The Use of Density-Law-Invariant Parameters For Criticality Safety Assessment

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Introduction

- Purpose : to assist the end users
- Complexity of Neutron Transport
- Many ways of characterizing a neutron system
- Not all parameters are of the same usefulness in providing physics insights
- Geometry versus Material
- Density-Law-Invariant Parameters

W. R. Stratton on The Density Law

“ This is the density law in criticality physics which simultaneously exact, simple, and useful. In a critical system, if the densities are increased everywhere to x times their initial value and all the linear dimensions are reduced 1/x times their value, the system will remain critical”

Ref: LA-3612, “ Criticality Data and Factors Affecting Criticality of Single Homogeneous Unit”, Sept, 1967

H. Paxton on the Density Law

- “ Criticality dimensions are inversely proportional to the density, provided the density changes are uniform”

Ref: H. Paxton, “ Criticality Control in Operations with Fissile Material”, LA-3366(rev), January, 1972

Review of the Density Law

- Actually, under the density law transformation, not only the new system remains to be critical, the neutron physics remains the same.
- We know the following:

1) Neutron diffusion equation stays invariance under the density law transformation
2) Neutron transport equation stays invariance under the density law transformation

Invariance of Diffusion Theory under The Density Law Transformation

- System I: $\dfrac{d^2\phi}{dx_1^{\,2}} + B_1^{\,2}\phi = 0 \qquad N_i \qquad \sigma_{ai} \qquad etc$
- System II: $\dfrac{d^2\phi}{dx_2^{\,2}} + B_2^{\,2}\phi = 0 \qquad N_i' \qquad \sigma_{ai}' \qquad etc$
- Relation between System I & System II
  1. Spatial parameters
  2. The densities
  3. The material buckling
  4. The diffusion equations

Invariance of Diffusion Theory under The Density Law Transformation

- The spatial parameters

$$x_2 = (g)x_1, \quad dx_2 = (g)dx_1, \quad \frac{d\phi}{dx_2} = \frac{1}{g}\frac{d\phi}{dx_1}, \quad \frac{d^2\phi}{dx_2^{\,2}} = \frac{1}{g^2}\frac{d^2\phi}{dx_1^{\,2}}$$

- The material buckling parameters

$$\because N_i' = \frac{1}{g}N_{i,} \qquad \forall i,$$

$$B_2^{\,2} = \frac{\sum_i \left(\nu_i'\Sigma_{fi}' - \Sigma_{ai}'\right)}{D'} = \frac{\sum_i N_i'\left(\nu_i'\sigma_{fi}' - \sigma_{ai}'\right)}{\frac{1}{3}\sum_i \frac{1}{N_i'\left(\sigma_t' - \mu_0'\sigma_s'\right)}} = \frac{\frac{1}{g}\sum_i N_i\left(\nu_i'\sigma_{fi}' - \sigma_{ai}'\right)}{\frac{g}{3}\sum_i \frac{1}{N_i\left(\sigma_t' - \mu_0'\sigma_s'\right)}} = \frac{1}{g^2}\cdot B_1^{\,2}$$

Invariance of Diffusion Theory under The Density Law Transformation

- The Diffusion Equations are the same with same boundary condition

$$\frac{d^2\phi}{dx_2^{\,2}} + B_2^{\,2}\phi = 0 \quad \rightarrow \frac{1}{g^2}\frac{d^2\phi}{dx_1^{\,2}} + \frac{1}{g^2}B_1^{\,2}\phi = 0 \quad \rightarrow \frac{d^2\phi}{dx_1^{\,2}} + B_1^{\,2}\phi = 0$$

- Similarly, The transport equation is invariant under the density law transformation

Examples of The Density-Law-Invariant Parameters

- The number of the neutron mean free paths
- The surface mass density
- The non-leakage fraction or the leakage fraction
- The normalized system k-eff/k-inf
- The average escape probability

Example 1- Use of Non-leakage Fraction or Leakage Fraction

- For example, it is customary to represent the neutron reproduction factor as follows:

$k_{eff} = k_{inf} * \text{(Nonleakage Fraction)}$

$= k_{inf} / (1+M^2B^2)$

where

M^2 is the migration area

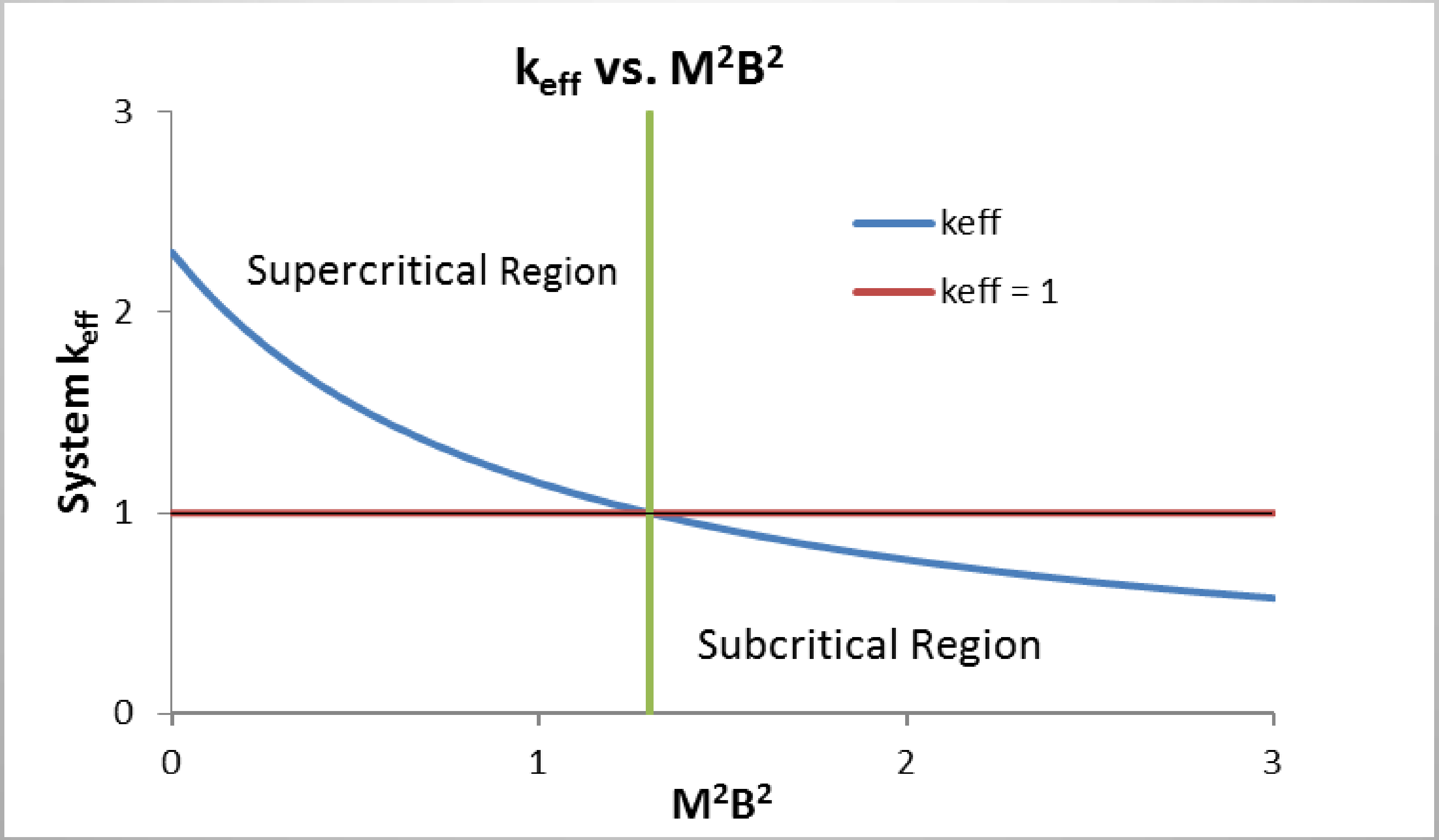
B^2 is the geometric buckling

M^2B^2 is conserved under the density law transformation. So is the non-leakage fraction or leakage fraction

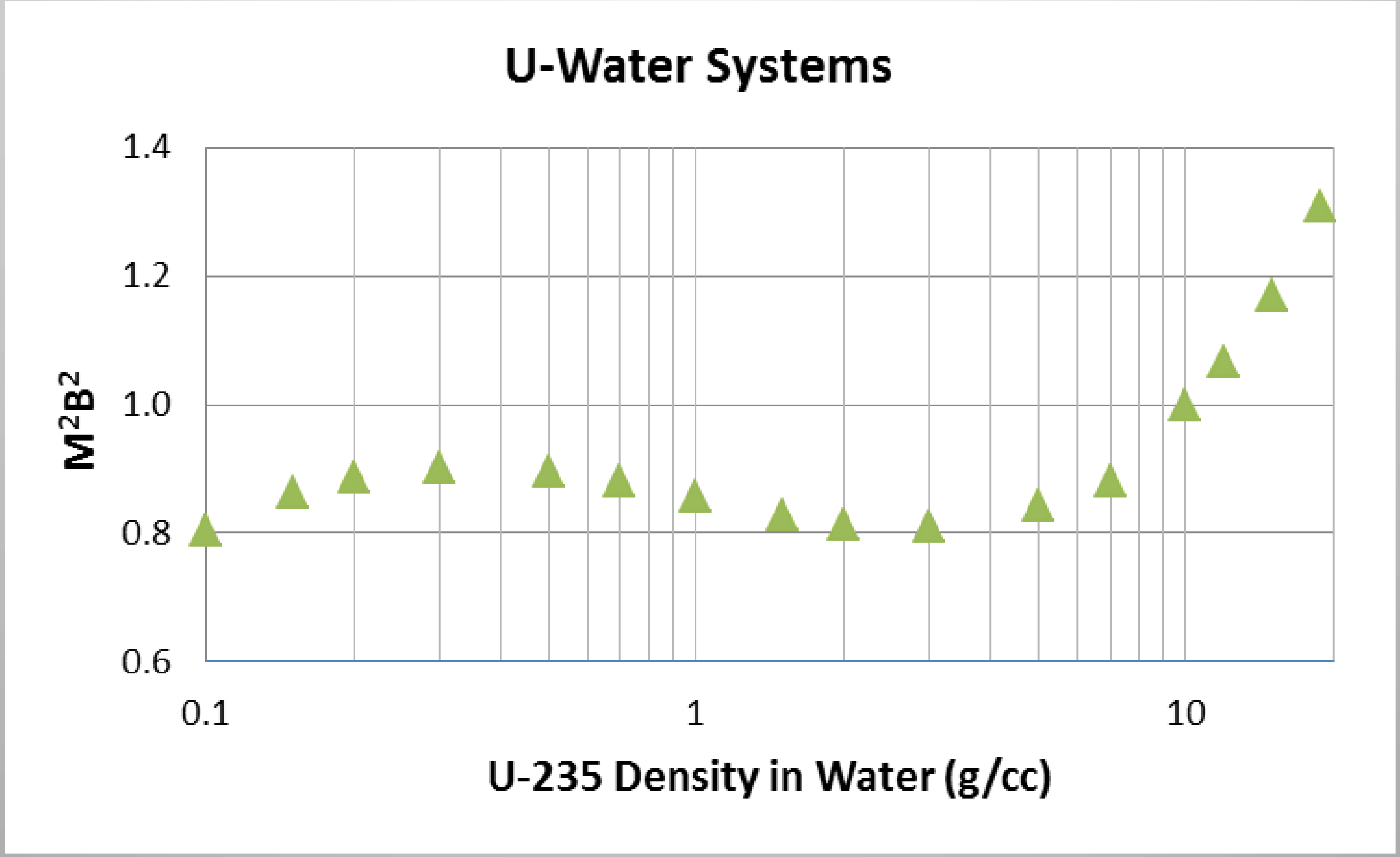
Example 1- Use of Non-leakage Fraction or Leakage Fraction (Continued)

- $M^2B^2 = k_{inf} - 1$ → Critical
- $M^2B^2 < k_{inf} - 1$ → Supercritical
- $M^2B^2 > k_{inf} - 1$ → Subcritical

Example 1- Use of Non-leakage Fraction or Leakage Fraction (Continued)



Example 1- Use of Non-leakage Fraction or Leakage Fraction (Continued)



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Example 2- Use of Number of Neutron Mean Free Paths

- For example, the average escape probability P_0 for the sphere with radius a and the mean free path length l is, per the Dirac chord method,

$$P_0 = \left(\frac{3}{8 \cdot \left(\frac{a}{l}\right)^3}\right) \cdot \left(2 \cdot \left(\frac{a}{l}\right)^2 - 1 + \left(1 + \frac{2a}{l}\right) \cdot e^{-\frac{2a}{l}}\right)$$

- Both the number of the mean free paths and the average escape probability P_0 are conserved under the density law transformation.

Example 2- Use of Number of Neutron Mean Free Paths (Continued)

a/l (no. of mfp)	1	2	3	4	5
P_0 (Average escape Probability)	0.52	0.33	0.23	0.18	0.15

Concluding Remarks

- The density law offers a few physics insights to the neutron transport process.
- The use of parameters which are invariant under the density laws offers an interesting way of looking at criticality safety issues.
- An end user may want to include some of the density-law-invariant parameters as tools in the tool box for criticality safety assessment.