

# **Determination of Correlations among Benchmark Experiments by Monte Carlo Sampling Techniques**

**Matthias Bock, Maik Stuke**

Gesellschaft für Anlagen- und Reaktorsicherheit (GRS) mbH

September 30, 2013

**NCSD 2013**

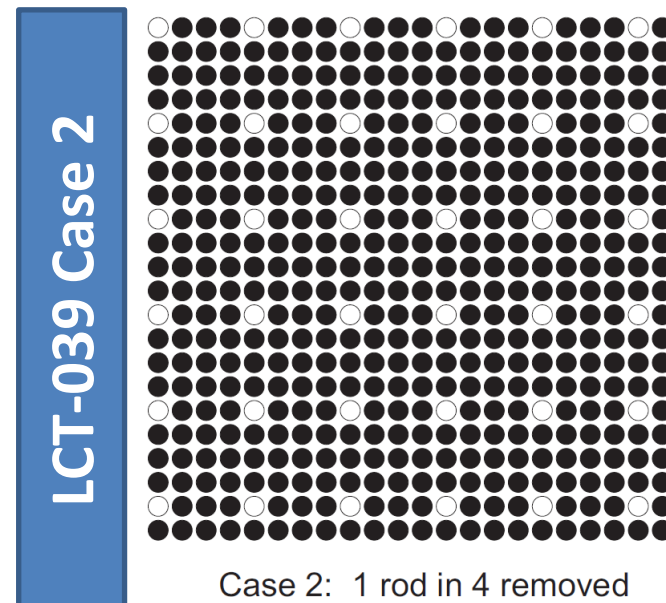
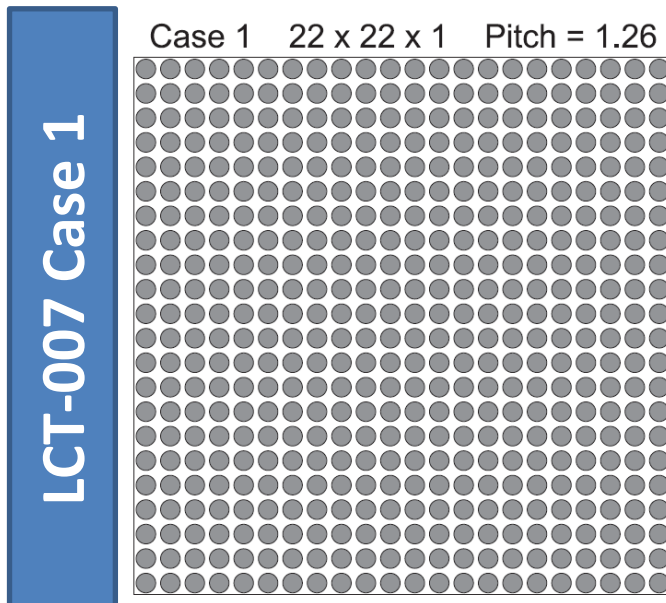
**Wilmington, NC, USA**

## Motivation

- Code validation is an important issue in criticality safety assessments
- Collections of benchmark experiments provide a source to test codes against qualified experimental results
- The ICSBEP Handbook contains additional information about the major experimental uncertainties of each experiment
  - Allows uncertainty analysis based on the Monte Carlo sampling technique
- Uncertainty analyses can be used to determine correlations among benchmark experiments
  - Cases sharing manufacturing tolerances due to the use of the same fuel rods
  - These correlations may influence the subsequent bias determination for an application case

## UACSA Phase IV Benchmark Proposal

- Discussed during last year's Expert Group meeting at the OECD/NEA's WPNCs
- Goal: Determine the "Impact of correlations between different criticality safety benchmark experiments on the estimation of the computational bias of  $k_{\text{eff}}$ "
- Benchmark experiments under consideration (taken from ICSBEP Handbook):
  - LEU-COMP-THERM-007: Cases 1 to 4
  - LEU-COMP-THERM-039: all 17 cases
  - The experiments share the experimental apparatus and the fuel rods



# Uncertain Parameters

- Analyzing manufacturing tolerances
- Two types of uncertain parameters:
  - Those common to all 21 cases
  - Those individual for each experiment
- Probability density functions:
  - Uniform distribution between a and b:  $U(a,b)$
  - Normal distribution:  $N(\mu,\sigma^2)$

## Common uncertain parameters

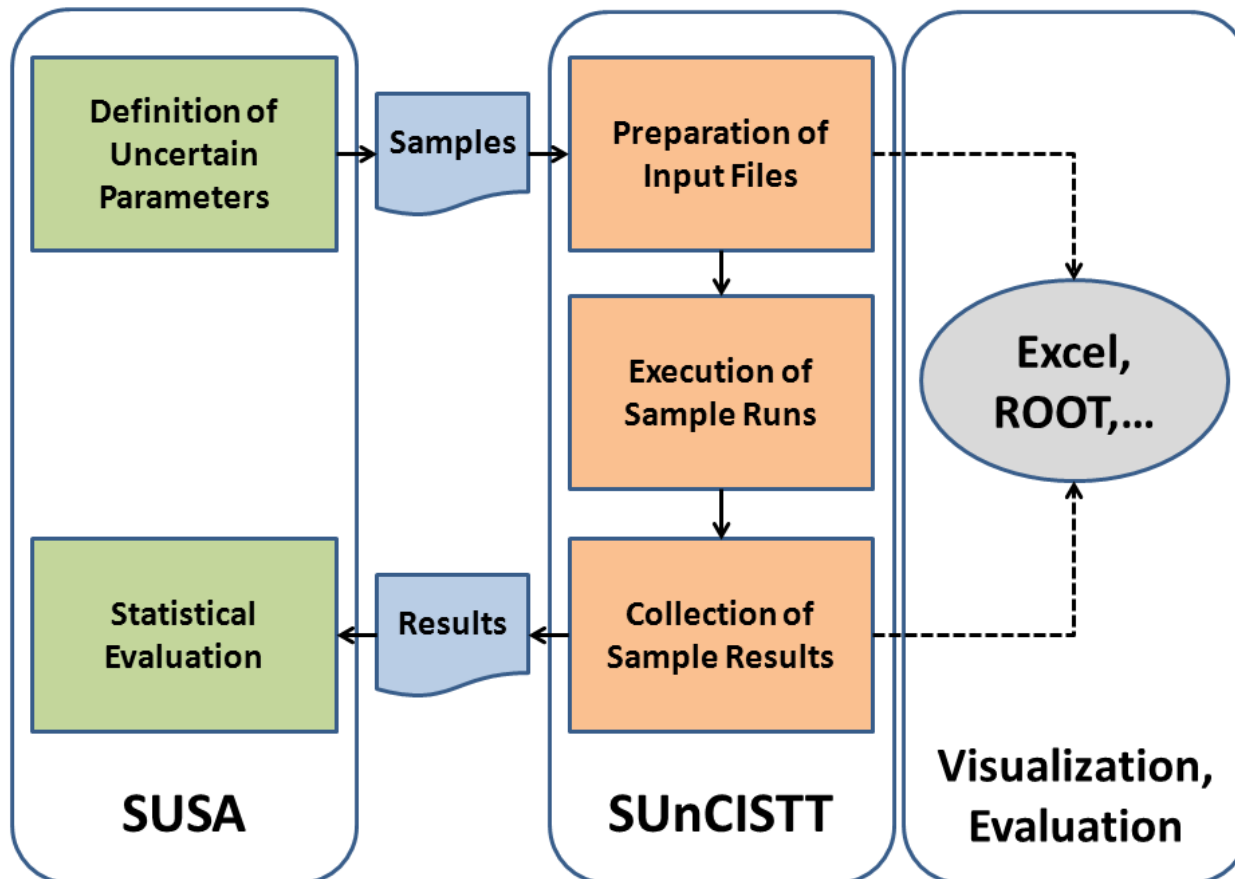
Parameter	Distribution Model	Model Parameters	
		a or $\mu$	b or $\sigma$
Fuel rod inner diameter	$U(a,b)$	0.81 cm	0.83 cm
Fuel rod thickness	$U(a,b)$	0.055 cm	0.065 cm
Fuel pellet diameter	$N(\mu,\sigma^2)$	0.7892 cm	0.0017 cm
Fuel density	$N(\mu,\sigma^2)$	10.38 g/cm <sup>3</sup>	0.0133 g/cm <sup>3</sup>
Height of fissile column	$N(\mu,\sigma^2)$	89.7 cm	0.3 cm
U-234 content	$N(\mu,\sigma^2)$	0.0307 At.-%	0.0005 At.-%
U-235 content	$N(\mu,\sigma^2)$	4.79525 At.-%	0.002 At.-%
U-236 content	$N(\mu,\sigma^2)$	0.1373 At.-%	0.0005 At.-%

## Individual uncertain parameter

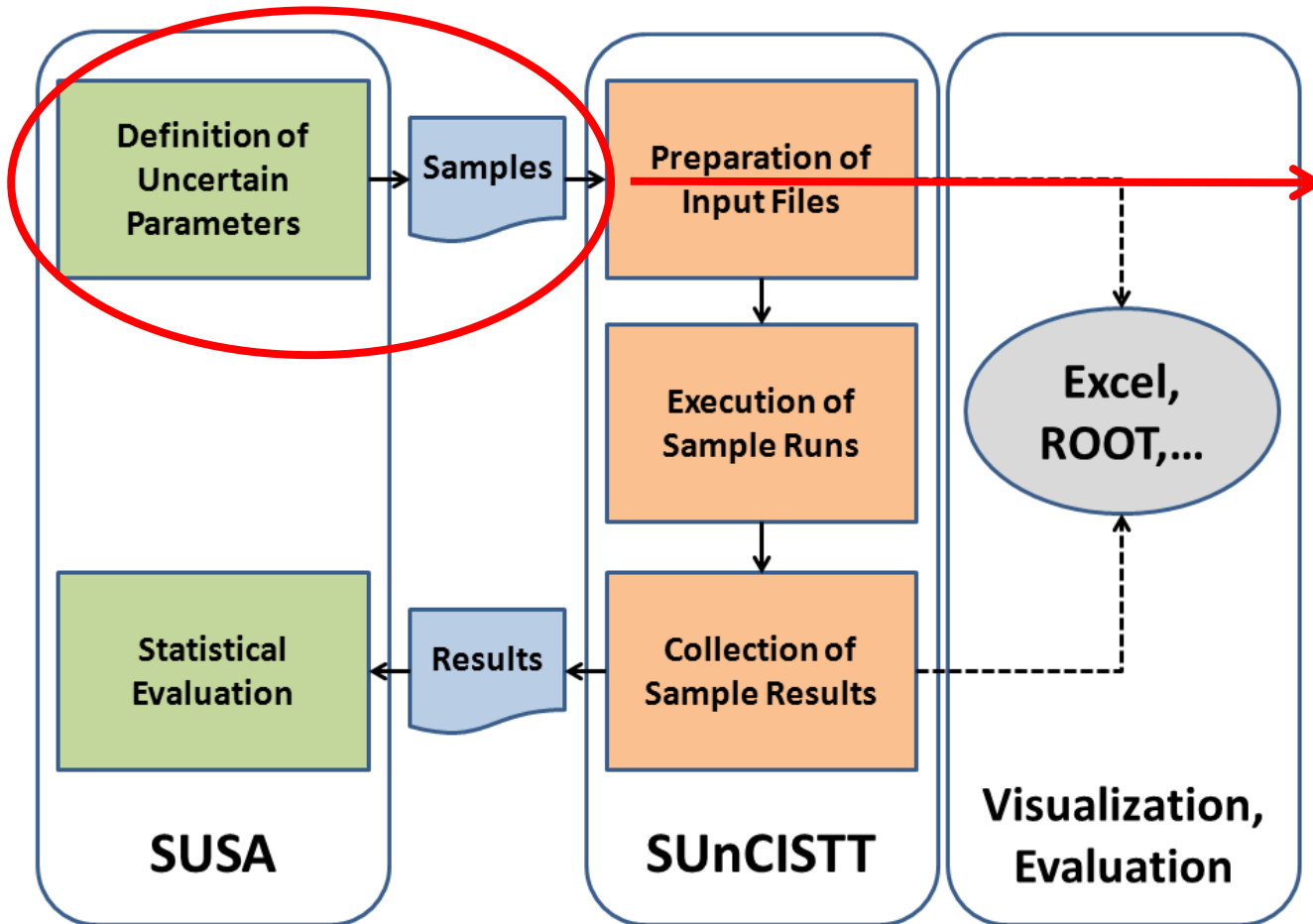
Parameter	Experiment	Distribution Model	Model Parameters	
			$\mu$	$\sigma$
Critical water height	LCT-007-001	$N(\mu,\sigma^2)$	90.69 cm	0.10 cm
	LCT-007-002		73.53 cm	0.10 cm
	LCT-007-003		77.98 cm	0.06 cm
	LCT-007-004		79.85 cm	0.10 cm
	LCT-039-001		81.36 cm	0.07 cm
	LCT-039-002		77.69 cm	0.06 cm
	LCT-039-003		73.05 cm	0.06 cm
	LCT-039-004		89.07 cm	0.06 cm
	LCT-039-005		84.37 cm	0.06 cm
	LCT-039-006		58.77 cm	0.06 cm
	LCT-039-007		69.71 cm	0.06 cm
	LCT-039-008		66.79 cm	0.06 cm
	LCT-039-009		64.47 cm	0.07 cm
	LCT-039-010		58.37 cm	0.07 cm
	LCT-039-011		81.34 cm	0.06 cm
	LCT-039-012		75.38 cm	0.07 cm
	LCT-039-013		72.52 cm	0.06 cm
LCT-039-014	71.14 cm	0.06 cm		
LCT-039-015	69.88 cm	0.06 cm		
LCT-039-016	69.40 cm	0.06 cm		
LCT-039-017	68.75 cm	0.06 cm		

# Individual Uncertainty Analysis with the SUnCISTT

- The SUnCISTT is the GRS tool to perform uncertainty analyses in criticality safety assessments
- It uses a MC sampling method and the GRS tool SUSAs to analyze the impact of uncertain technical parameters

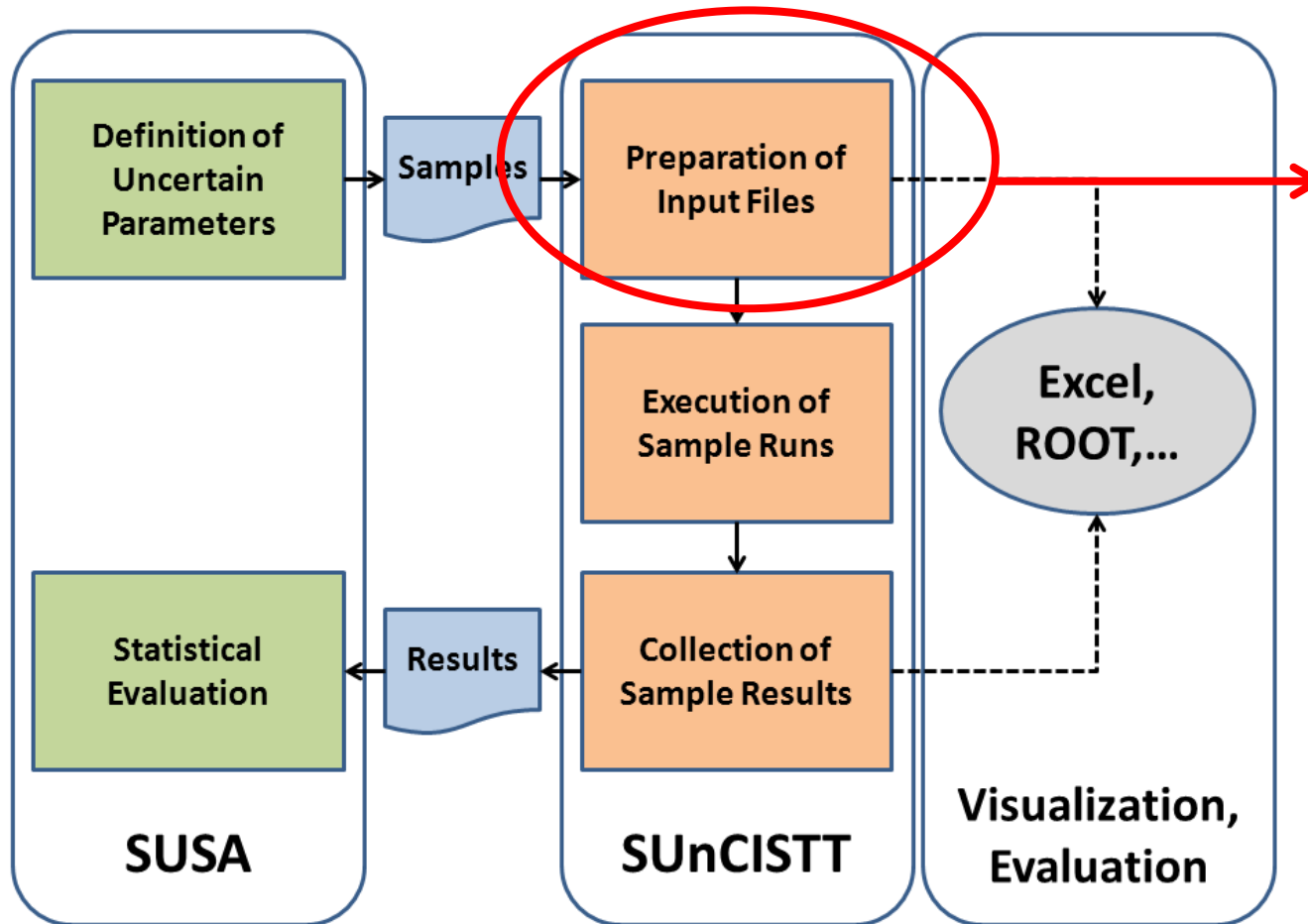


# Individual Uncertainty Analysis with the SUnCISTT



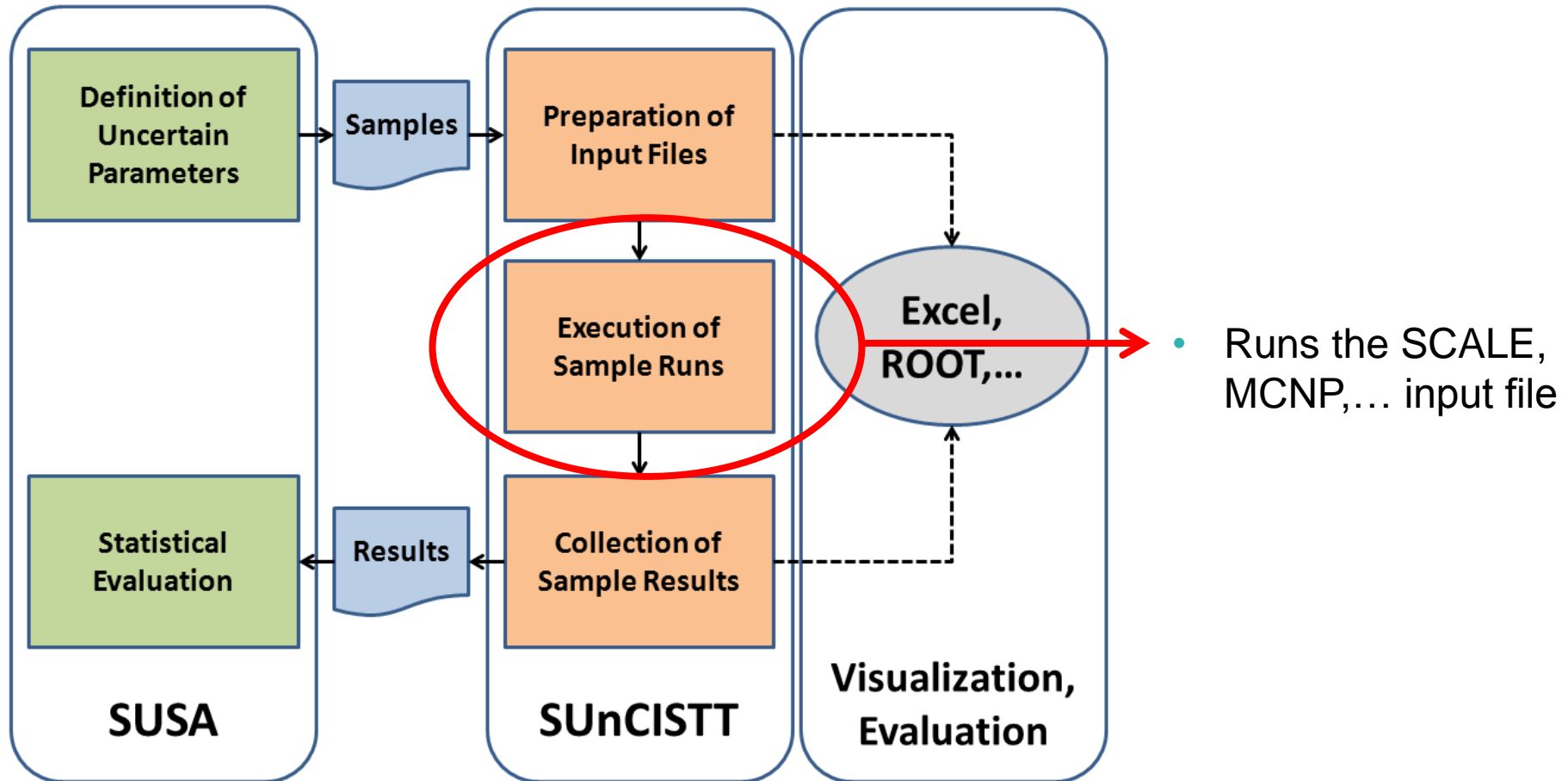
- Creates a list of randomly chosen values for each uncertain parameter according to the user defined probability function
- User defined template file: Input file for SCALE, MCNP, ... which contains keywords at the places of the uncertain parameters

# Individual Uncertainty Analysis with the SUnCISTT



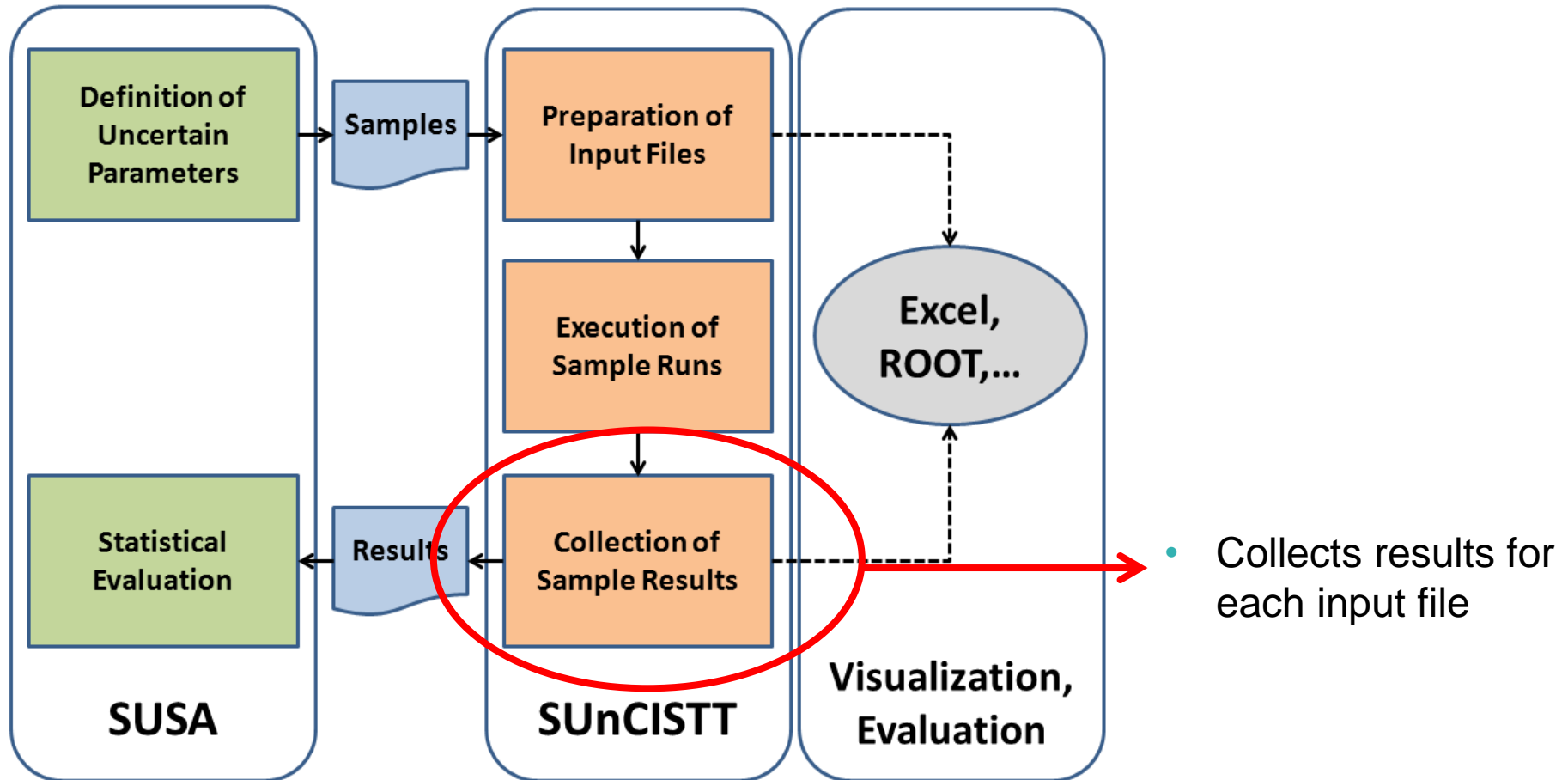
- For each sample: Copy of the template file is created and the keywords are replaced by the corresponding values to get a valid SCALE, MCNP,... input file

# Individual Uncertainty Analysis with the SUnCISTT

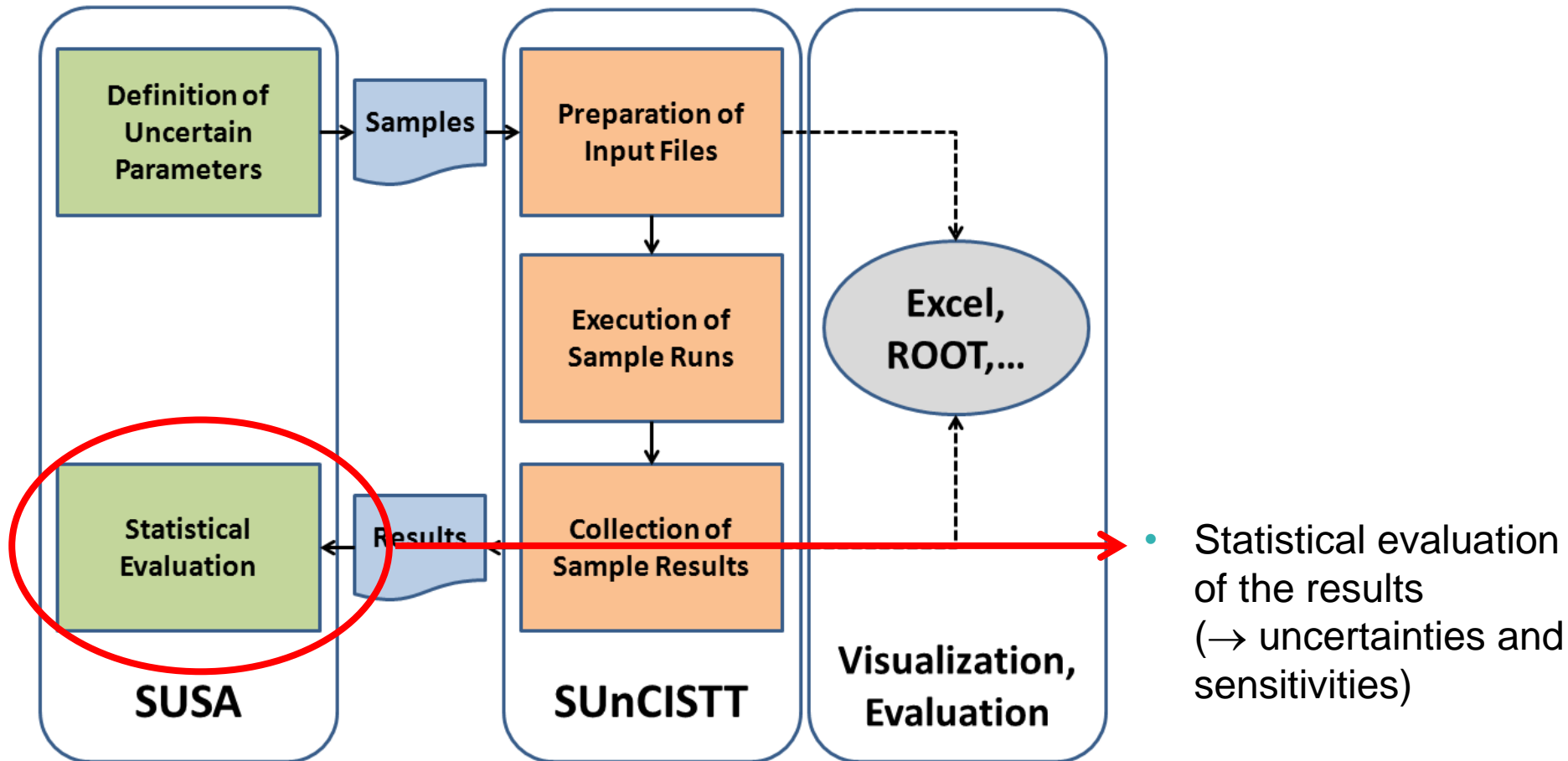




# Individual Uncertainty Analysis with the SUnCISTT

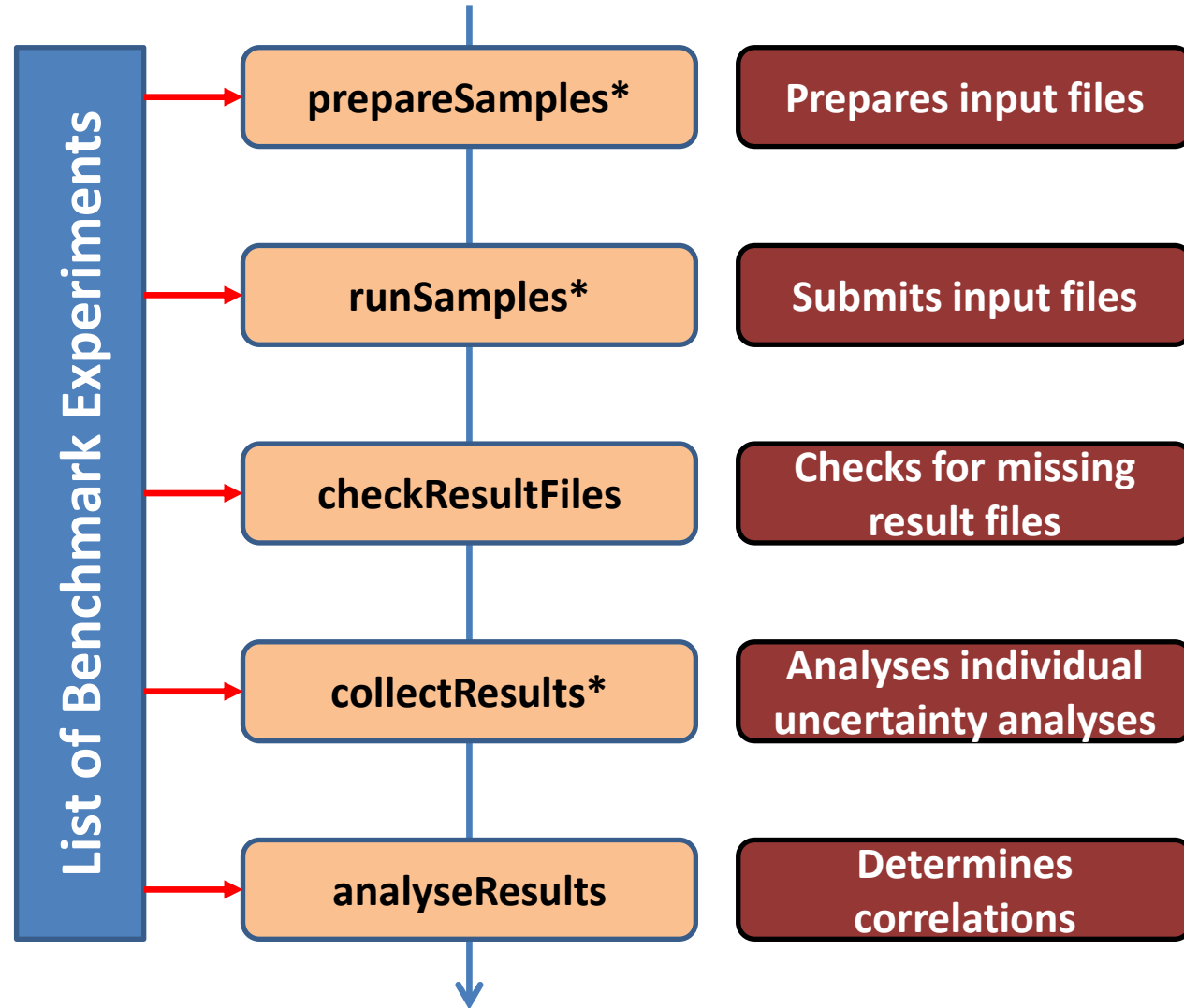


# Individual Uncertainty Analysis with the SUnCISTT



## Extending the SUnCISTT for Correlation Analyses

- Several uncertainty analyses are steered at the same time
- In the first mode, the option is included to consider the common variation of uncertain parameters (using the same values in all experiments)



## Determination of correlation

- Pearson correlation of benchmark experiments  $A$  and  $B$  is determined using the calculated  $k_{\text{eff}}$  values

$$\Sigma_{AB} = \frac{1}{\sigma_A \sigma_B} \sum_{i=1}^n (k_{A,i} - \bar{k}_A)(k_{B,i} - \bar{k}_B)$$

with  $i$ : sample number

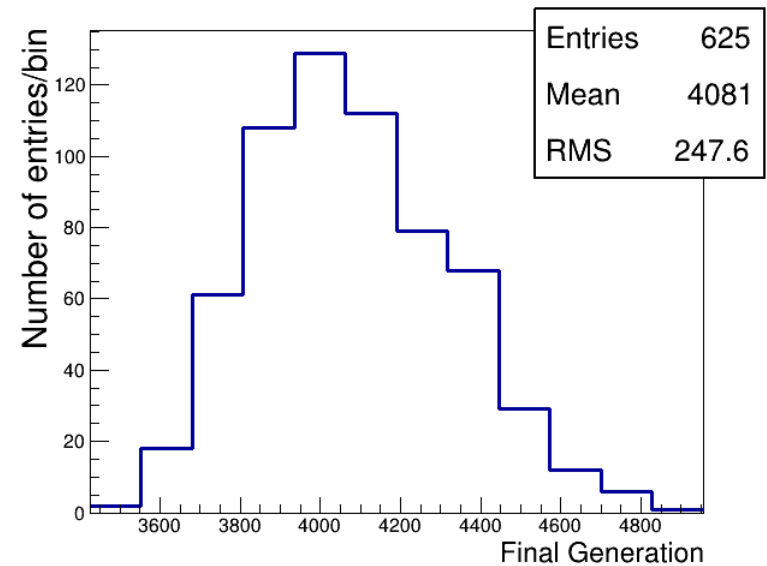
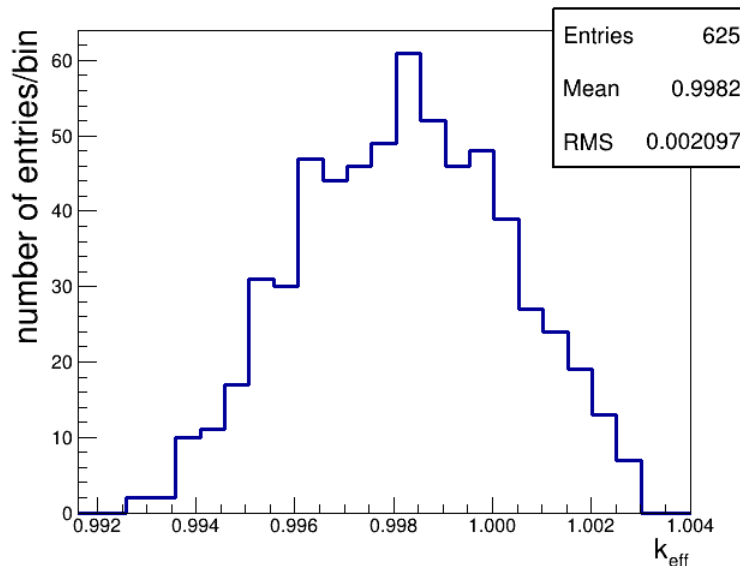
$$\bar{k}_{A/B} = \frac{1}{n} \sum_{i=1}^n k_{A/B,i}$$

$$\sigma_{A/B} = \sqrt{\sum_{i=1}^n (k_{A/B,i} - \bar{k}_{A/B})^2}$$

## Benchmark analysis

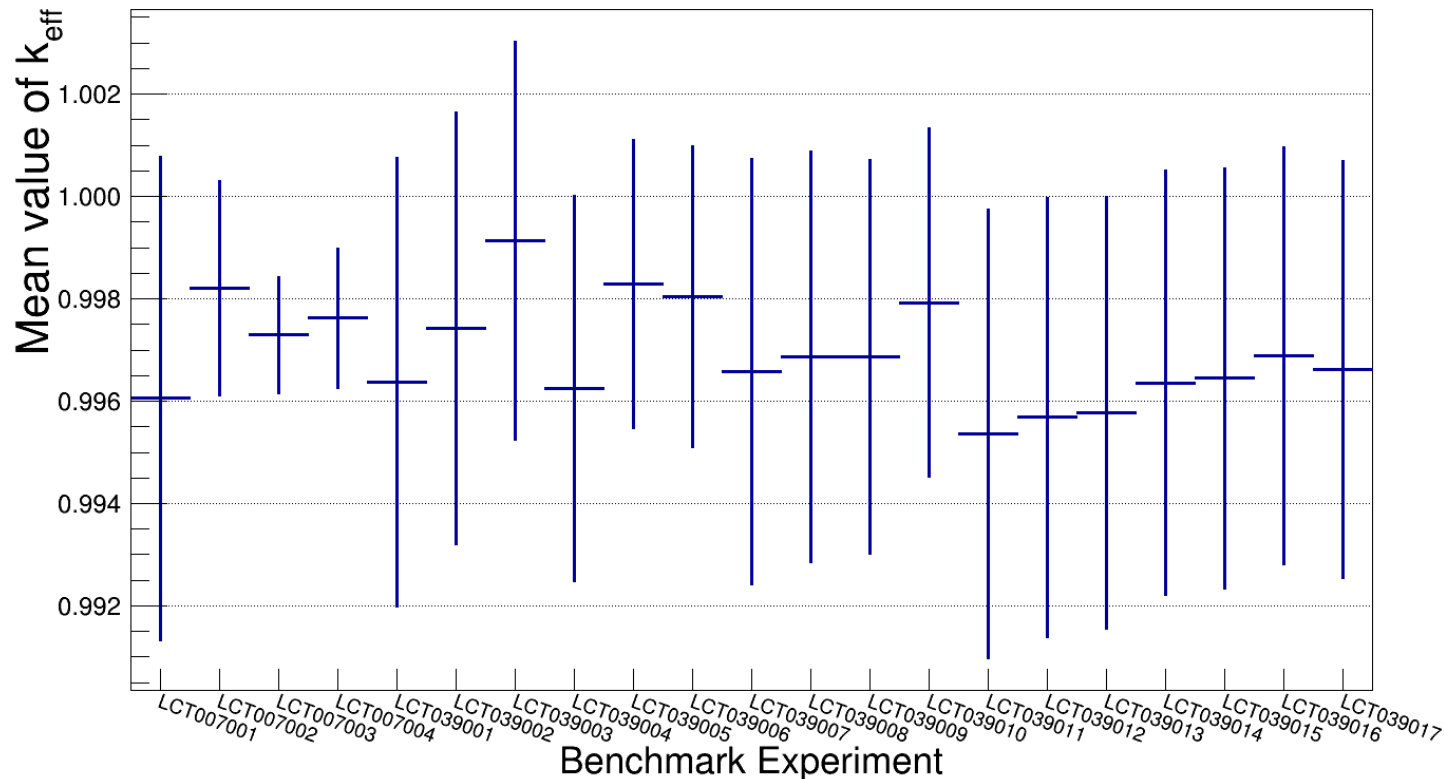
- The criticality calculations are performed with SCALE's CSAS5 sequence of SCALE 6.1.2
- 625 samples were prepared and run for each experiment
- Neutrons per generation: 100k, Convergence criterion: 5.0E-05
- Sufficiently small to be negligible w.r.t. the uncertainty arising from the manufacturing tolerances

Example results of  
LCT-007-002



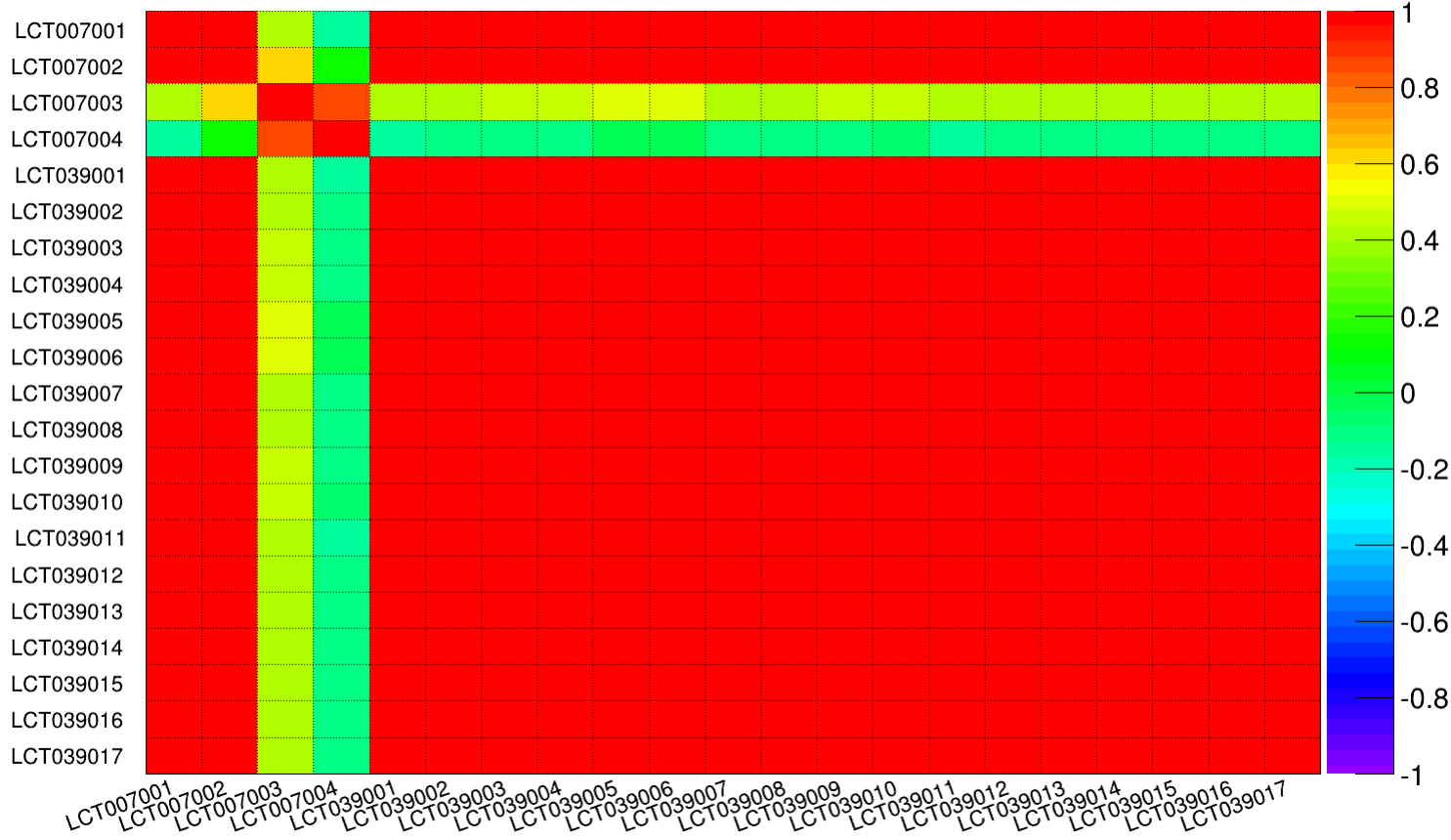
## Results: Mean Value and Standard Deviation

- All mean value below  $k_{\text{eff}} = 1$
- In agreement with the calculations reported in the ICSBEP Handbook
- Standard deviations are between 1.1E-03 and 4.7E-03
- Well above the convergence limit, but in most cases greater than the uncertainties reported in the ICSBEP Handbook (up to factor  $\sim 3$ )



## Correlation Results, Common variation of uncertain parameters

- High correlations up to almost 100% between benchmark experiments
- However, two cases have significantly lower correlation coefficients
- Effect of the pitch and thus of the neutron spectrum



LEU-COMP-THERM-007	
Case number	Pitch [cm]
1	1.26
2	1.60
3	2.10
4	2.52

## Summary & Outlook

- GRS has extended the capabilities of its SUnCISTT to determine correlations between  $k_{\text{eff}}$  results of benchmark experiments
- This new utility was applied to a benchmark proposal of the UACSA Expert Group
- Results indicate:
  - Qualitative correlation information in the DICE database is insufficient for further use
    - LCT-007 and LCT-039 are highly correlated
    - However, w.r.t. correlation coefficients spectral effects of the individual configuration dominate the impact of the common variation of shared technical uncertainties
    - Case-by-case correlation coefficients are required
  - Differences in the uncertainties for the individual analyses compared to the ICSBEP Handbook
- The next step is to take the derived correlation matrices into account and investigate their influence on the bias determination



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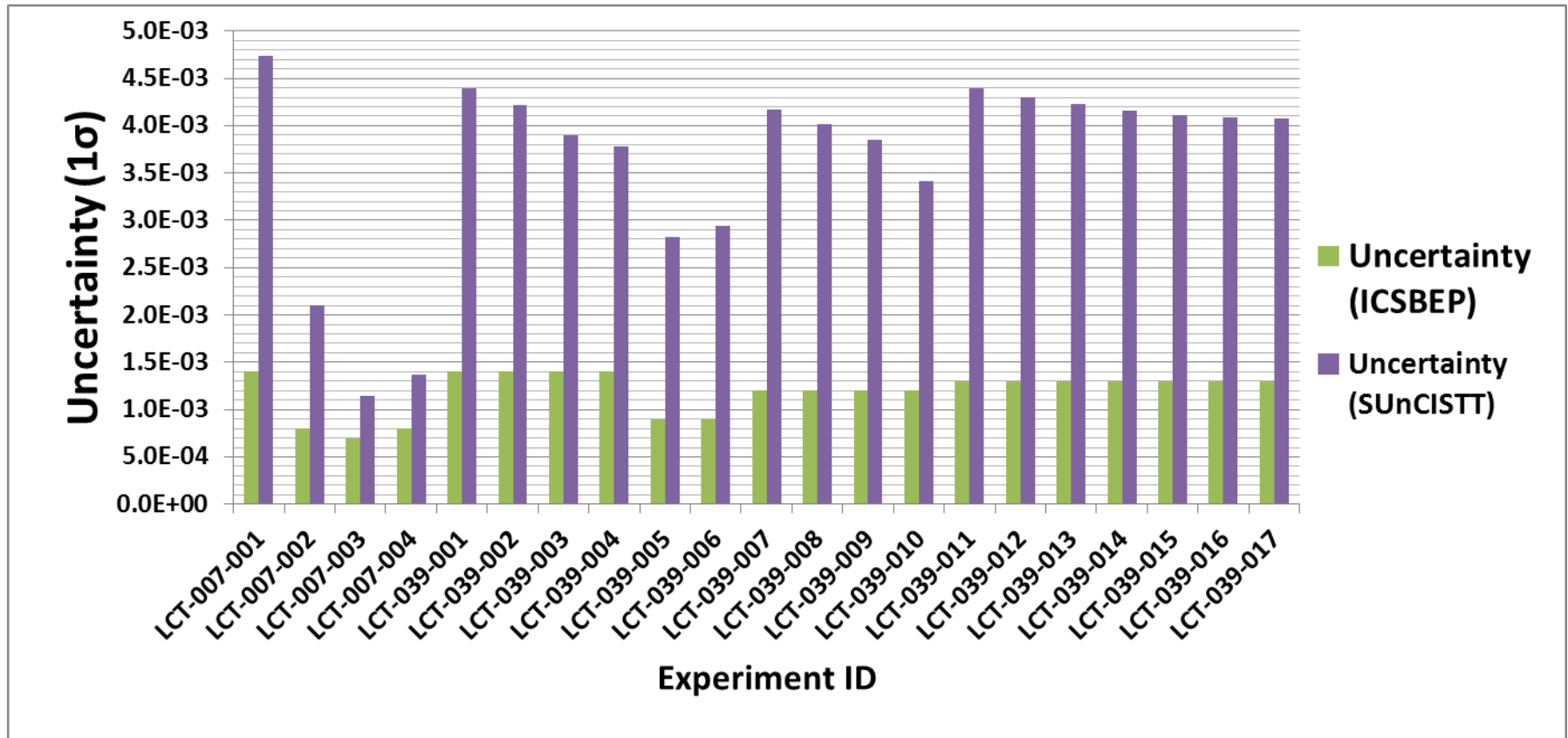
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## Comparison of Uncertainties

- The ICSBEP Handbook states uncertainties for each benchmark experiment
- They have often been used as input to the validation procedure
- However, there is a discrepancy between calculated (SUnCISTT) and the reported (ICSBEP) uncertainty estimates



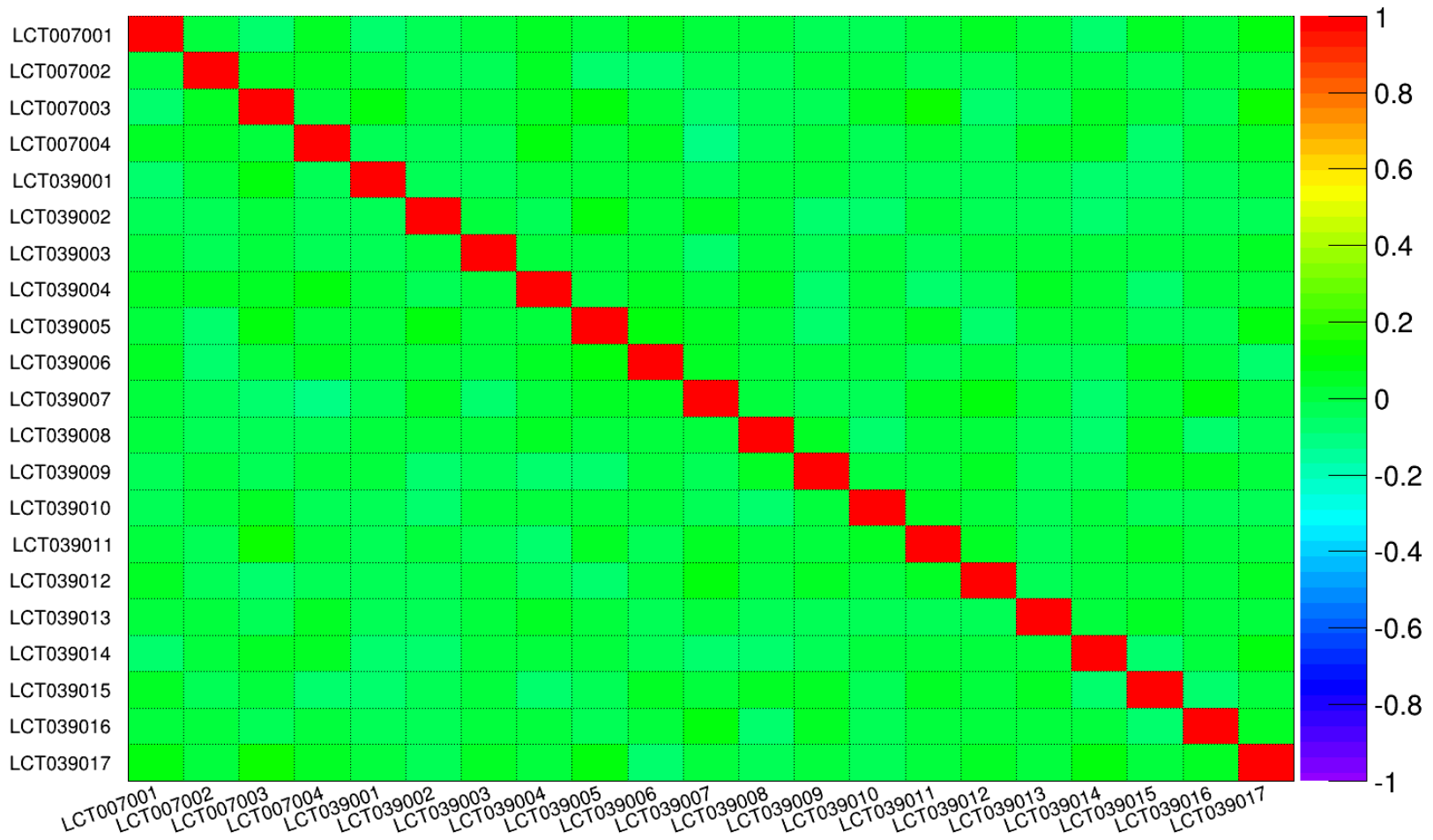
## Publicly Available Correlation Information

- The DICE database contains information about the correlations among benchmark experiments
- However, for most cases – especially those interesting for us – the available information is
  - only qualitatively
  - only for the whole experiment series
- Goal: determine the correlation coefficients between the different cases of the experimental series

	LCT 004	LCT 005	LCT 006	LCT 007	LCT 009	LCT 010	LCT 011
LCT032							
LCT034				+			
LCT035			+				
LCT036							
LCT037				+			
LCT038				+			
LCT039				+			
LCT040				+			
LCT041							
LCT042	+	+			+	+	
LCT043							

## Correlation Results I: Individual Variation

- Definition of correlation coefficient according to Pearson
- Off-diagonal elements are in agreement with statistical fluctuations



## Results for the Trend Parameter EALF

- EALF = Energy of average lethargy causing fission
- Common parameter to describe thermal neutron spectra
- Displays the spectral effect of the increasing fuel rod pitch

