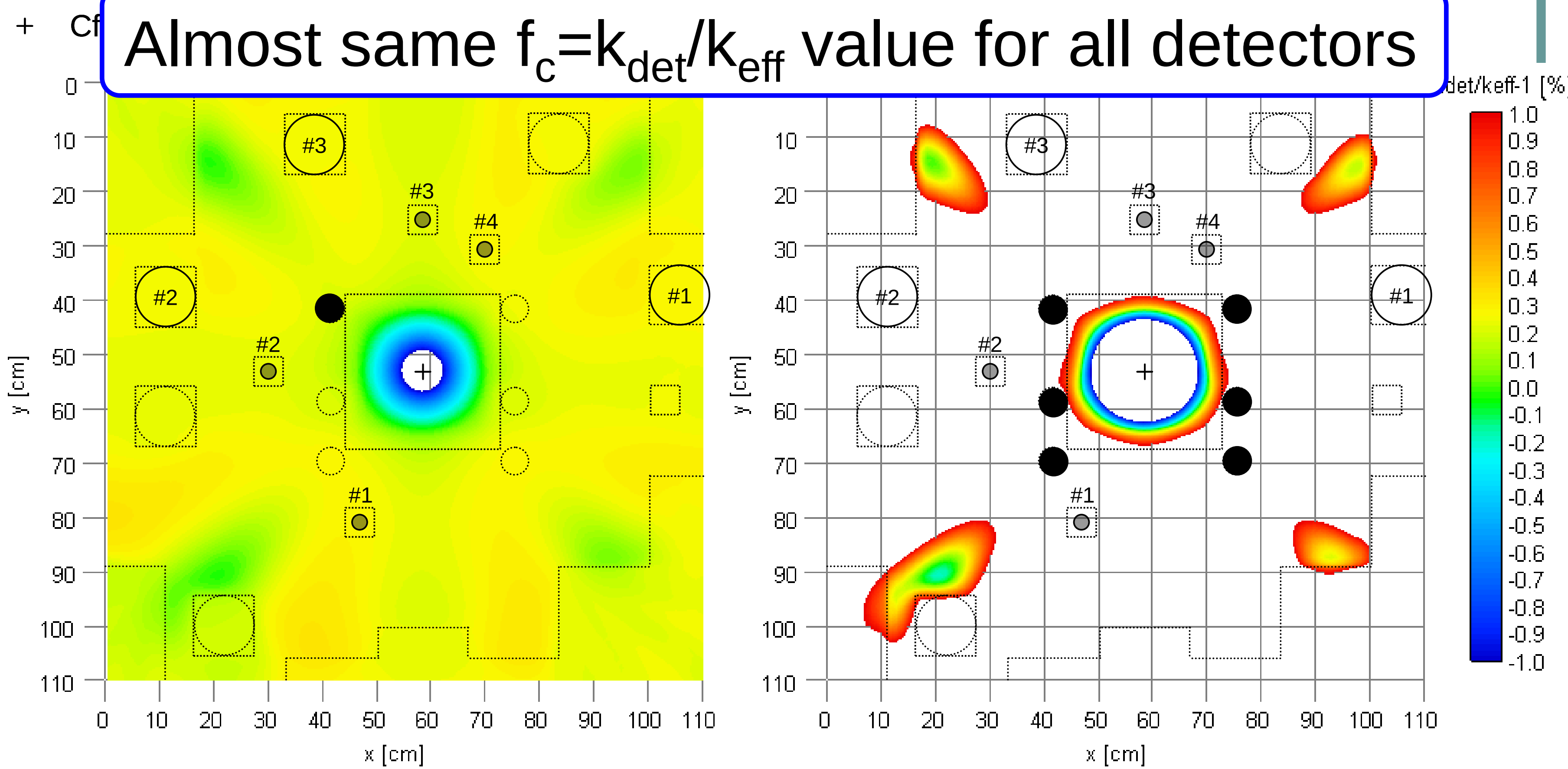


(−ρ) by NSM can be improved with f_s & f_c

with f_s & f_c correction

$k_{det}=k_{eff}$ map for Cf-souce
by SRAC/DANTSYS,16-group Sn transport calc.

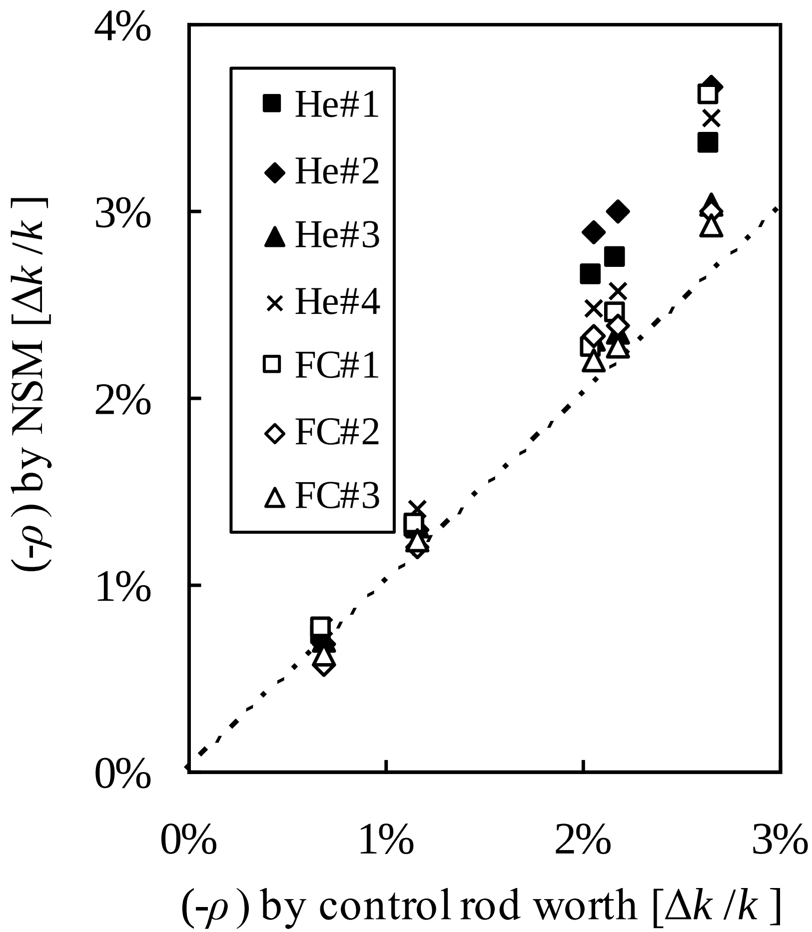
Almost same $f_c=k_{det}/k_{eff}$ value for all detectors



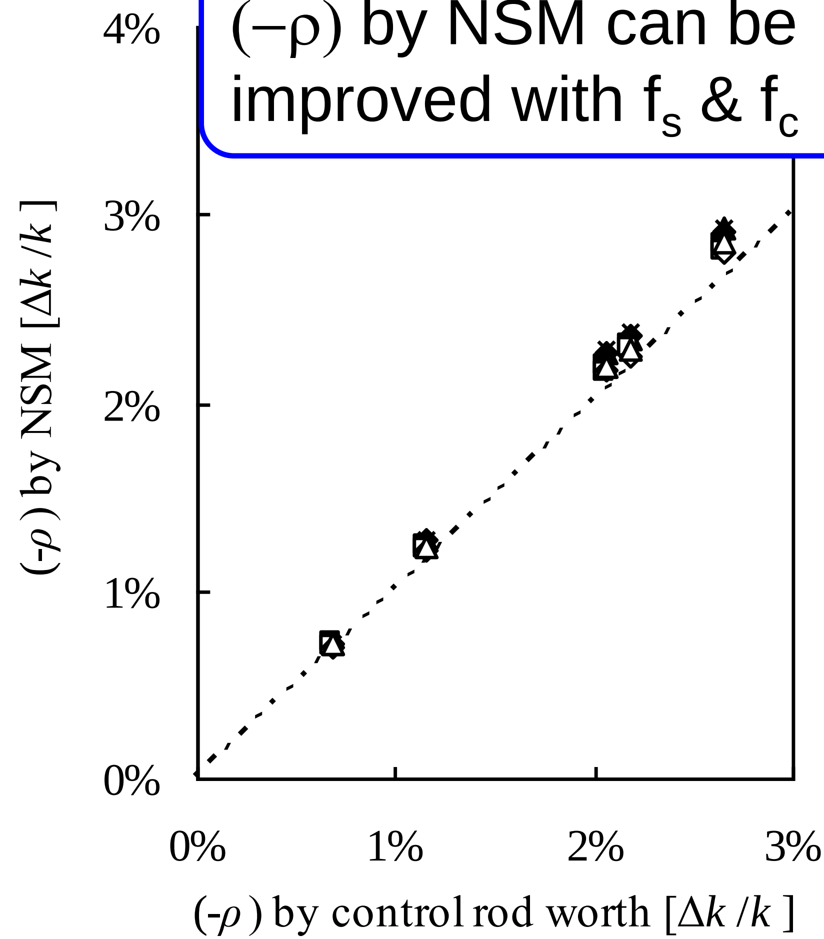
only C3 rod in reference state

All rods in

Result of NSM for Cf-source



without corerction



(−ρ) by NSM can be improved with f_s & f_c

with f_s & f_c correction

Conclusion

- NSM based on k_{det}
  - Good results without corrections at detector position where $k_{det} = k_{eff}$
    - position can be searched by calculation for reference state
  - Correction factors can be evaluated by only forward calculations without adjoint ones

- Future works
  - Evaluation of k_{det} and correction factors using continuous energy Monte Carlo code
  - Uncertainty Analysis for k_{det} and correction factors

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