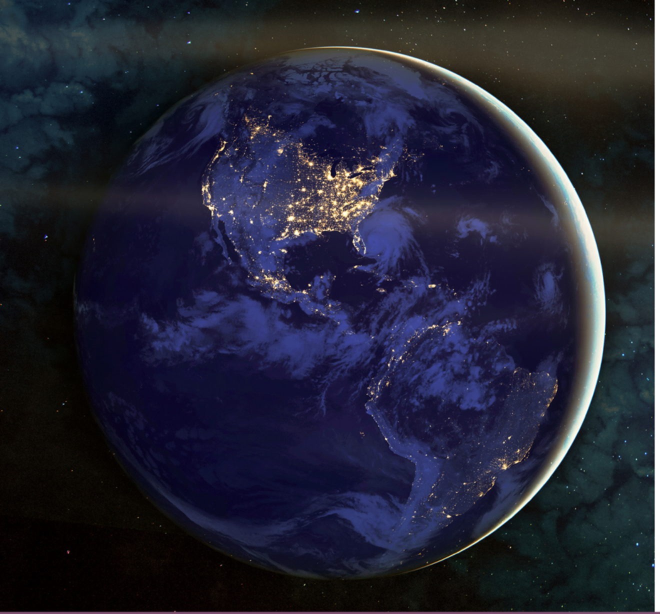

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Subcriticality Measurement using Feynman-α with a Fully Random Sampling and Second-Order Filtering Technique for AGN-201K

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CONTENTS

INTRODUCTION

- Most nuclear facilities are designed to have conservative subcritical margin to prevent accidentally uncontrolled neutron multiplications.
- Therefore, an accurate real-time measurement of subcriticality can provide a helpful way to guarantee the safe operation of nuclear facilities.
- Noise analysis methods have been studied for a long time for this purpose.
- In this work, subcriticality experiment is performed with the Feynman-α method at AGN-201K which is zero-power research and training reactor in our country.
- To reduce computing time and for improve accuracy near five critical states in estimating the prompt neutron decay constant, a fully random sampling technique coupled with the second order differential filtering is devised to effectively process the data obtained with a fine gate time within reduced computing time.

THEORY AND METHOD

❖ *Noise analsysis method*

- Noise analsysis method are based on the same basic premise that the properites of a subcritical system can be determined by measuring the fluctuations in the fission chain processes that depend on the stochastic nature of the birth and death of neutrons.
- So, if the time of the source or detection event are measurable, the distribution event of the times between the source (or detection) event and detection event would proved a direct indication of the dynamic properties of the subcritical system.

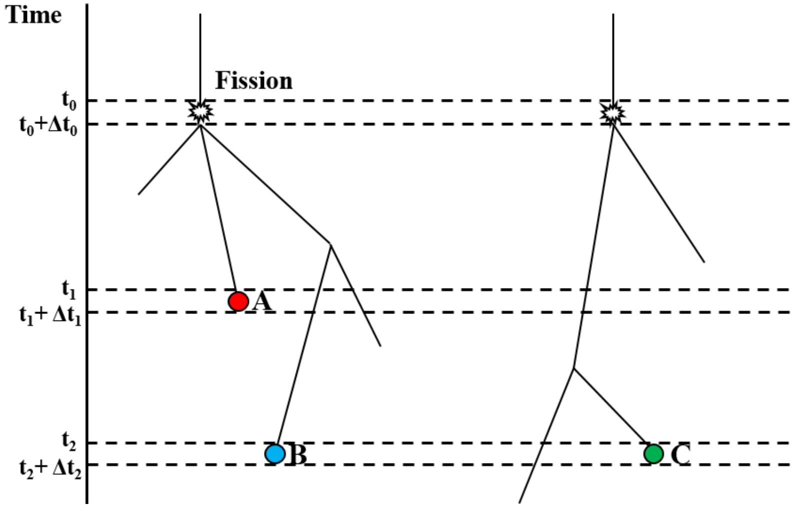


Fig. 1 Random branching process of fission neutron

THEORY AND METHOD

Feynman-α method

- **The Feynman-α method can be derived from the Rossi-α method. This method can determine the prompt neutron decay constant (α) by considering the ratio of the variance to the mean of neutron counts collected in a fixed time interval (i.e., gate time).**

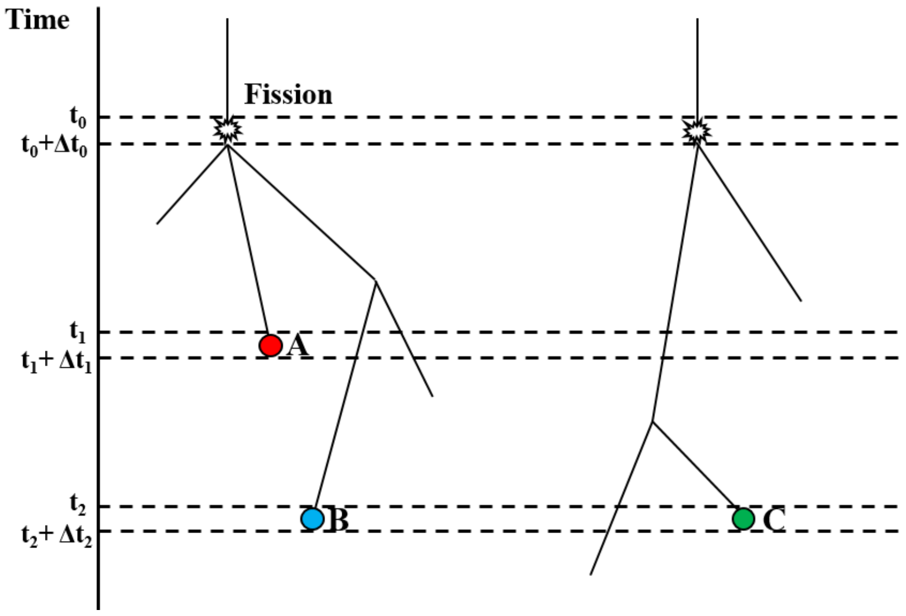


Fig. 1 Random branching process of fission neutron

Rossi-α method

$$p(t_1,t_2)dt_1dt_2 = F\varepsilon^2\left(F+\frac{D_v k_p^2}{2(1-k_p)\Lambda}e^{-\alpha(t_2-t_1)}\right)dt_1dt_2$$

Integration of it over time

$$\int_{t_2=0}^{\tau}\int_{t_1=0}^{t_2} p(t_1,t_2)dt_1dt_2 = 1+\frac{\varepsilon D_v}{\rho_p^2}\left(1-\frac{1-e^{-\alpha\tau}}{\alpha\tau}\right); \quad \rho_p=\frac{k_p-1}{k_p}$$

Feynman-α method

$$Y(\tau)=\frac{\overline{C_k^2}-(\overline{C_k})^2}{\overline{C_k}}-1=Y_\infty\left[1-\frac{1-e^{-\alpha\tau}}{\alpha\tau}\right]$$

THEORY AND METHOD

❖ *Feynman-α method*

$$Y(\tau)=\frac{\overline{C_k^2}-(\overline{C_k})^2}{\overline{C_k}}-1=Y_\infty\left[1-\frac{1-e^{-\alpha\tau}}{\alpha\tau}\right],$$

- **where Y is defined as the variance-to-mean ratio of a series of neutron counts (C_k) with a gate time τ subtracted by 1. The saturated correlation amplitude Y_∞ includes detector efficiency ε, Diven's factor D_ν and prompt reactivity ρ_p.**

$$Y(\tau)=\frac{\overline{C_k^2}-(\overline{C_k})^2}{\overline{C_k}}-1=Y_\infty\left[1-\frac{1-e^{-\alpha\tau}}{\alpha\tau}\right] \Rightarrow$$

Fitting Curve = $1+Y_\infty\left[1-\dfrac{1-e^{-\alpha\tau}}{\alpha\tau}\right]$

$$Y=\frac{\overline{C_k^2}-(\overline{C_k})^2}{\overline{C_k}}$$

$$\alpha=\frac{\beta-\rho}{\Lambda} \Rightarrow \boxed{k_{eff}=\frac{1}{1-\beta+\alpha\Lambda}} \xleftarrow{\alpha}$$

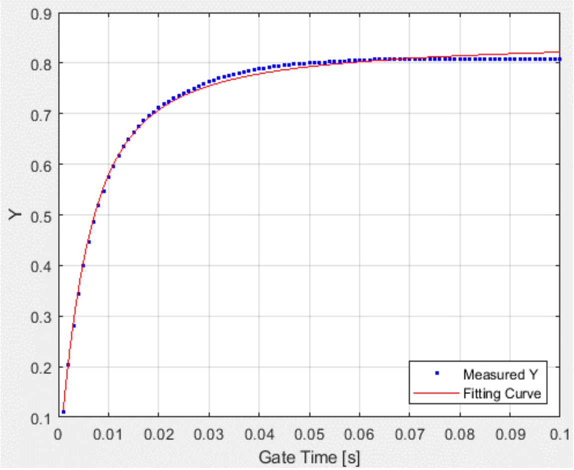


Fig. 2 Determination of prompt neutron decay constant by using Feynman-α fitting curve

THEORY AND METHOD

2nd order Feynman-α differential filtering method

- However, the conventional Feynman-α method suffers from the divergence of the variance near the critical state.
- To circumvent the divergence of the variance, Bennett (1960) proposed an improved method with differences of the counts between adjacent gates.
- Hashimoto et al. (1997) generalized the Bennett's method to develop a difference-filtering technique and proposed a usage of the higher-order filtering for Feynman-α method to reduce the effect of reactor-power drift during a measurement.

$$\sigma_2(\tau) = \overline{\left(C_k - \frac{1}{2} C_{k-1} - \frac{1}{2} C_{k+1} \right)^2} \Big/ \overline{C_k} - \frac{3}{2}$$

$$= Y_\infty \left(1 - \frac{\frac{5}{3} - \frac{5}{2} e^{-\alpha\tau} + e^{-2\alpha\tau} - \frac{1}{6} e^{-3\alpha\tau}}{\alpha\tau} \right)$$

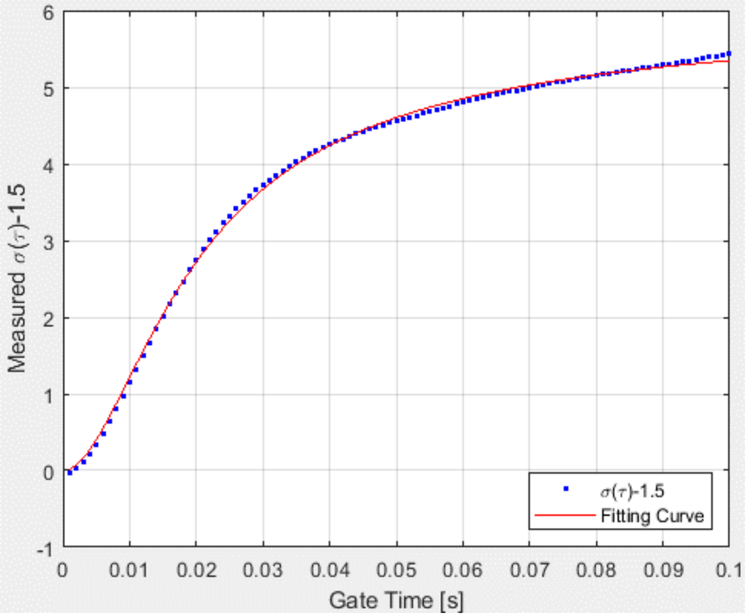
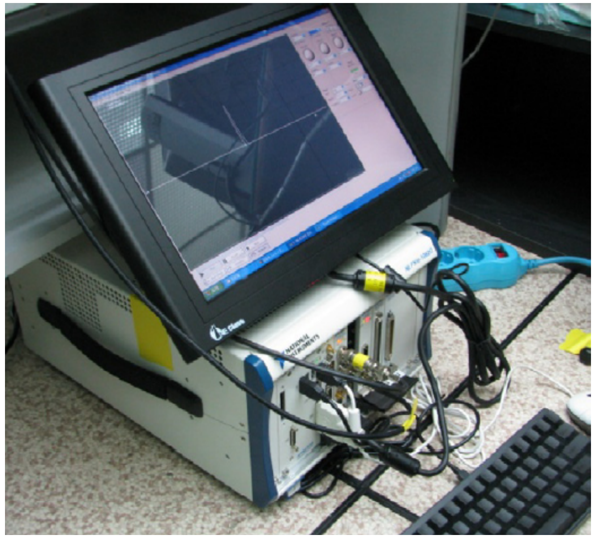


Fig. 3 2nd order Feynman fitting for measured σ_2

THEORY AND METHOD

❖ *Subcriticality Measurement System (SMS)*

- In this study, a time-series data of neutron counts within a **fine unit gate time of 10 μsec** is acquired using the SMS which was developed by Korea Electric Power Research Institute (KEPRI) for measureing the ex-core detector signal from commercial PWR to get the condition of large subcriticality.
- Since the neutron generation time (Λ) is estimated about 50~60 μsec, **the shorter gate time can acquire more detailed information for estimating α value.**



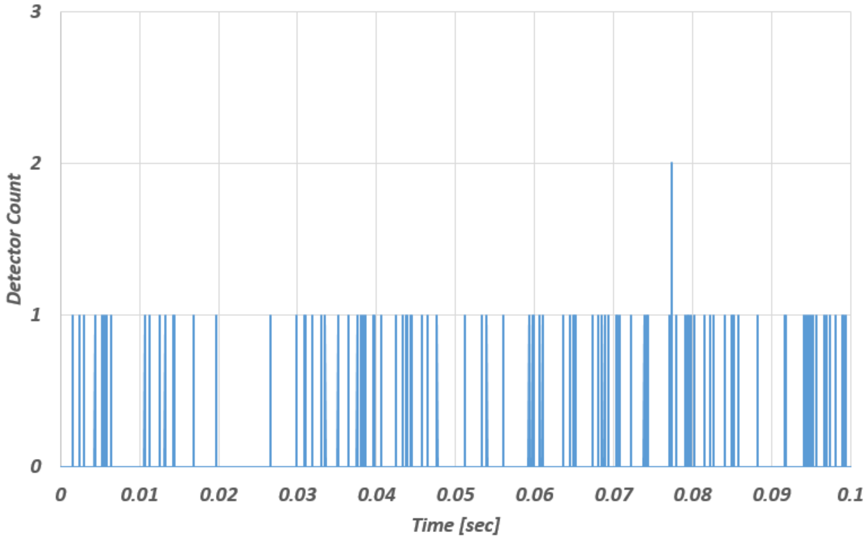


Fig. 4 Subcriticality Measurement System (SMS) and time series data of neutron counts for 0.1 sec

THEORY AND METHOD

❖ *Data Processing with a Fully Random Sampling*

- In general, the Feynman-α method requires sufficient number of measurement data for the reliable accuracy of curve fitting.
- A method called "Bunching-technique (time-swap)" increases the number of data by using shifted data even for long gate times.
- However, those method have some disadvantages that the number of the processing data is too big with a fine gate time, which drastically increases computing time.

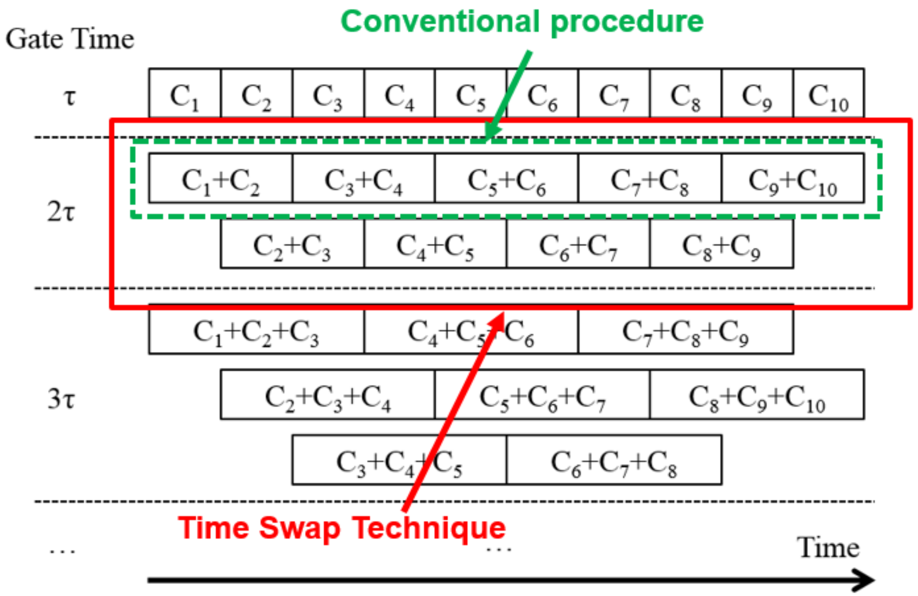


Fig. 5 Schematic view of the bunching-technique

THEORY AND METHOD

❖ *Data Processing with a Fully Random Sampling*

- **In this work, a simple efficient fully random sampling technique is suggested to overcome these drawbacks.**
- **In this method, for a given gate time, a given number of starting time points are randomly sampled over the whole data range and then the consecutive count data within the gate time for each sampled starting time bin : C_k.**

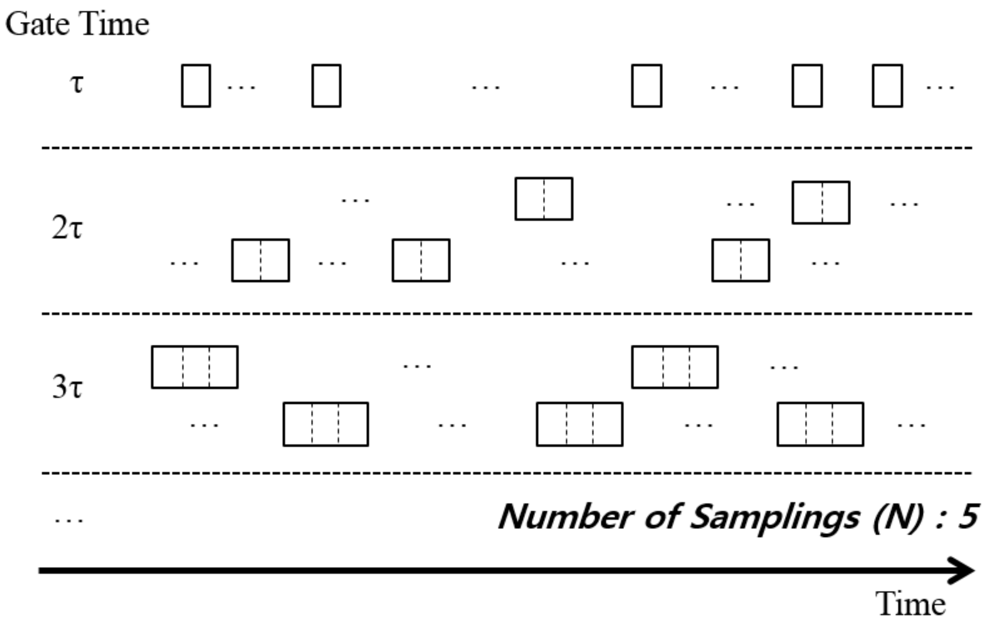


Fig. 6 Schematic view of the fully random sampling technique

THEORY AND METHOD

❖ *Data Processing with a Fully Random Sampling*

- The only inputs to be specified are the length of gate times (or number of gate times) and the number of the random samplings for each time bin.
- As the number of sampling data increases, the measured Y or σ_2 approaches a single value and dispersion decreases.

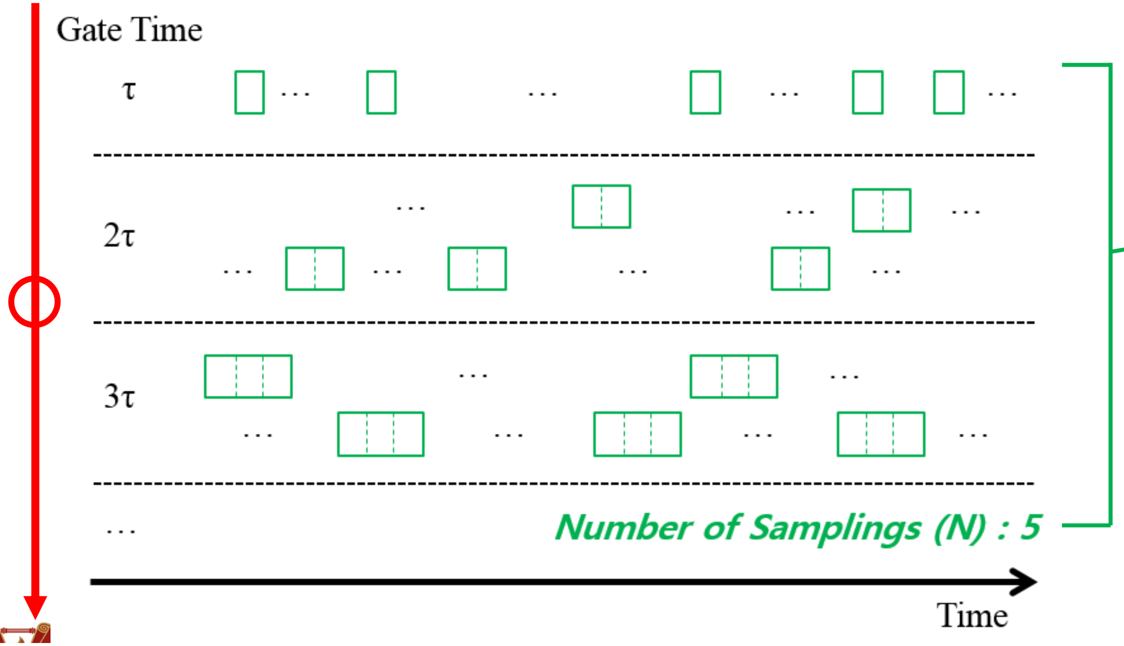


Fig. 6 Schematic view of the fully random sampling technique

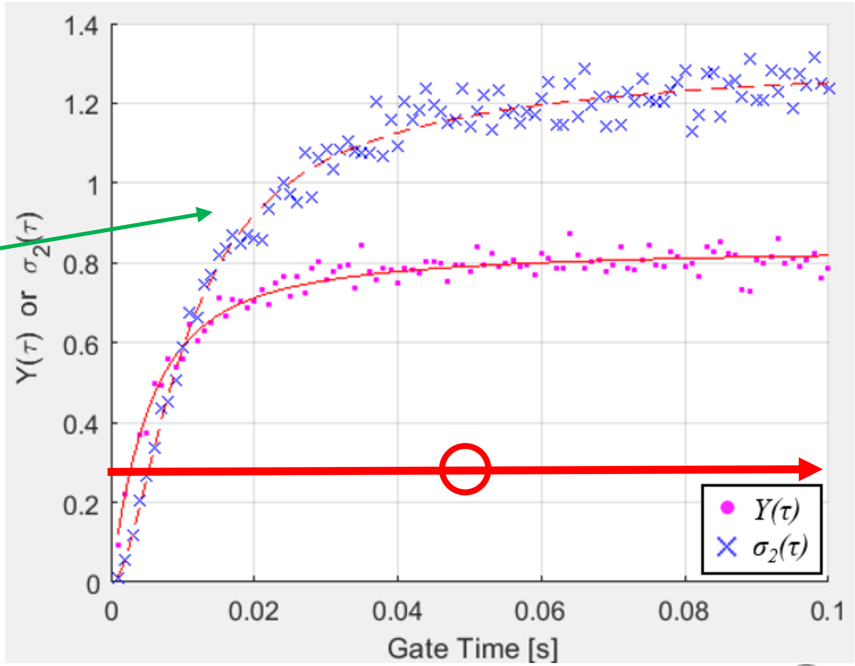


Fig. 7 Feynman fitting using random sampling
(# of gate time times : 100, # of samplings : 10 000)

AGN-201K AND SUBCRITICAL STATES

❖ *AGN-201K*

- AGN-201K is a zero-power research and training reactor built by Aerojet General Nucleonics (AGN).
- It is solid moderated reactor using polyethylene and licensed maximum power is 10 Watt.
- The fuel is a homogeneous mixture of UO_2 and polyethylene.
- The fuel is comprised of 10 disks with 12.8 cm radius and 25 cm active core height.
- Uranium enrichment of the fuel is about 19.5 w/o.
- The active core is surrounded by 25 cm thick graphite reflector followed by a 10 cm thick lead gamma shield.

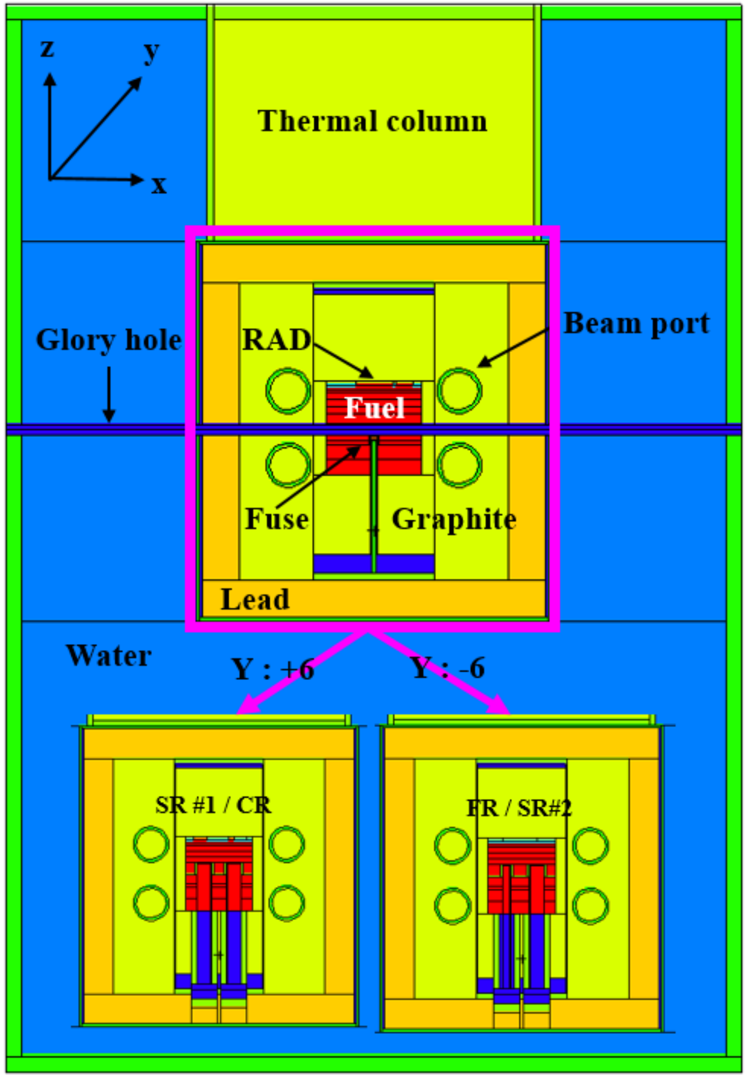


Fig. 8 Axial configuration of the AGN-201K

AGN-201K AND SUBCRITICAL STATES

❖ *AGN-201K*

- For fast neutron shielding, the outside of the core tank is filled with water of ~47.5 cm thickness.
- The control rod consists of 2 Safety Rods (SR), 1 Coarse Rod (CR), and 1 Fine Rod (FR) that have the same composition as the fuel material.
- During operation reactor power is controlled by CR and FR.
- In particular, an external Ra-Be source located in the left upper beam port supplies neutrons with an intensity of 10 mCi.
- A He-3 ex-core detector conneted with SMS is located in right-lower beam port

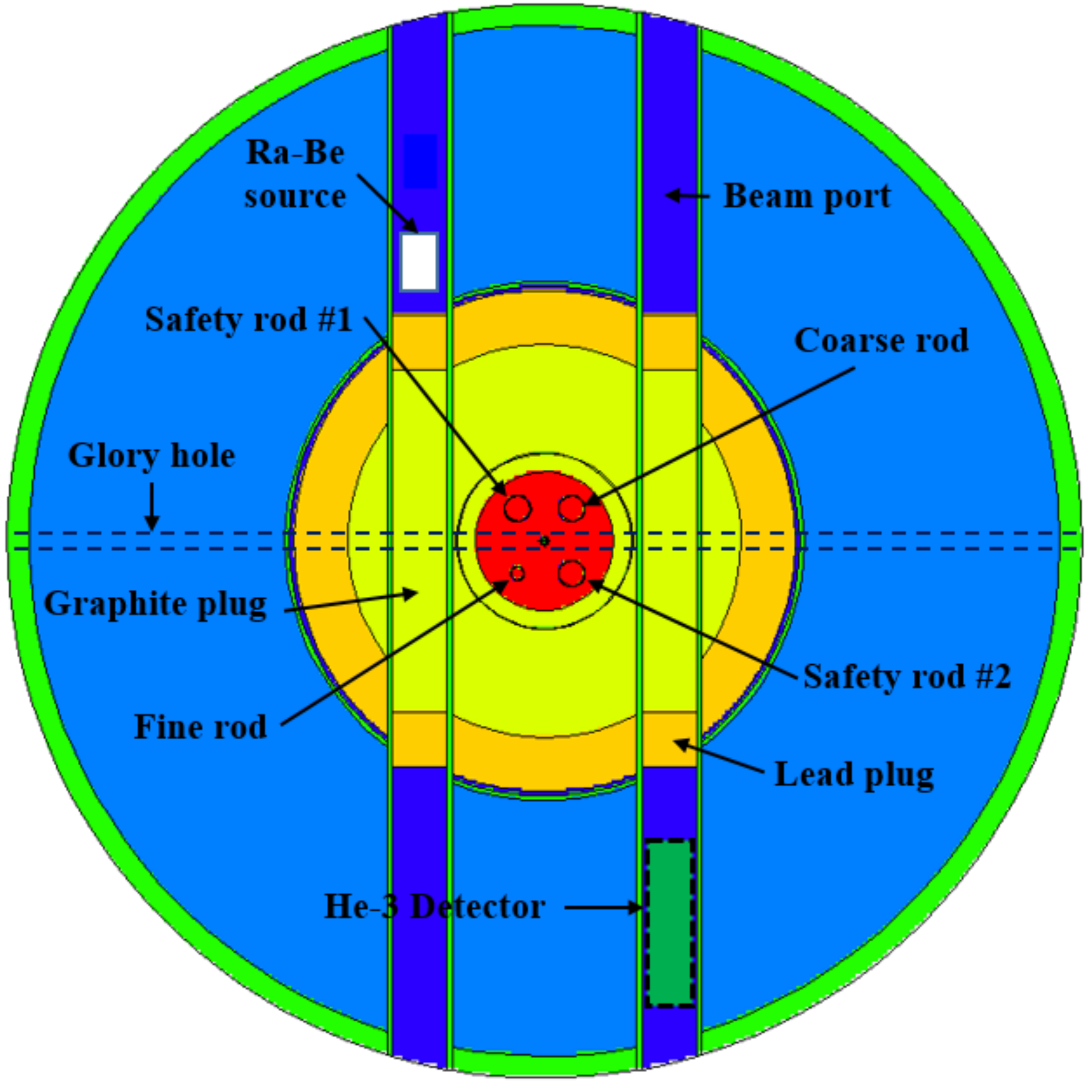


Fig. 10 Radial configuration of the AGN-201K

AGN-201K AND SUBCRITICAL STATES

❖ *Selected Subcritical States*

- **Before the subcriticality measurement, five Sub-Critical condition (SCR) are determined.**
- **By using the MCNP6 eigenvalue calculations, we obtain the reference k_{eff} values and kinetic parameters.**
- **The MCNP6 eigenvalue calculations are performed with ENDF/B-VII.1 cross sections, and with 100 inactive and 5 000 active cycles of 100 000 histories to minimize the statistical error of k_{eff} and kinetic parameters.**

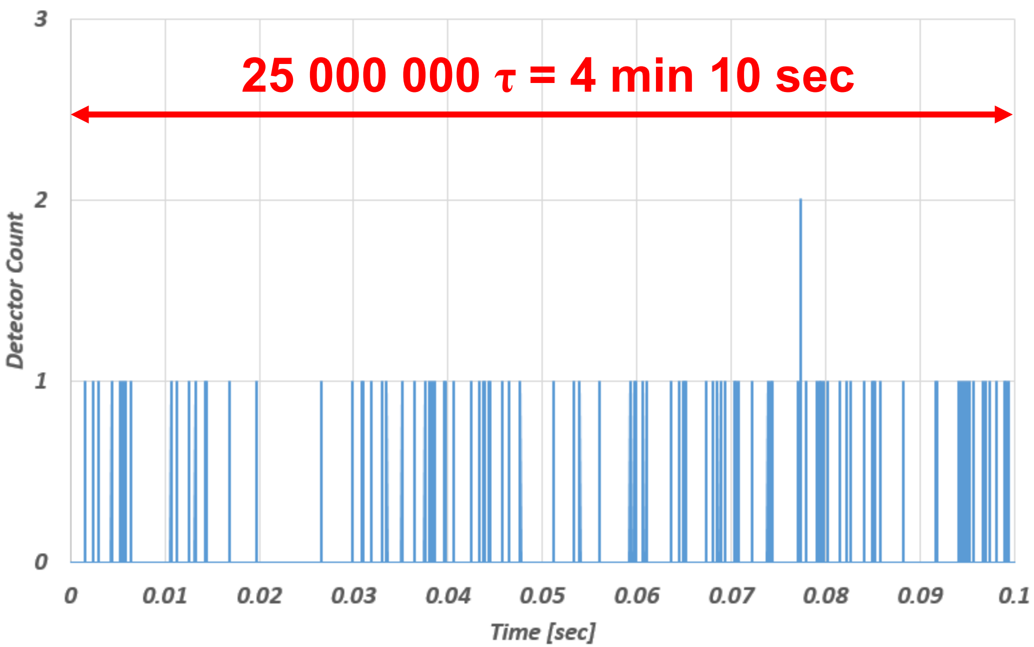
TABLE I. Reference multiplication factors and kinetic parameters estimated with MCNP6

Condition	k_{eff}	σ (pcm)	β_{eff}	Λ (μsec)	Inserted rod position (cm)			
					SR#1	SR#2	CR	FR
SCR1	0.98764	3	0.00755	55.89873	23.07	23.44	0	12.56
SCR2	0.99668	3	0.00761	54.55938	23.07	23.44	17.25	12.56
SCR3	0.99737	3	0.00746	54.27638	23.07	23.44	18.25	12.56
SCR4	0.99811	3	0.00757	53.91283	23.07	23.44	19.25	12.56
SCR5	0.99885	3	0.00763	54.00546	23.07	23.44	20.25	12.56

RESULTS AND DISCUSSION

❖ *Experiment Condition*

- A series of neutron count was obtained for 5 subcritical conditions by using SMS with a unit gate time of 10 μsec during 4 minutes to 5 miniutes.
- The number of time bins considered was 25 million counts (i.e., 25 000 000 τ, τ = 10 μsec).
- For curve fitting, the length of gate time was considered up to 0.1 sec (i.e., 10 000 τ).



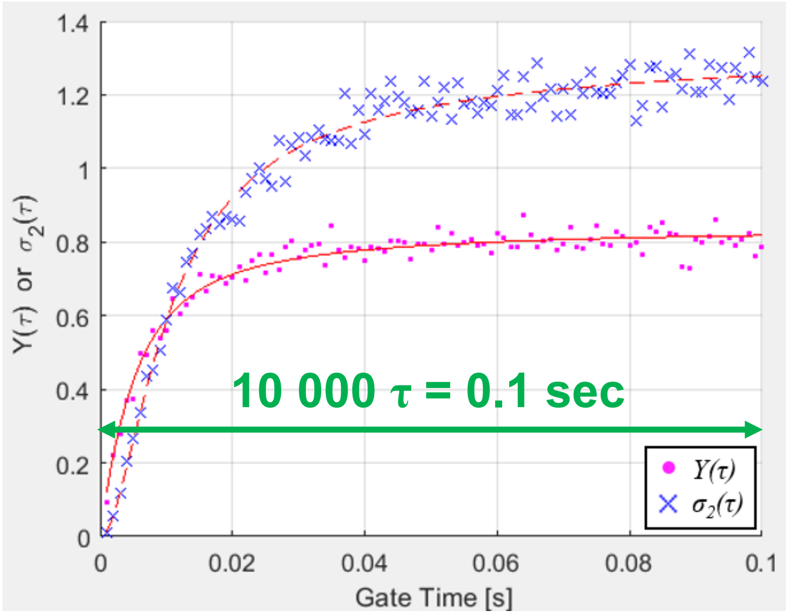
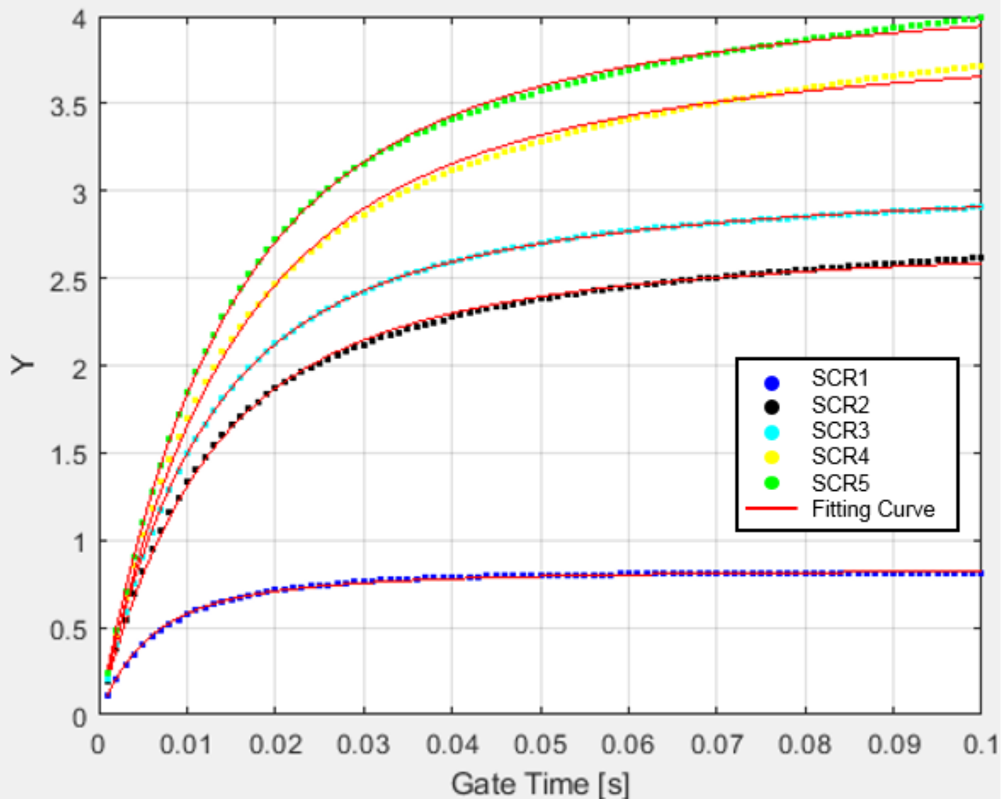


Fig. 11 Experiment condition for measuring subcriticality

RESULTS AND DISCUSSION

❖ *Feynman-α method*

- **Fig. 12 shows the reference k_{eff} and α value (α-PKE) and difference between reference value and estimated value.**



Technique			Time-swap	
# of gate times			100	
# of samplings			whole data	
Condition	k_{eff}	α-PKE	k-est	α-est
SCR1	0.98764	358.95	0.99103	296.99
			[a] -338.98	[b] 61.96
SCR2	0.99668	200.53	0.99976	143.92
			-307.80	56.61
SCR3	0.99737	186.03	0.99940	148.51
			-202.96	37.52
SCR4	0.99811	175.53	1.00120	118.13
			-309.26	57.40
SCR5	0.99885	162.6	1.00092	124.19
			-207.39	38.41
Average CPU time (sec)			108479	

[a] [(k_{eff}) – (k-est)] (pcm Δk), [b] [(α-PKE) – (α-est)] (1/s)

Fig. 12 Feynman fitting for five subcritical states using bunching-technique

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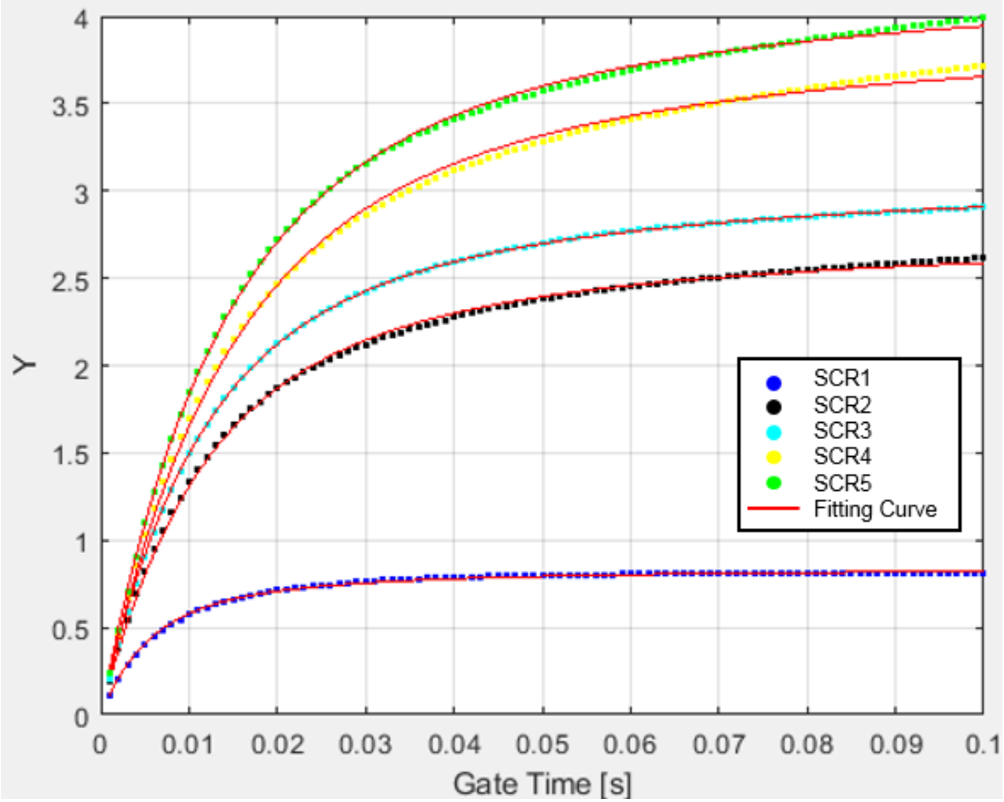
RESULTS AND DISCUSSION

❖ *Feynman-α method*

- Feynman-α method with time swap gives accurate k_{eff} results less than 340 pcm, but long computing times.



Technique			Time-swap	
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# of samplings			whole data	
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			-207.39	38.41
Average CPU time (sec)			108479	

[a] [(k_{eff}) – (k-est)] (pcm Δk), [b] [(α-PKE) – (α-est)] (1/s)

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Fig. 12 Feynman fitting for five subcritical states using bunching-technique

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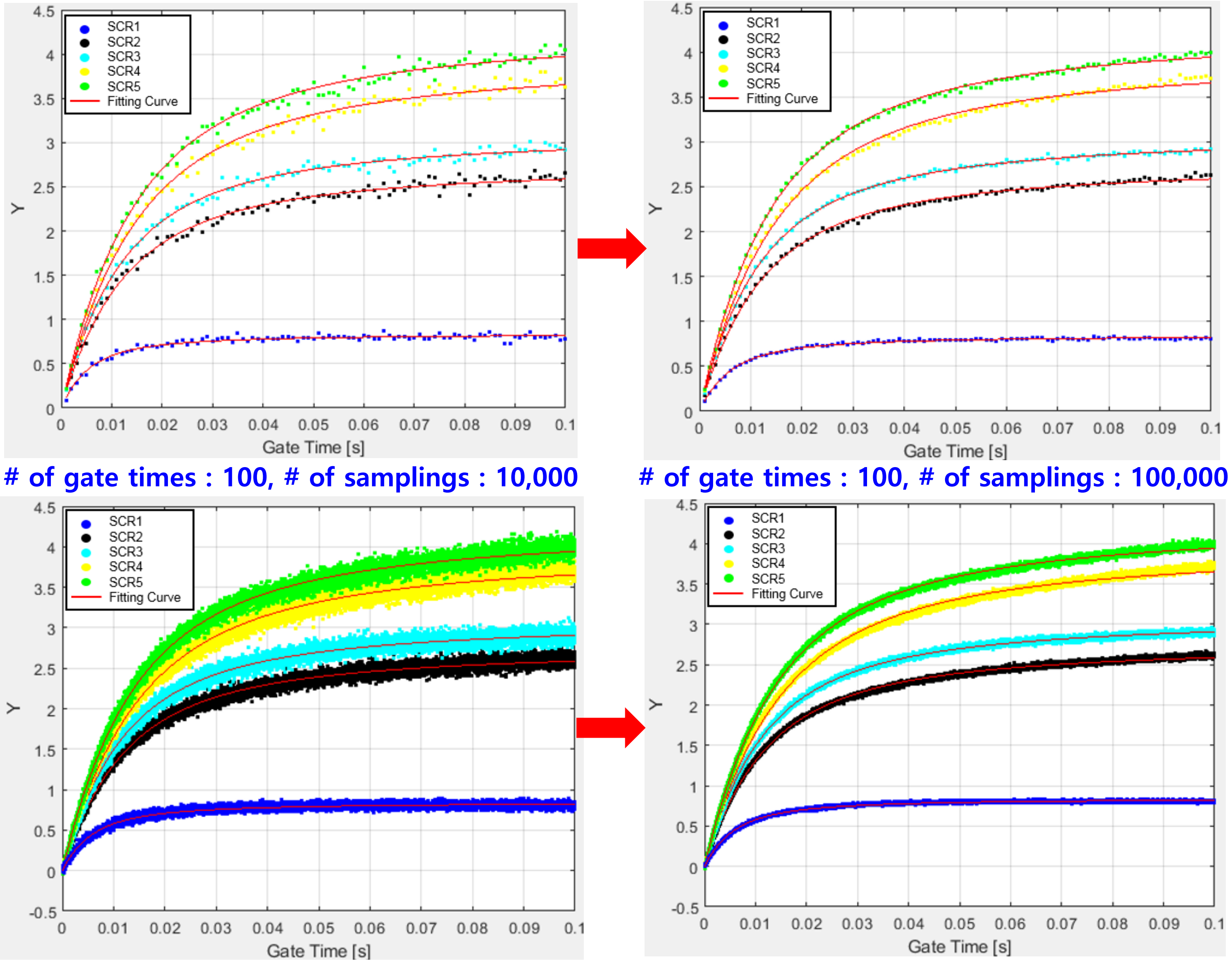
RESULTS AND DISCUSSION

❖ *Feynman-α method*

- The largest subcritical state SCR1 shows the lowest slope of the fitting curve.
- As shown in Fig. 13, as the number of sampling increases, the measured *Y* value converges toward a specific value.



# of gate times : 100, # of samplings : 10,000

# of gate times : 100, # of samplings : 100,000

# of gate times : 10,000, # of samplings : 10,000

# of gate times : 10,000, # of samplings : 100,000

Fig. 13 Feynman fitting for five subcritical states using fully random sampling tehcnique

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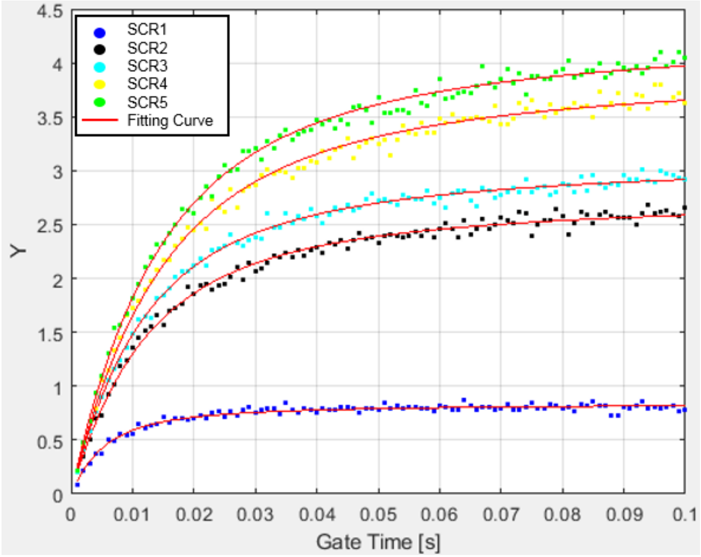
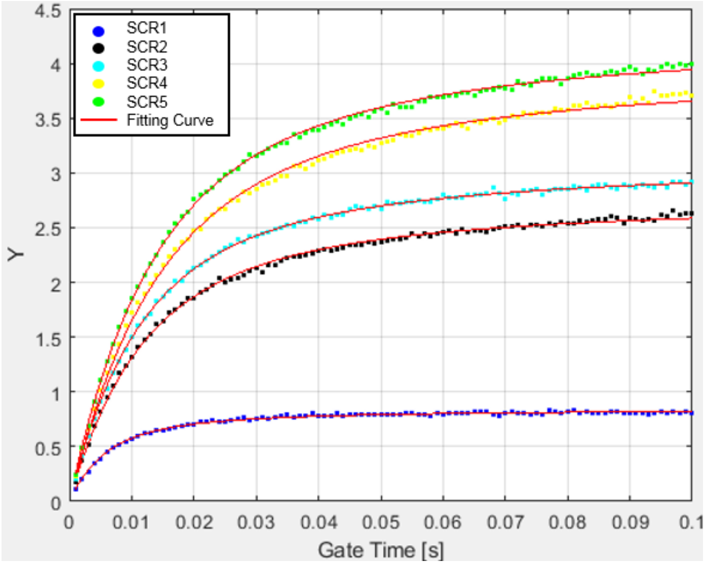
RESULTS AND DISCUSSION

❖ *Feynman-α method*

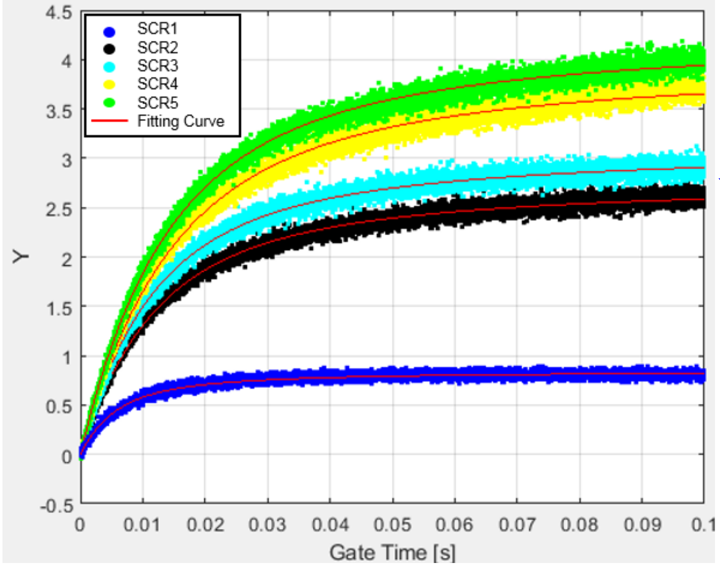
- **As the number of gate times increases, the measured α value can be estimated more elaborately.**
- **Therefore, the last case shows almost the same results as bunching technique even it takes shorter times.**



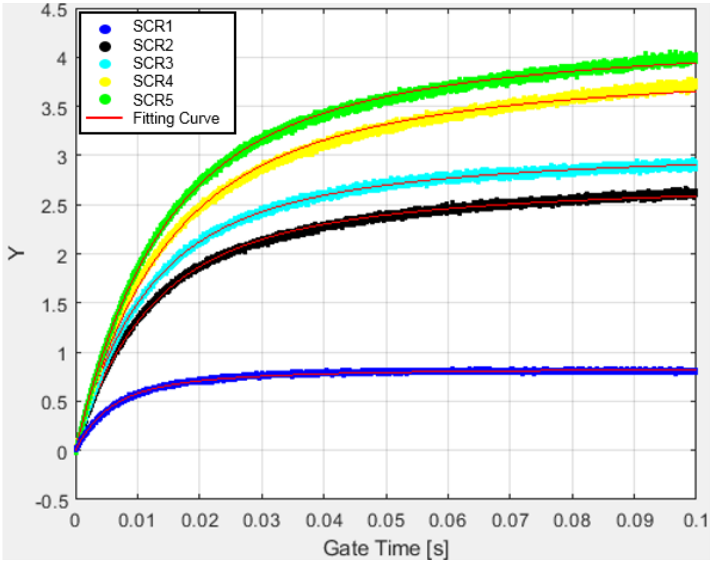
\# of gate times : 100, # of samplings : 10,000



\# of gate times : 100, # of samplings : 100,000



\# of gate times : 10,000, # of samplings : 10,000



\# of gate times : 10,000, # of samplings : 100,000

Fig. 13 Feynman fitting for five subcritical states using fully random sampling tehcnique

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RESULTS AND DISCUSSION

❖ *Feynman-α method with time-swap and fully random sampling methods*

- **It is noted that fully random sampling method even with a much smaller number of gate times gives comparable accuracies and its computing times are much shorter than those of time-swap method.**

TABLE II. Results of Feynman-α method using the time-swap and fully random sampling techniques

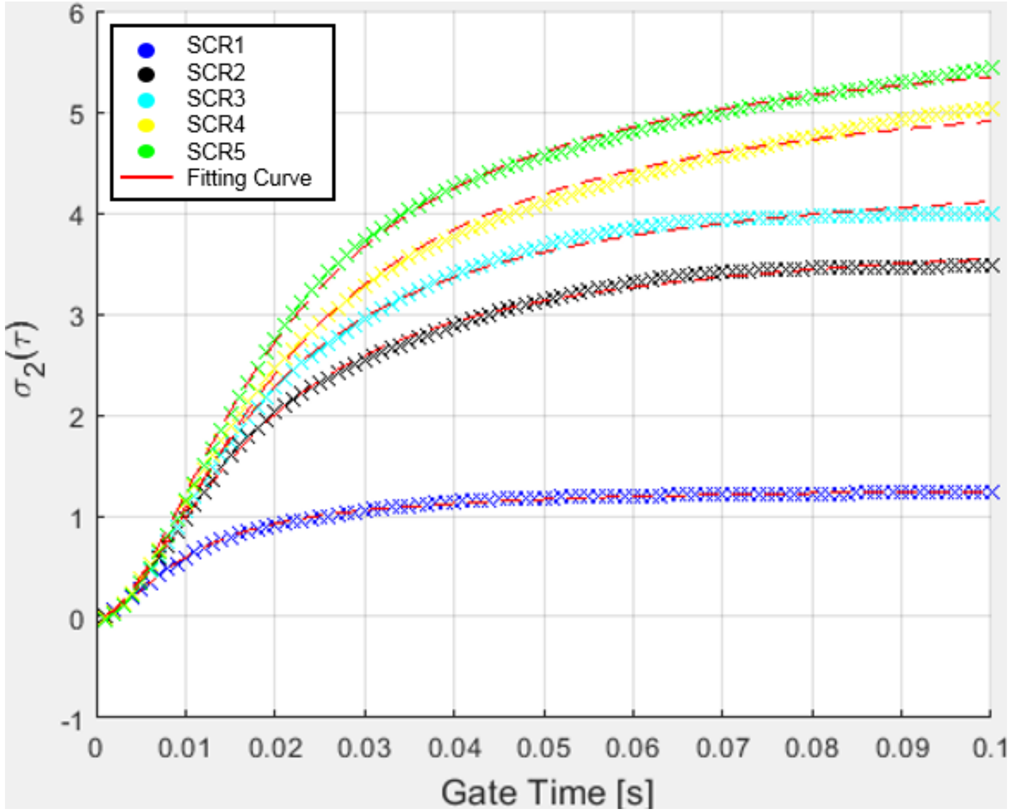
Technique			Time-swap		Fully random sampling							
# of gate times			100		100		100		100,000		100,000	
# of samplings			whole data		10,000		100,000		10,000		100,000	
Condition	k_{eff}	α-PKE	k-est	α-est	k-est	α-est	k-est	α-est	k-est	α-est	k-est	α-est
SCR1	0.98764	358.95	0.99103	296.99	0.98985	318.45	0.99115	294.83	0.99102	297.19	0.99103	297.00
			[a] -338.98	[b] 61.96	-221.31	40.50	-350.84	64.12	-337.86	61.76	-338.95	61.95
SCR2	0.99668	200.53	0.99976	143.92	0.99985	142.27	0.99973	144.35	0.99976	143.80	0.99975	143.98
			-307.80	56.61	-316.80	58.26	-305.46	56.18	-308.45	56.73	-307.45	56.55
SCR3	0.99737	186.03	0.99940	148.51	0.99961	144.61	0.99939	148.69	0.99940	148.51	0.99939	148.64
			-202.96	37.52	-224.12	41.42	-201.99	37.34	-202.95	37.52	-202.29	37.39
SCR4	0.99811	175.53	1.00120	118.13	1.00119	118.36	1.00123	117.62	1.00121	118.01	1.00120	118.23
			-309.26	57.40	-308.01	57.17	-312.03	57.91	-309.91	57.52	-308.74	57.30
SCR5	0.99885	162.6	1.00092	124.19	1.00109	121.10	1.00091	124.36	1.00092	124.25	1.00092	124.19
			-207.39	38.41	-224.10	41.50	-206.49	38.24	-207.06	38.35	-207.41	38.41
Average CPU time (sec)			***108479***		***703***		***1142***		***5737***		***45353***	

[a] [(k_{eff}) – (k-est)] (pcm Δk), [b] [(α-PKE) – (α-est)] (1/s)

RESULTS AND DISCUSSION

❖ 2nd order Feynman-α differential filtering method

- 2nd Feynman-α method with time swap gives accurate k_{eff} results less than 480 pcm, but long computing times.



Technique			Time-swap	
# of gate times			100	
# of samplings			whole data	
Condition	k_{eff}	α-PKE	k-est	α-est
SCR1	0.98764	358.95	0.99247	270.73
			[a] -483.34	[b] 88.22
SCR2	0.99668	200.53	0.99897	158.29
			-229.46	42.24
SCR3	0.99737	186.03	0.99914	153.31
			-176.95	32.72
SCR4	0.99811	175.53	1.00059	129.50
			-247.84	46.03
SCR5	0.99885	162.6	1.00029	136.00
			-143.56	26.60
Average CPU time (sec)			292271	

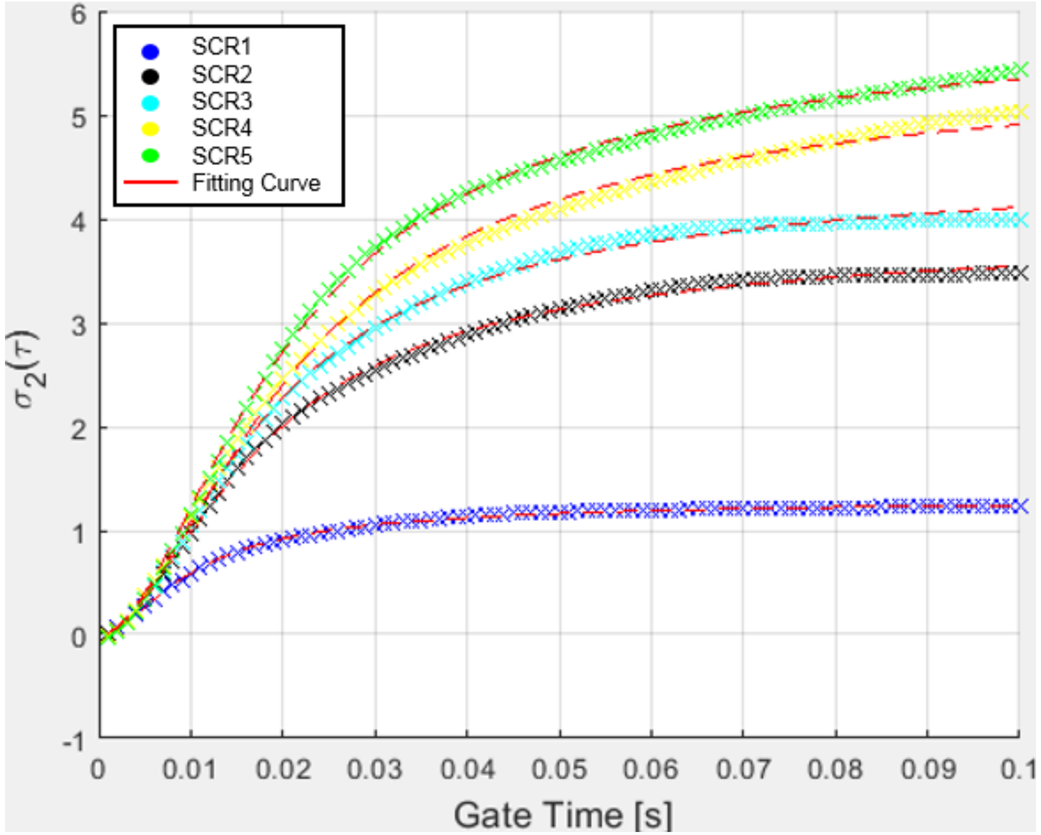
[a] [(k_{eff}) – (k-est)] (pcm Δk), [b] [(α-PKE) – (α-est)] (1/s)

Fig. 14 2nd order Feynman-α fitting for five subcritical states using bunching-technique

RESULTS AND DISCUSSION

❖ *2nd order Feynman-α differential filtering method*

- **2nd Feynman-α method shows more accurate measurement near critical states than conventional Feynman-α method.**



Method			2nd order F-α		Feynman-α	
Technique			Time-swap		Time-swap	
# of gate times			100		100	
# of samplings			whole data		whole data	
Condition	k_{eff}	α-PKE	k-est	α-est	k-est	α-est
SCR1	0.98764	358.95	0.99247	270.73	0.99103	296.99
			[a] -483.34	[b] 88.22	-338.98	61.96
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SCR3	0.99737	186.03	0.99914	153.31	0.99940	148.51
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Average CPU time (sec)			292271		108479	

[a] [(k_{eff}) – (k-est)] (pcm Δk), [b] [(α-PKE) – (α-est)] (1/s)

Fig. 14 2nd order Feynman-α fitting for five subcritical states using bunching-technique

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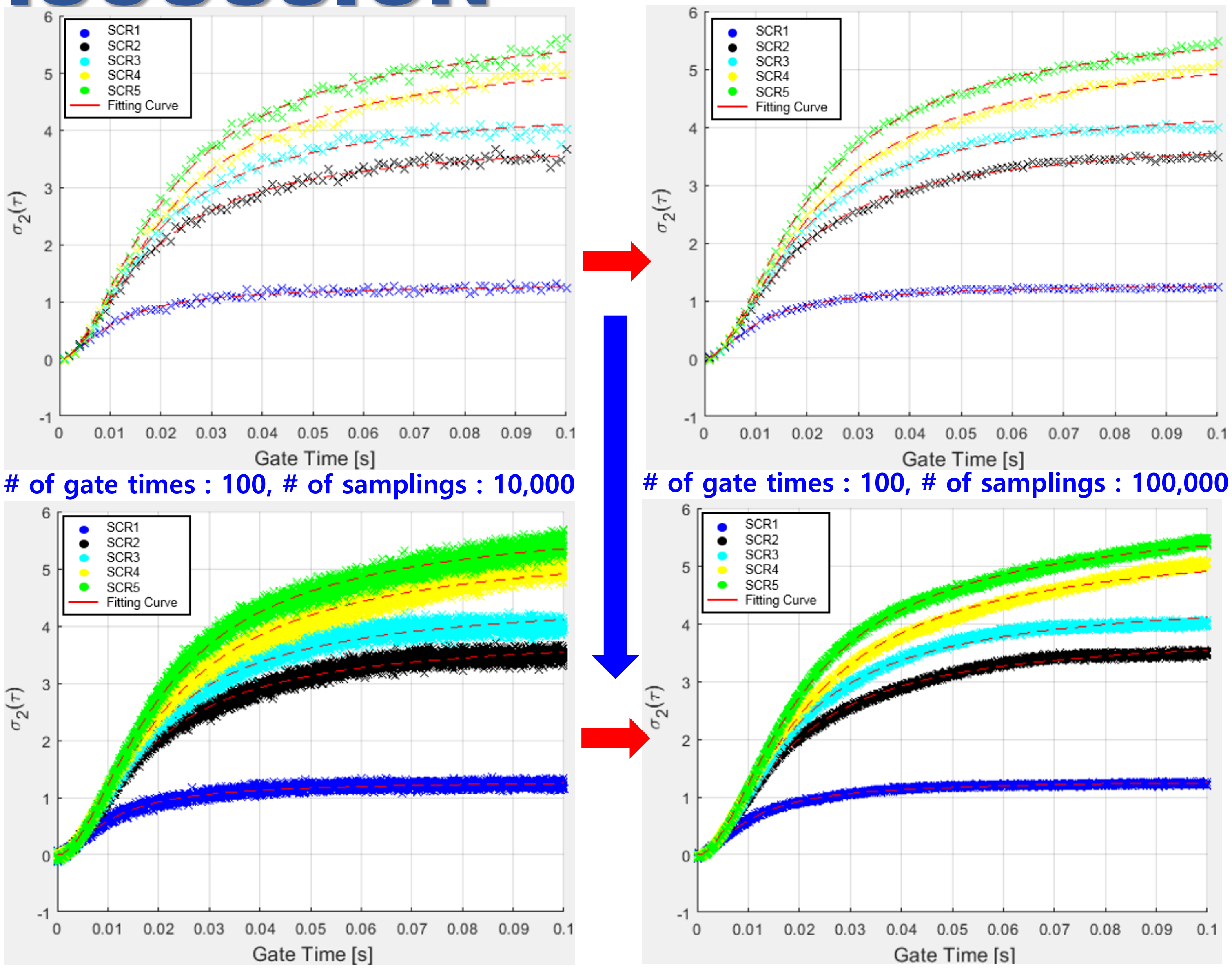
RESULTS AND DISCUSSION

❖ 2nd Feynman-α method

- The shape of the graph looks similar compared with the Feynman-α method, but slight difference at the front.
- As shown in Fig. 15, the number of sampling increases, the measured σ_2 value converges toward a specific value.
- The α value can be estimated more elaborately if we increase the number of gate times.



# of gate times : 100, # of samplings : 10,000

# of gate times : 100, # of samplings : 100,000

# of gate times : 10,000, # of samplings : 10,000

# of gate times : 10,000, # of samplings : 100,000

Fig. 15 2nd order Feynman-α fitting for five subcritical states using fully random sampling tehcnique

RESULTS AND DISCUSSION

2nd order Feynman-α differential filtering method

- **It is noted that fully random sampling method even with a much smaller number of gate times gives comparable accuracies and its computing times are much shorter than those of time-swap method.**

TABLE III. k_{eff} estimated with 2nd order Feynman-α differential filtering method using the time-swap and fully random sampling techniques

Technique			Time-swap		Fully random sampling							
# of gate times			100		100		100		100,000		100,000	
# of samplings			whole data		10,000		100,000		10,000		100,000	
Condition	k_{eff}	α-PKE	k-est	α-est	k-est	α-est	k-est	α-est	k-est	α-est	k-est	α-est
SCR1	0.98764	358.95	0.99247	270.73	0.99271	266.40	0.99249	270.36	0.99247	270.77	0.99249	270.45
			[a] -483.34	[b] 88.22	-507.22	92.55	-485.39	88.59	-483.16	88.18	-484.89	88.50
SCR2	0.99668	200.53	0.99897	158.29	0.99897	158.33	0.99896	158.50	0.99897	158.32	0.99898	158.22
			-229.46	42.24	-229.26	42.20	-228.36	42.03	-229.35	42.21	-229.85	42.31
SCR3	0.99737	186.03	0.99914	153.31	0.99906	154.78	0.99913	153.45	0.99915	153.14	0.99914	153.27
			-176.95	32.72	-169.00	31.25	-176.22	32.58	-177.88	32.89	-177.16	32.76
SCR4	0.99811	175.53	1.00059	129.50	1.00057	129.81	1.00060	129.23	1.00060	129.36	1.00058	129.63
			-247.84	46.03	-246.19	45.72	-249.35	46.30	-248.62	46.17	-247.16	45.90
SCR5	0.99885	162.6	1.00029	136.00	1.00031	135.54	1.00030	135.67	1.00029	135.95	1.00028	136.03
			-143.56	26.60	-146.01	27.06	-145.30	26.93	-143.81	26.65	-143.38	26.57
Average CPU time (sec)			*292271*		*719*		*1401*		*7224*		*61796*	

[a] $[(k_{eff}) - (k\text{-est})]$ (pcm Δk), [b] [(α-PKE) – (α-est)] (1/s)

RESULTS AND DISCUSSION

2nd order Feynman-α differential filtering method

- **It is noted that the 2nd order Feynman-α method shows a more accurate value than the conventional Feynman-α method near critical states (SCR2~SCR5).**

TABLE III. k_{eff} estimated with 2nd order Feynman-α differential filtering method using the time-swap and fully random sampling techniques

Method			**2nd order Feynman-α**					**Conventional Feynman-α**				
Technique			Time-swap	Fully random sampling				Time-swap	Fully random sampling			
# of gate times			100	100	100	100,000	100,000	100	100	100	100,000	100,000
# of samplings			whole data	10,000	100,000	10,000	100,000	whole data	10,000	100,000	10,000	100,000
Condition	k_{eff}	α-PKE	k-est	k-est	k-est	k-est	k-est	k-est	k-est	k-est	k-est	k-est
SCR1	0.98764	358.95	0.99247	0.99271	0.99249	0.99247	0.99249	0.99103	0.98985	0.99115	0.99102	0.99103
			[a] -483.34	-507.22	-485.39	-483.16	-484.89	[a] -338.98	-221.31	-350.84	-337.86	-338.95
SCR2	0.99668	200.53	0.99897	0.99897	0.99896	0.99897	0.99898	0.99976	0.99985	0.99973	0.99976	0.99975
			-229.46	-229.26	-228.36	-229.35	-229.85	-307.80	-316.80	-305.46	-308.45	-307.45
SCR3	0.99737	186.03	0.99914	0.99906	0.99913	0.99915	0.99914	0.99940	0.99961	0.99939	0.99940	0.99939
			-176.95	-169.00	-176.22	-177.88	-177.16	-202.96	-224.12	-201.99	-202.95	-202.29
SCR4	0.99811	175.53	1.00059	1.00057	1.00060	1.00060	1.00058	1.00120	1.00119	1.00123	1.00121	1.00120
			-247.84	-246.19	-249.35	-248.62	-247.16	-309.26	-308.01	-312.03	-309.91	-308.74
SCR5	0.99885	162.6	1.00029	1.00031	1.00030	1.00029	1.00028	1.00092	1.00109	1.00091	1.00092	1.00092
			-143.56	-146.01	-145.30	-143.81	-143.38	-207.39	-224.10	-206.49	-207.06	-207.41
Average CPU time (sec)			***292271***	***719***	***1401***	***7224***	***61796***	***108479***	***703***	***1142***	***5737***	***45353***

[a] $[(k_{eff}) - (k\text{-est})]$ (pcm Δk)

CONCLUSION

- In this work, subcriticality experiment is performed with the Feynman-α and 2nd Feynman-α differential method at AGN-201K.
- A fully random sampling technique is devised to overcome the drawbacks that bunching-technique with fine unit gate time drastically increases computing time,.
- For measuring subcriticality, eigenvalue calculations are performed with MCNP6 to obtain reference k_{eff} and kinetic parameters.
- In conclusion, it was shown that the new fully random sampling technique suggested in this work can provide accurate subcriticality estimations with computationally efficient way for AGN-201K and this method coupled with the second-order differential method gives slightly better estimation for the near-critical cases.

THANK YOU FOR YOUR ATTENTION !!

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