

# SUB-CRITICAL MASS DETERMINATION FOR CYLINDRICAL SYSTEMS OF U-233, U-235 AND PU-239

Criticality Safety Analysis  
Nhu Thuy Tran and Eugene Masala

**NCSD 2013**

Wilmington, NC

“Criticality Safety in the Modern Era: Raising the Bar”



# OUTLINE

- ❖ Introduction
- ❖ Sub-Critical Mass (SCM) Calculation Method
- ❖ Calculations and Results of SCM for Water-Moderated U-233, U-235 and Pu-239 in Storage Configuration
- ❖ Verification of Calculation Method and Discussion of SCMs for Different Geometric Systems
- ❖ Conclusions

# Introduction

- The effective multiplication factor ( $k_{\text{eff}}$ ), the criticality indicator of a system, depends on parameters associated with the materials and geometry of the system.
- Criticality safety can be achieved by controlling one or more parameters of the system within sub-critical limits (SCLs).
- Mass is a key controlled parameter → Determination of SCM (or SCL on mass), especially for U-233, U-235 and Pu-239 (i.e. Special Fissionable Material - SFM ), is an important part of criticality safety analysis.
- Because Pu-239 has been known as the fissile nuclide which has the smallest minimum critical mass (MCM), it is often considered to be the representative fissile nuclide in SCM calculations for SFM mixtures.

## Introduction (Cont)

- However, the results of SCM calculated for a particular storage cylindrical configuration have shown that U-233 is the most reactive fissile nuclide.

$$\text{SCM(U-233)}:\text{SCM(Pu-239)} \approx 1:2$$

*This result is unexpected given that Pu-239 has the smallest MCM.*

- Is the SCM calculation method applied for the storage configuration correct?
- What is the parameter contributing the apparent discrepancy between the SCMs of different geometric systems?

# SCM Calculation Method

- Storage configuration – SFM mixtures (U-233, U-235 and Pu-239), loaded in cylindrical containers of 13.5 cm diameter, and up to seven containers stacked in a cylindrical pipe of 15 cm diameter.
- The way to determine SCM (mass per container) for the storage configuration is:
  - calculate SCM for individual fissile nuclides (U-233, U-235 or Pu-239) using a computer code, and
  - derive SCM for SFM mixtures (U-233, U-235 and Pu-239) from the SCMs of the individual fissile nuclides.
- The SCMs for individual fissile nuclides were calculated in compliance with Canadian Nuclear Safety Commission (CNSC) regulatory requirements for criticality safety.

## SCM Calculation Method (Cont)

- As per the CNSC regulatory requirements, if a computer code is used to calculate  $k_{\text{eff}}$  for a system, an upper subcritical limit (USL) must be established for the calculated  $k_{\text{eff}}$  and the subcriticality of the system must be assessed as follows:

- $k_p + |\Delta k_p| \leq \text{USL}$  - Subcritical
- $k_p + |\Delta k_p| > \text{USL}$  - Possible Critical

$k_p$  = the  $k_{\text{eff}}$  resulted from calculation

$\Delta k_p$  = uncertainty of the  $k_p$  calculation ( $2 \sigma$ )

$\text{USL} = k_c - |\Delta k_c| - |\Delta k_m|$

$k_c$  = the mean  $k_{\text{eff}}$  resulting from calculation of the benchmark criticality experiments (bias)

$\Delta k_c$  = a margin for  $k_c$  bias and bias uncertainty

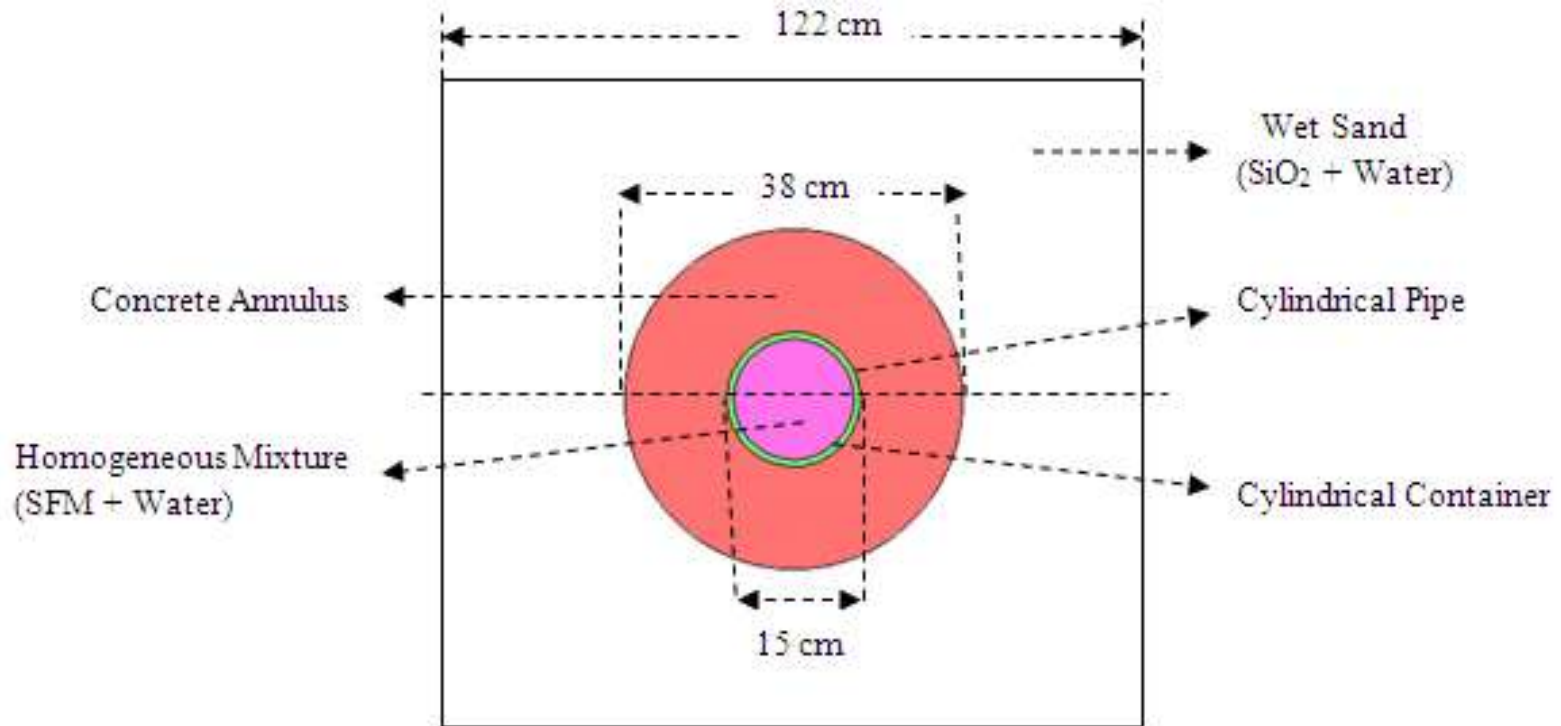
$\Delta k_m$  = an administrative margin of 50 mk

## SCM Calculation Method (Cont)

- The computer code used is the SCALE/KENO V.a with the 238-group ENDF/B-VI neutron-cross-section library. Based on AECL validation of the computer code, the USLs are established as follows:
  - For U-233:  $USL = 0.997 - 0.004 - 0.050 = \mathbf{0.943}$
  - For U-235 (HEU):  $USL = 0.997 - 0.006 - 0.050 = \mathbf{0.941}$
  - For Pu-239:  $USL = 1.003 - 0.004 - 0.050 = \mathbf{0.949}$
- The fissile mass that yields a calculated value of  $(k_p + 2\sigma)$  that is equal to the respective USL is defined as SCM.

$$\mathbf{SCM = Mass @ (k_p + 2\sigma) = USL}$$

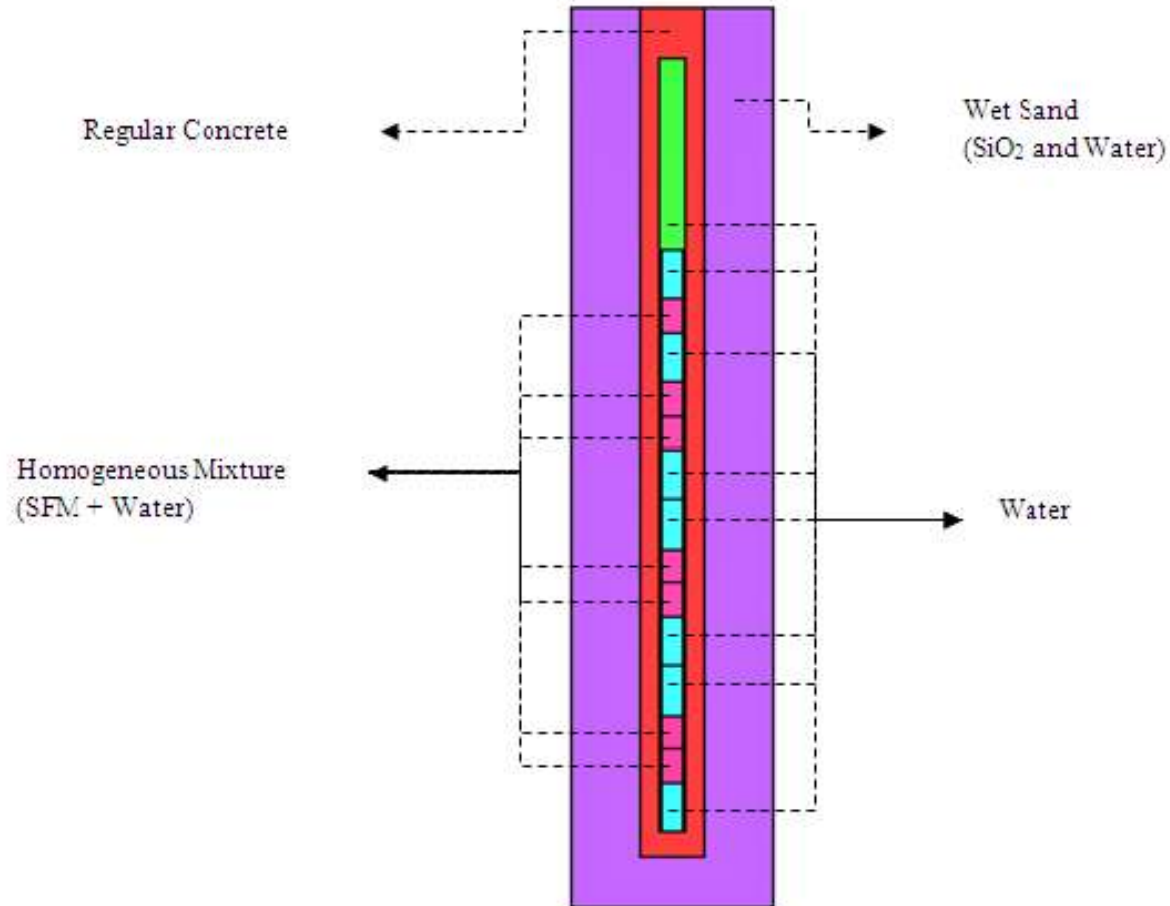
# Calculations and Results of SCM for Water-Moderated SFM in Storage Cylindrical Configuration



**Figure 1 - Plan View of The Storage Model**



# Calculations and Results of SCM for Water-Moderated SFM in Storage Cylindrical Configuration (Cont)

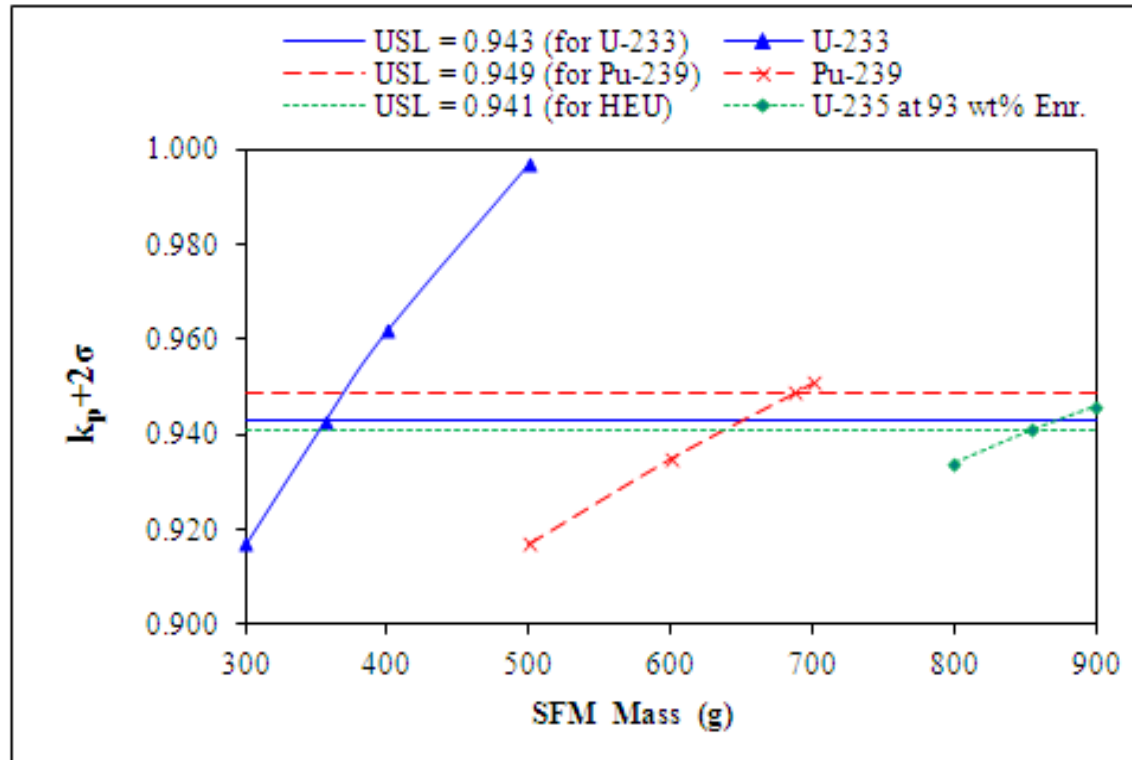


**Figure 2 - Side View of The Storage Model (with Seven Containers of SFM Mixture)**

# Calculations and Results of SCM for Water-Moderated SFM in Storage Cylindrical Configuration (Cont)

- The SCMs for individual fissile nuclides (U-233, U-235 or Pu-239) were determined by carrying out the following steps:
  - For each fissile nuclide, select loading masses.
  - For each selected loading mass, identify the highest  $(k_p+2\sigma)$  value at the optimum moderation by running a set of calculations, in which the fuel mixture diameter is kept constant but the fuel mixture height is varied.
  - From the curve of highest  $(k_p+2\sigma)$  versus loading mass, interpolate the SCM, i.e., the loading mass corresponding to a  $(k_p+2\sigma)$  equal to the respective USL.

# Calculations and Results of SCM for Water-Moderated SFM in Storage Cylindrical Configuration (Cont)



**Figure 3: Highest ( $k_p+2\sigma$ ) vs. Loading Mass for Cylindrical Storage Geometry**

# Calculations and Results of SCM for Water-Moderated SFM in Storage Cylindrical Configuration (Cont)

**Table I. Interpolation of SCM (Mass per Container) for  
Cylindrical Storage Geometry**

U-233 Mass (g)	$k_p+2\sigma$	Pu-239 Mass (g)	$k_p+2\sigma$	U-235 Mass (g)	$k_p+2\sigma$
300	0.917	500	0.917	800	0.934
<b>356</b>	<b>0.943 (USL)</b>	600	0.935	<b>855</b>	<b>0.941 (USL)</b>
400	0.962	<b>688</b>	<b>0.949 (USL)</b>	900	0.946
500	0.997	700	0.951		

**SCM (U-233) = 356 g**

**SCM (Pu-239) = 688 g**

**SCM (U-235) = 855 g**

# Calculations and Results of SCM for Water-Moderated SFM in Storage Cylindrical Configuration (Cont)

- SCM (mass per container) for SFM mixtures can be derived from the SCMs of the individual fissile nuclides by:
  - **using the smallest SCM (U-233) as representative of SFM****OR**
  - **calculating “equivalent” U-235, and using the SCM(U-235)**

“Equiv” U-235 mass = U-235 mass + [SCM(U-235):SCM(U-233)] × U-233 mass  
+ [SCM(U-235):SCM(Pu-239)] × Pu-239 mass

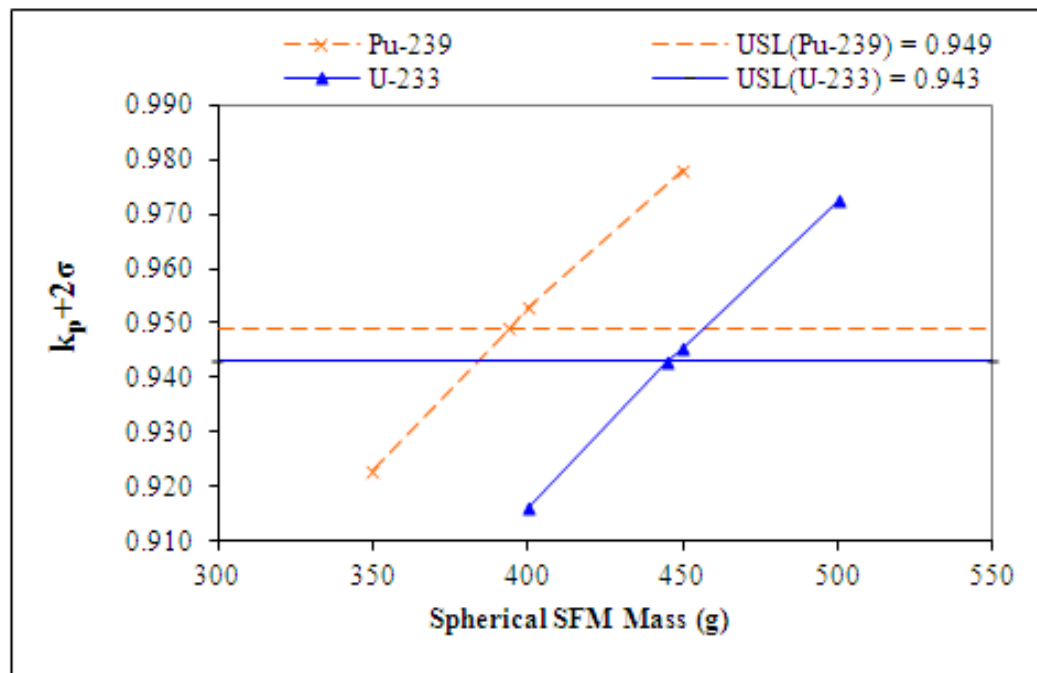
$$\text{SCM(U-235):SCM(U-233)} = 855 \text{ g}:356 \text{ g} \approx 2.40$$

$$\text{SCM(U-235):SCM(Pu-239)} = 855 \text{ g}:688 \text{ g} \approx 1.25$$

“Equiv” U-235 mass = U-235 mass + **2.40** × U-233 mass + **1.25** × Pu-239 mass

# Verification of Calculation Method and Discussion of SCM Values for Different Geometric Systems

- The way to verify the SCM calculation method is determine SCMs for U-233 and Pu-239 in water-moderated spherical systems using the same computer code, and then compare the resulted SCMs to standard data (ANSI/ANS-8.1).



**Figure 4. Highest ( $k_p + 2\sigma$ ) vs. Mass for Spherical Geometry**

# Verification of Calculation Method and Discussion of SCM Values for Different Geometric Systems (Cont)

**Table II. Interpolation of SCM for Spherical Geometry**

U-233 Mass (g)	$k_p+2\sigma$	Pu-239 Mass (g)	$k_p+2\sigma$
400	0.916	350	0.923
<b>445</b>	<b>0.943 (USL)</b>	<b>394</b>	<b>0.949 (USL)</b>
450	0.945	400	0.953
500	0.973	450	0.978

$$\text{SCM (U-233)} = 445 \text{ g}$$

$$\text{SCM (Pu-239)} = 394 \text{ g}$$

- Comparing these results to the SCMs of U-233 and Pu-239 in solutions provided by ANSI/ANS-8.1 [SCM(U-233) = 550 g and SCM(Pu-239) = 480 g], the ratios of SCM(Pu-239):SCM(U-233) are almost the same.

$$\text{SCM(Pu-239):SCM(U-233)} = 394 \text{ g} : 445 \text{ g} \approx 480 \text{ g} : 550 \text{ g} \approx 0.88$$

Thus, the SCM calculation method applied for the storage configuration is correct.

# Verification of Calculation Method and Discussion of SCM Values for Different Geometric Systems (Cont)

- What is the parameter contributing to the apparent discrepancy between the SCMs of the cylindrical storage configuration and those of the spherical systems?
  - *The answer IS density*
- The calculations of the spherical systems show that regardless of the mass, the optimum density is  $\approx 0.06 \text{ g/cm}^3$  for U-233 OR  $\approx 0.03 \text{ g/cm}^3$  for Pu-239
- The spherical diameter corresponding to the optimum density at the SCM value is  $\approx 24 \text{ cm}$  for U-233 OR  $\approx 29 \text{ cm}$  for Pu-239
- These results are corroborated by the SCLs of cylindrical diameter provided in ANSI/ANS-8.1 Standard
  - $\text{SCL(U-233)} = 10.5 \text{ cm}$  [for  $^{233}\text{UO}_2\text{F}_2$ ] or  $11.7 \text{ cm}$  [for  $^{233}\text{UO}_2(\text{NO}_3)_2$ ]
  - $\text{SCL(Pu-239)} = 15.4 \text{ cm}$  [for  $^{239}\text{Pu}(\text{NO}_3)_4$ ]
- Because  $\text{SCL(U-233)} < \text{Storage Container ID} = 13.5 \text{ cm} < \text{SCL(Pu-239)}$ , U-233 could be much more reactive than Pu-239 in the cylindrical storage configuration.



# Conclusions

- The ratios of SCM of the fissile nuclides change with the fissile density and thus the geometric dimension.
  - Though Pu-239 has the smallest MCM, it is not always appropriate to consider Pu-239 as representative for SFM mixtures.
  - Calculations of SCM for SFM mixtures in a specific configuration should be performed individually for each fissile nuclide contained in the mixtures.
  - For the storage configuration, the SFM mass loading can be controlled by:
    - using the smallest SCM (U-233) as representative of SFMOR
    - calculating “equivalent” U-235, and using the SCM( U-235)
- “Equiv” U-235 mass = U-235 mass + **2.40** × U-233 mass + **1.25** × Pu-239 mass

# ACKNOWLEDGMENTS

- AECL staff and colleagues
  - Jeremy Stewart,
  - Colleen Degagne,
  - Jingjing Wang,
  - Laura Blomeley,
  - Fred Adams,
  - Tracy Pearce, and
  - Darren Radford

Thank you!

 **AECL EACL**

