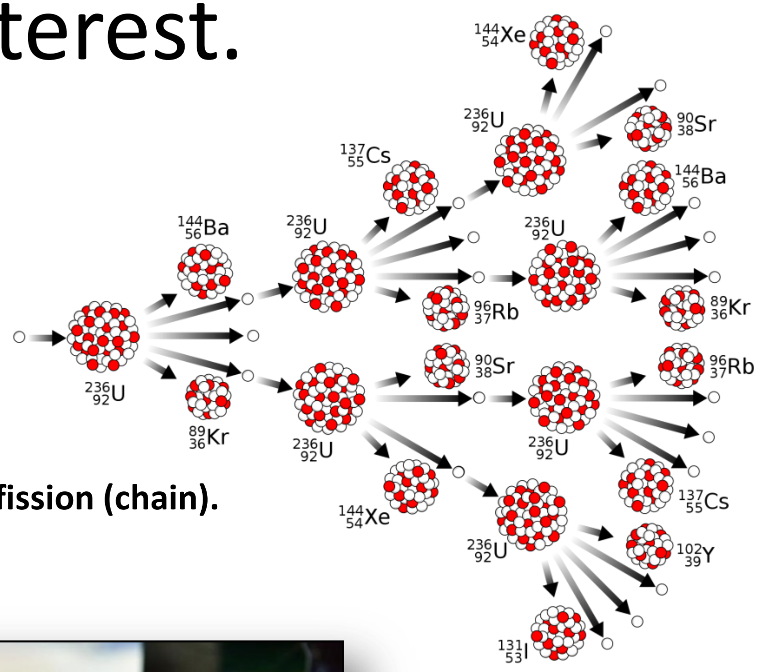
Two-Exponential Rossi-alpha Analysis of Copper- and Polyethylene-Reflected Plutonium

ANS Winter Meeting
Washington, DC USA
20 November 2019

Michael Y. Hua[1,2], Caiser A. Bravo[1], Jesson D. Hutchinson[2], George E. McKenzie[2], and Sara A. Pozzi[1]
mikwa@umich.edu

1. *Department of Nuclear Engineering and Radiological Sciences, University of Michigan, Ann Arbor, MI, 48109, USA*
2. *NEN-2: Advanced Nuclear Technology Group, Los Alamos National Laboratory, Los Alamos, NM, 87544, USA*

Reactivity is a quantity of widespread interest.



Nuclear fission (chain).

- Fissionable material may undergo fission; neutrons born from fission can cause another fission…

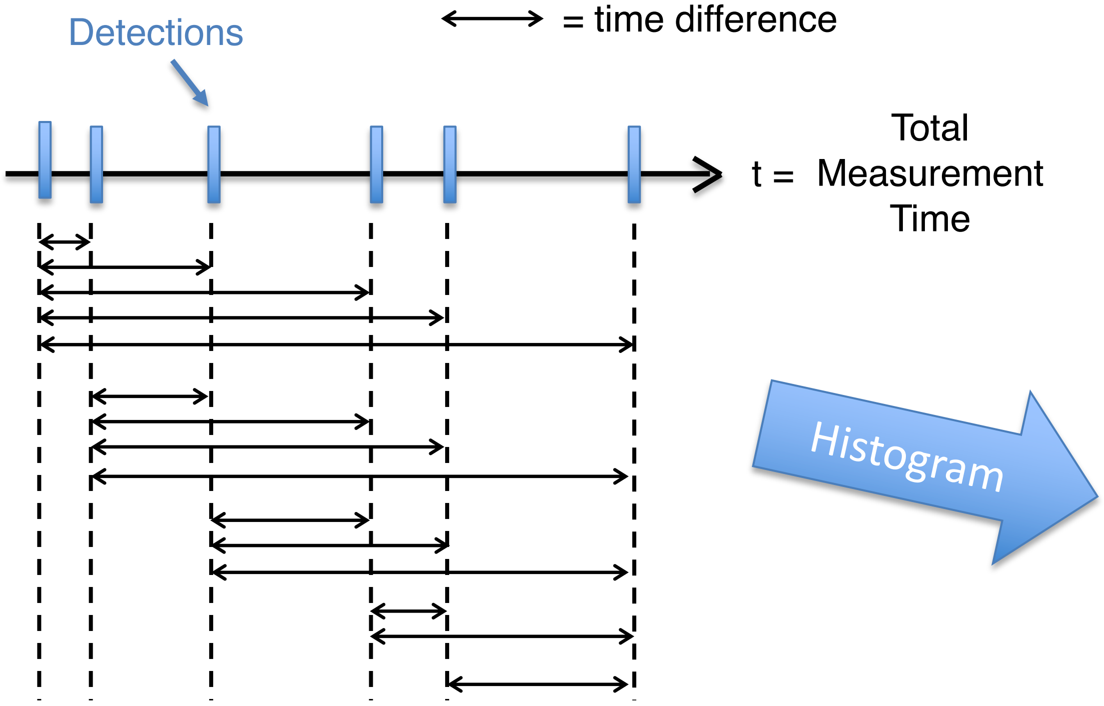
  ⇒ Neutron-multiplying system (characterized by reactivity).

- The reactivity estimates are of interest in:
  - **Criticality Safety**: will the assembly remain subcritical during procedures and upset conditions? In-situ measurements.
  - **Nuclear Nonproliferation and Safeguards**: assay of fuel assemblies and detection of material diversion.
  - **Emergency Response**: determine if a sample is fissionable or if neutrons are from another source e.g., (alpha,n).

- We cannot directly estimate a system's subcritical reactivity… infer from Rossi-alpha.

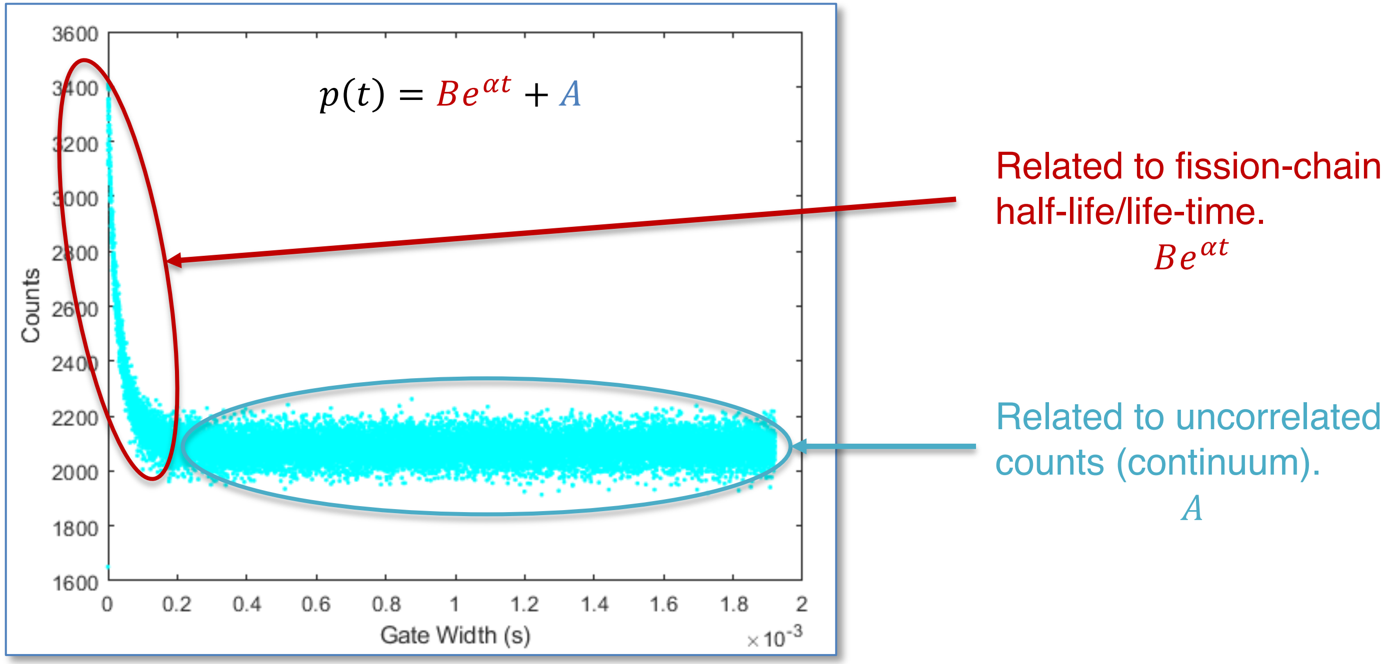
IAEA Inspection

The Traditional Rossi-alpha Method.



α = Prompt neutron decay constant

Obtained from fitting a Rossi-alpha histogram



Sample Rossi-alpha plot: NoMAD ^{3}He-based detector measuring the BeRP ball.

Inferring/Estimating Reactivity from the Rossi-alpha.

α = Prompt neutron decay constant

Obtained from fitting a Rossi-alpha histogram

One measurement with assumptions

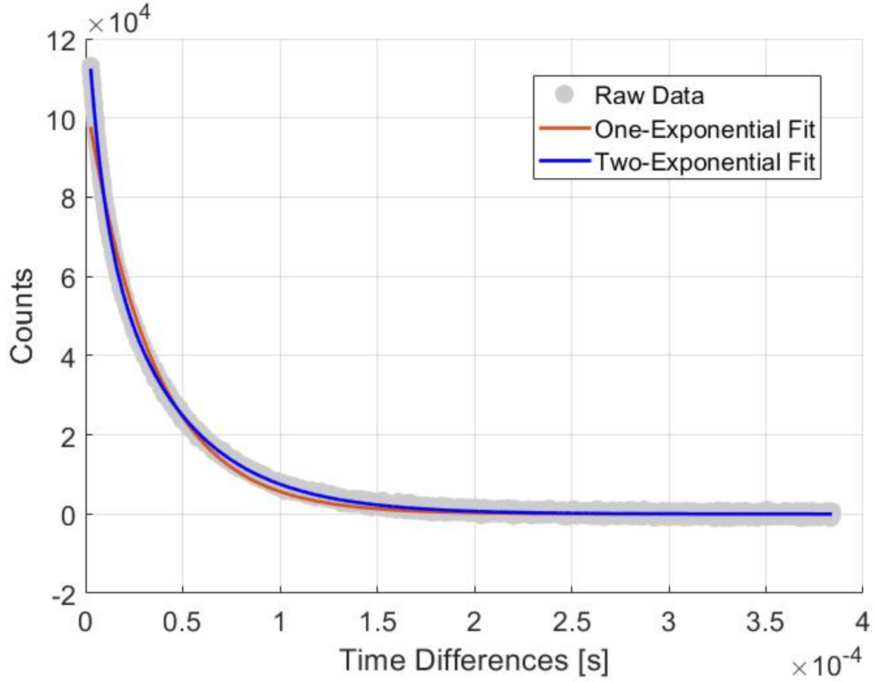
Multiple replacement measurements

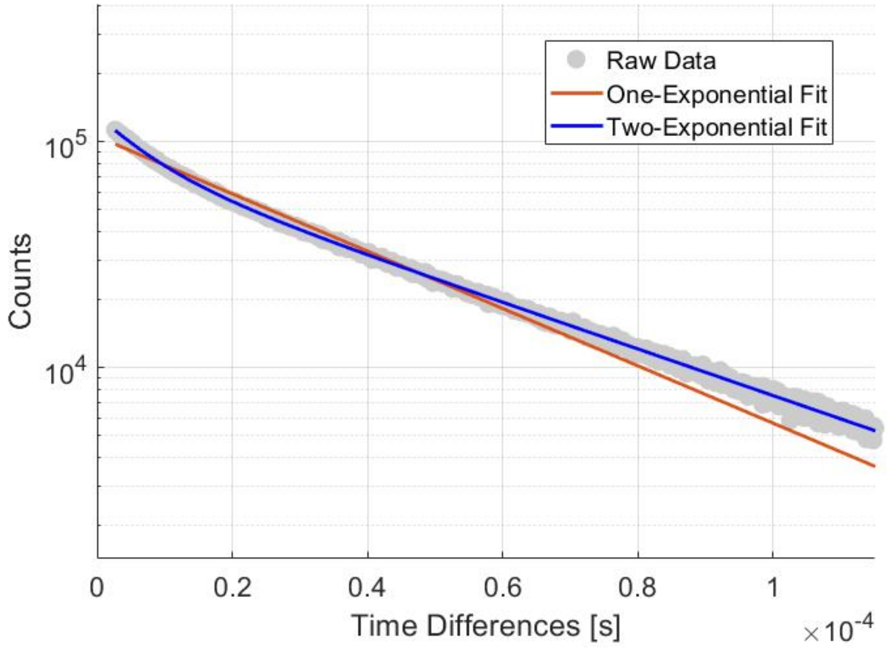
$$\rho = \beta_{\text{eff}} + \alpha\Lambda \quad \text{or} \quad \rho = \rho(\alpha, \alpha_{DC})$$

Then:

$$k_{\text{eff}} = \frac{1}{1-\rho}$$

Rossi-alpha measurements of moderated samples are better fit by a two-exponential model.





Predominant interaction: **scatter**
Neutrons are still detected, but at a later time.

The shielding introduces new, energy-dependent correlations.

Fitting sums of exponentials is a mathematically ill-posed problem.
→ Inherent limitation of ^3He systems.

Accidentals obscure the second exponential.

The two-exponential Rossi-alpha model for reflected assemblies.

One-Exp

$$p(t) = Be^{\alpha t} + A$$

Two-Exp

$$p(t) = A - C\left[e^{r_1 t}\rho_1 + e^{r_2 t}\rho_2\right]$$

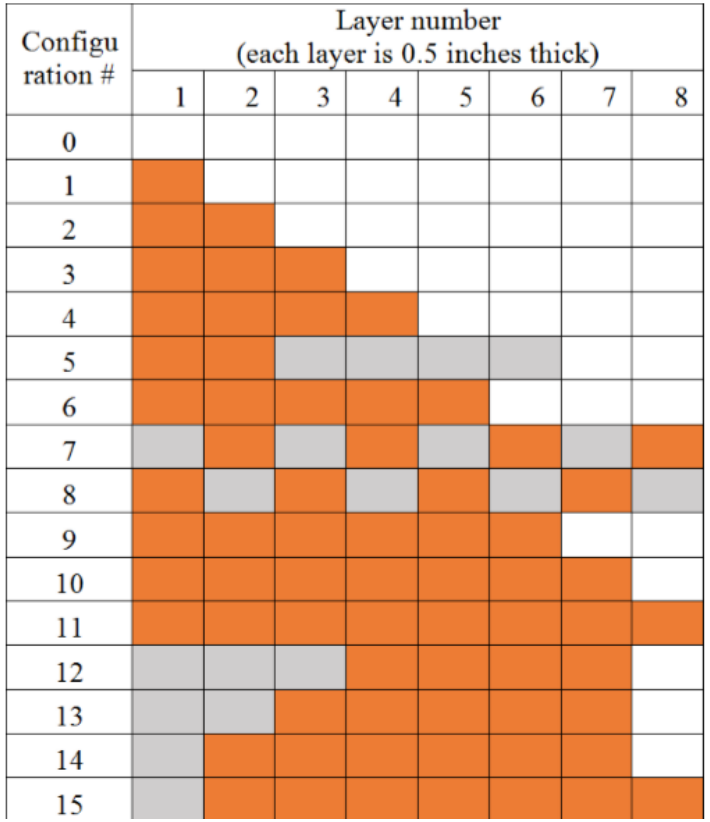
$$\rho_1 = \frac{(1-R)^2}{r_1} + \frac{2(1-R)(R)}{r_1 + r_2}, \quad \text{and}$$

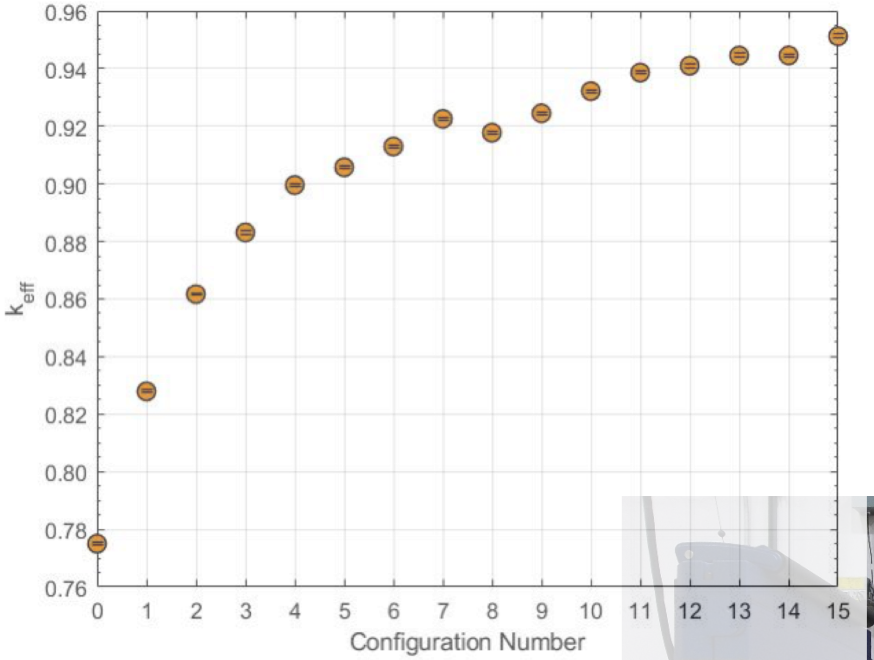
$$\rho_2 = \frac{R^2}{r_2} + \frac{2(1-R)(R)}{r_1 + r_2}.$$

$$\alpha = r_1(1-R) + r_2 R. \qquad \ell_{ctd} = -\frac{1}{r_2(1-R) + r_1 R}$$

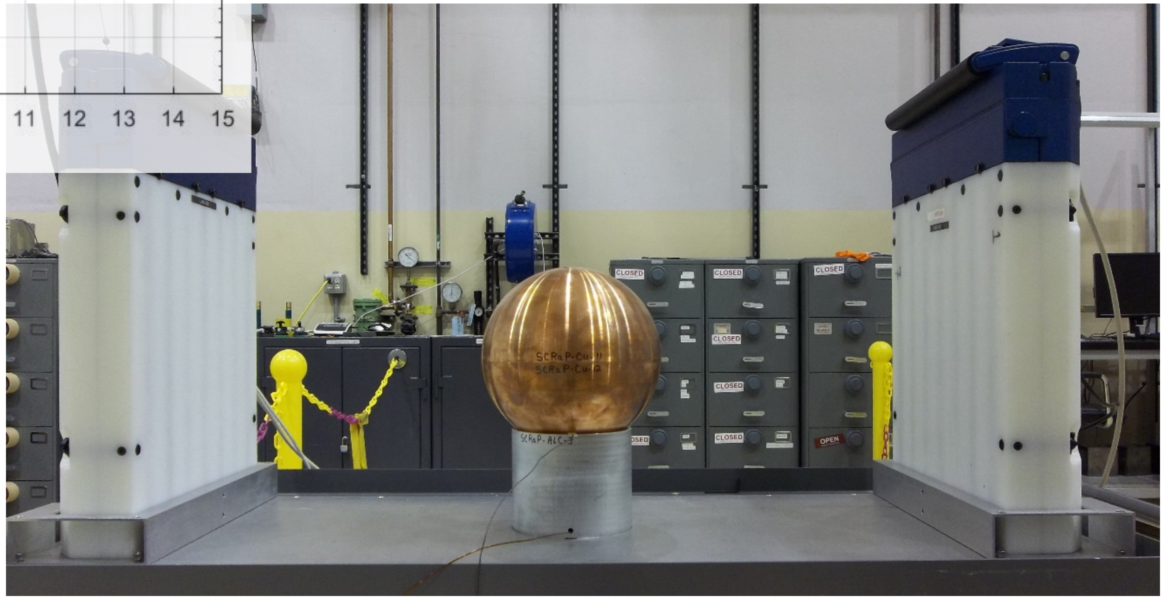
- In one-exp, α is the exponential.
- In two-exp, α is a linear combination of the exponentials.
- Also, $-1/\ell_{ctd}$ is a linear combination of the exponentials.

The SCRαP benchmark measurements.

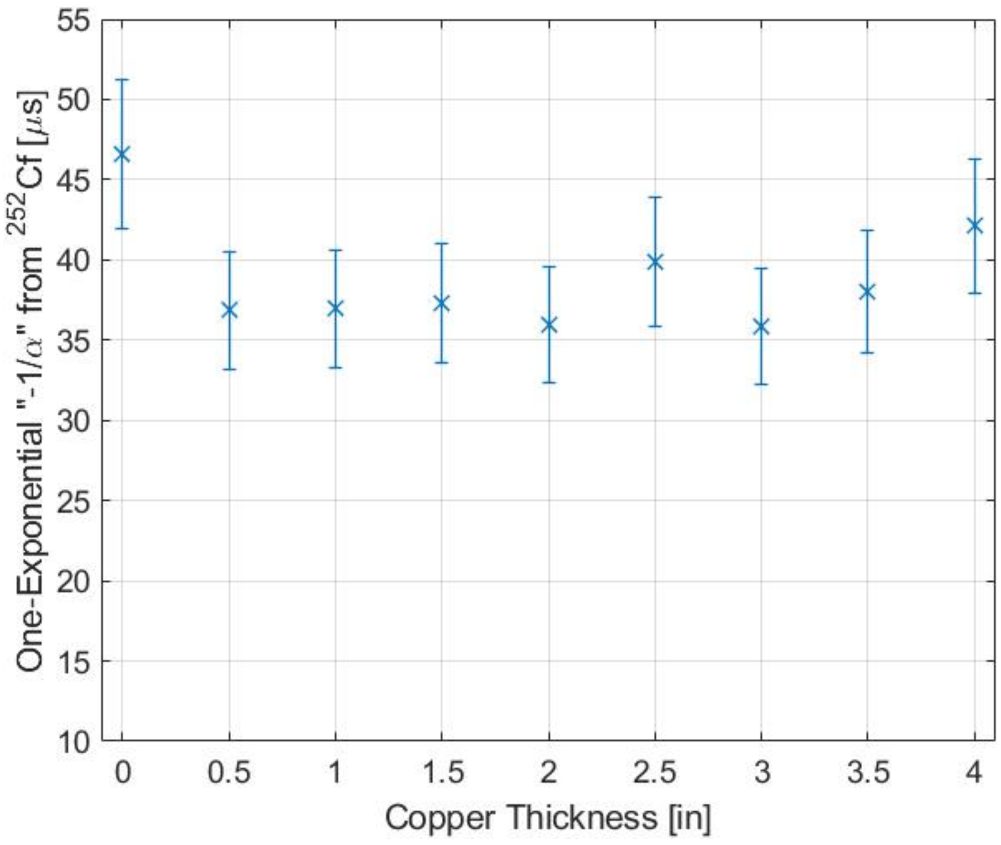




- ~4.5 kg, α-phase Pu: BeRP Ball
- Replacement measurements with ^{252}Cf
- Reflected by copper and polyethylene shells (0.5-inch increments)
- Measured passively with NoMAD: ^{3}He tubes in polyethylene matrix



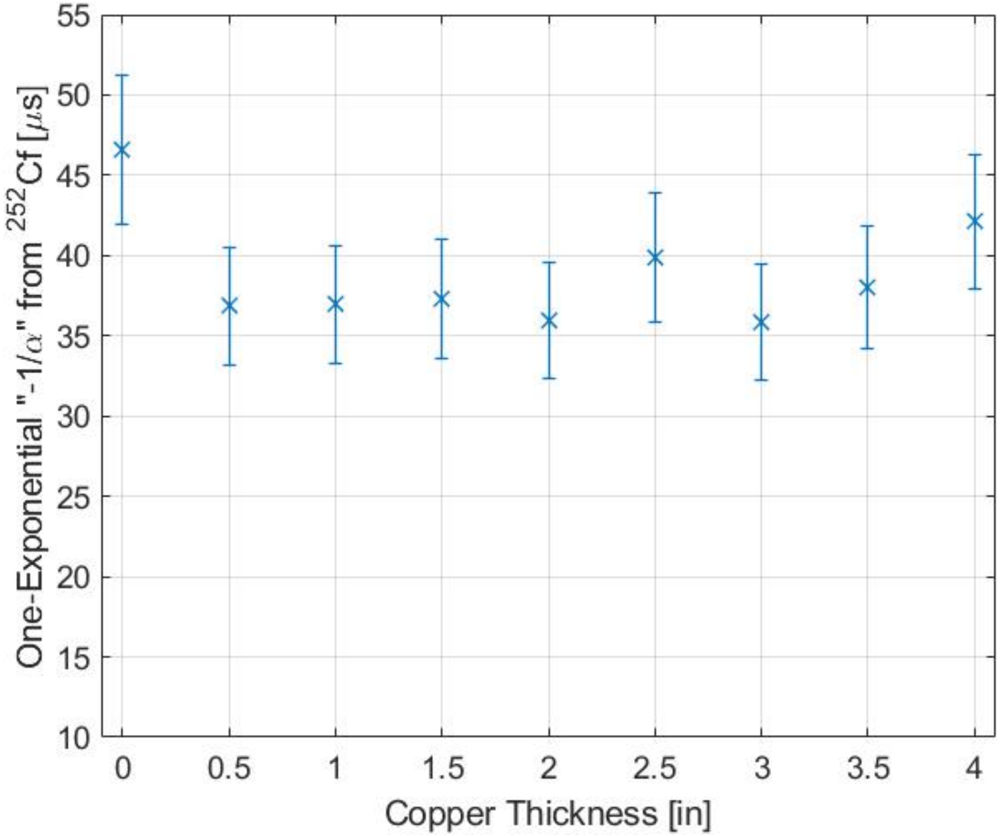
Copper Negligibly Affects ℓ_{ctd}

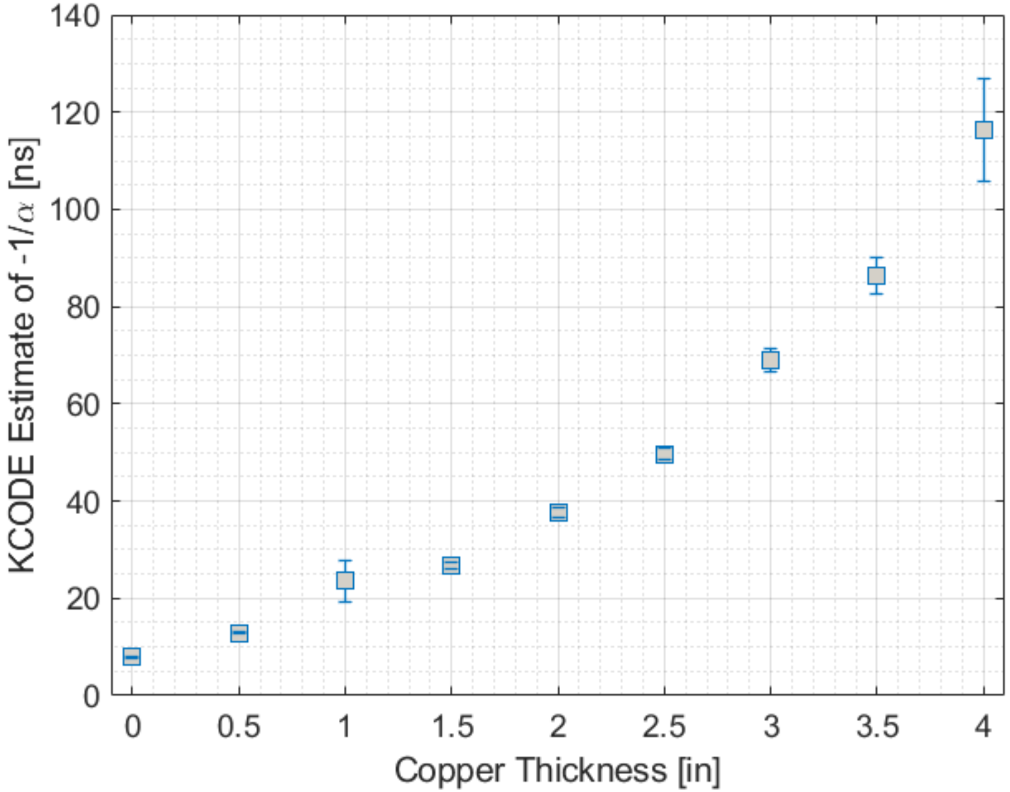


- Uncertainty comes from fit uncertainty only.
- Does copper have a non-negligible contribution to the time a neutron spends in the reflector before detection after leaving the core for the last time?
  - If so, we would see an increasing trend in the plot on the left.
  - We do not.
- Values are dominated by the known slowing-down-time of NoMAD detectors (35-40 μs).

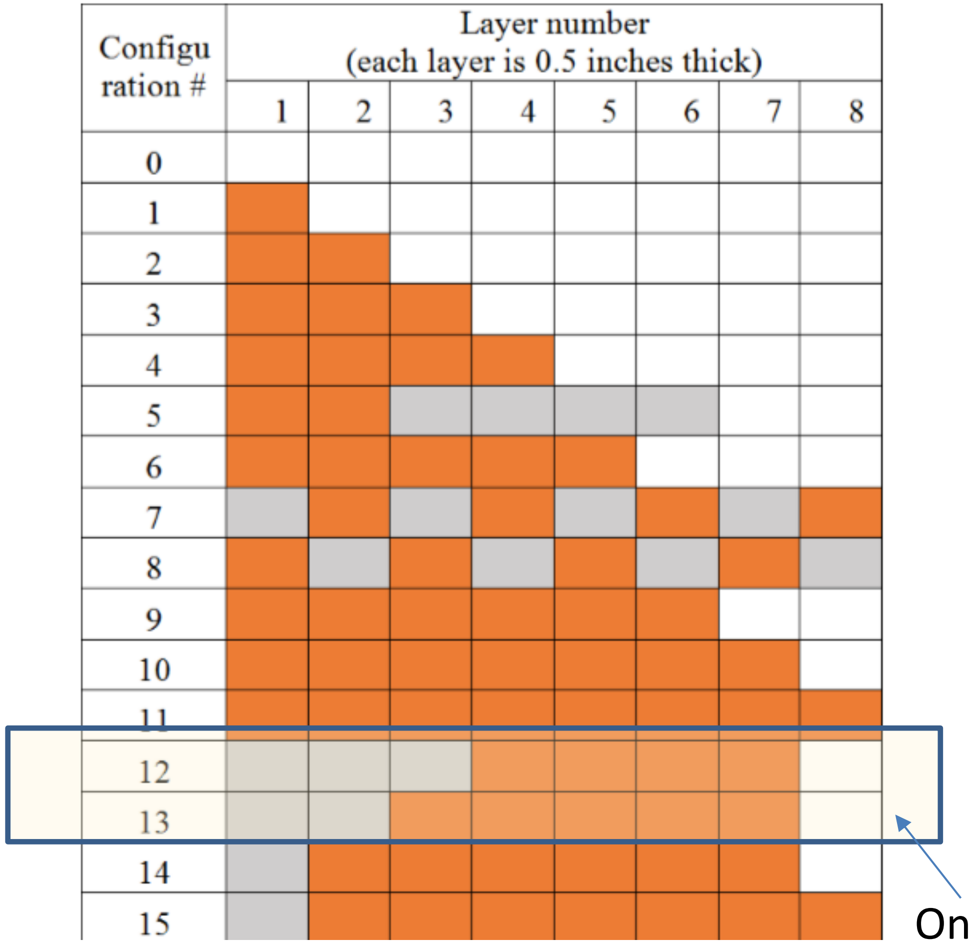
The α values are dominated by the NoMAD slowing down time.

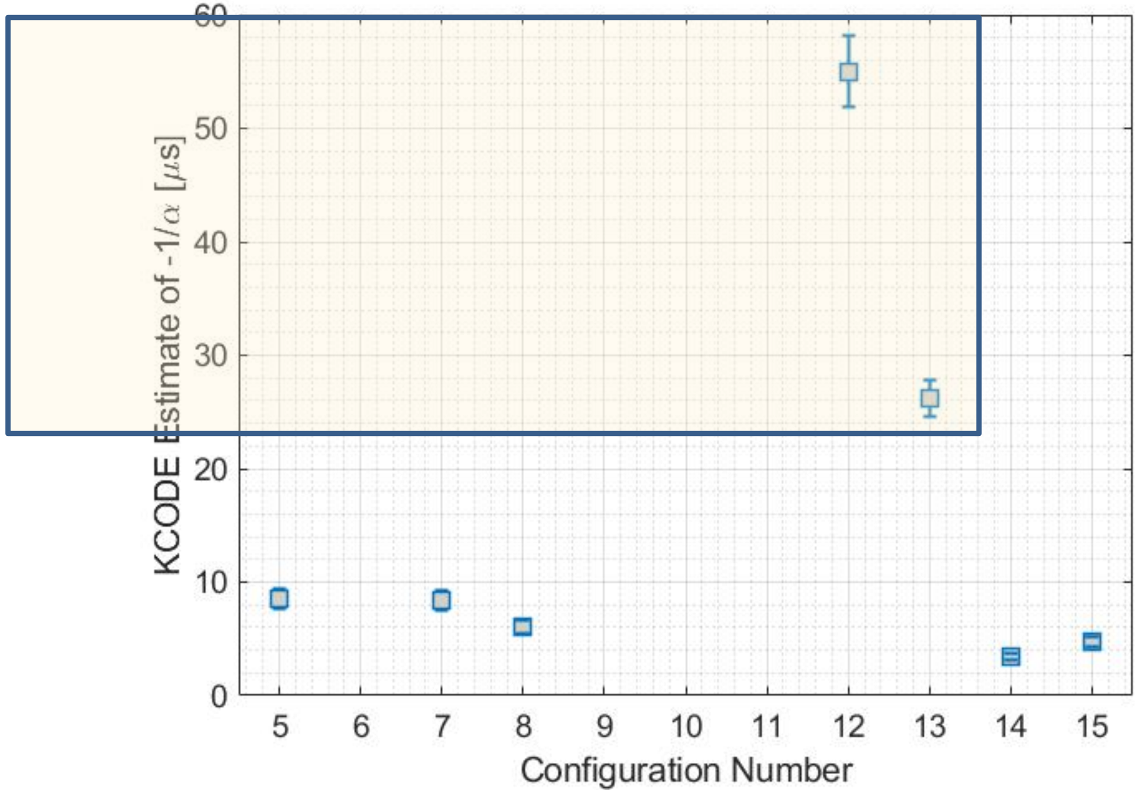
Copper-only configurations are not suitable for two-exp/Rossi-alpha analysis.





The KCODE α values from configurations 12 and 13 are comparable to the known slowing down time.



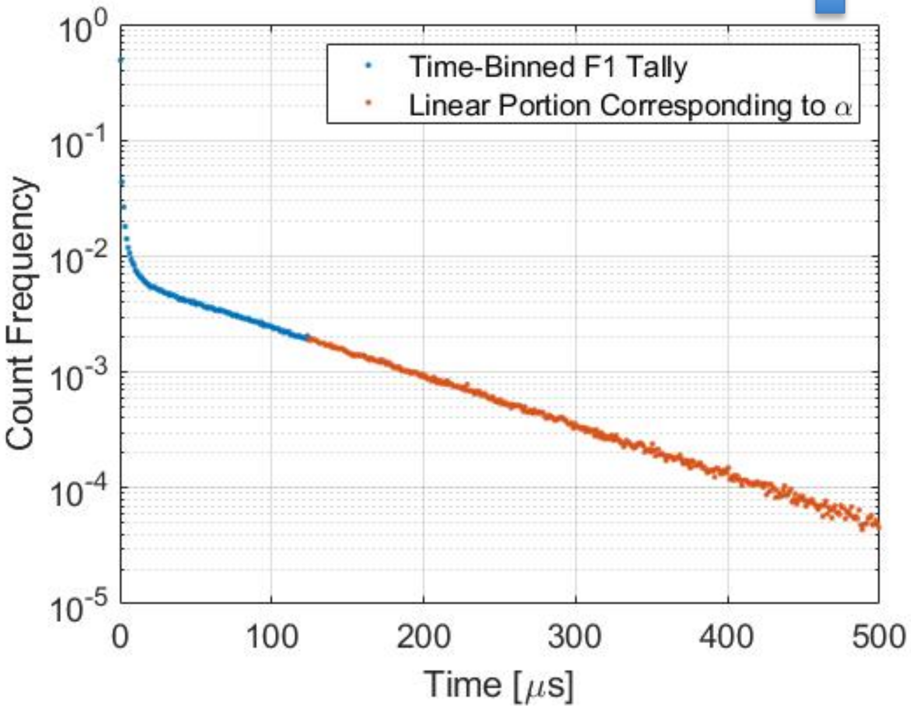


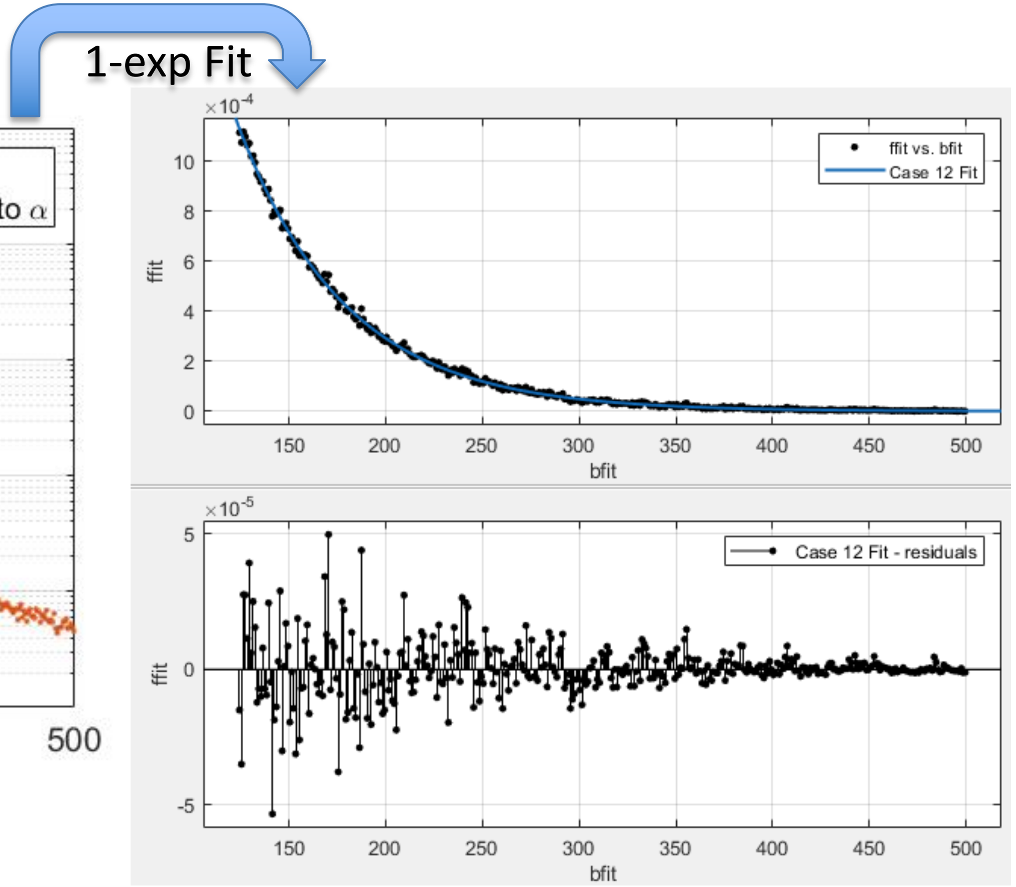
Only cases 12 and 13 have $\alpha \propto \mu s$ (KCODE)

……the other α are too fast for the NoMADs

Simulation of the prompt neutron decay constant α.

- Previous work shows KOPTS does not perform as well for deeply subcritical, reflected assemblies.
- Instead, obtain α estimate from time-binning an F1 tally and looking at large times: fit the exponential die-away.
  - "Time-Bin, Tail-Fit"
- In this work, F1 tally is on the surface of the BeRP ball (location mostly affects convergence time).

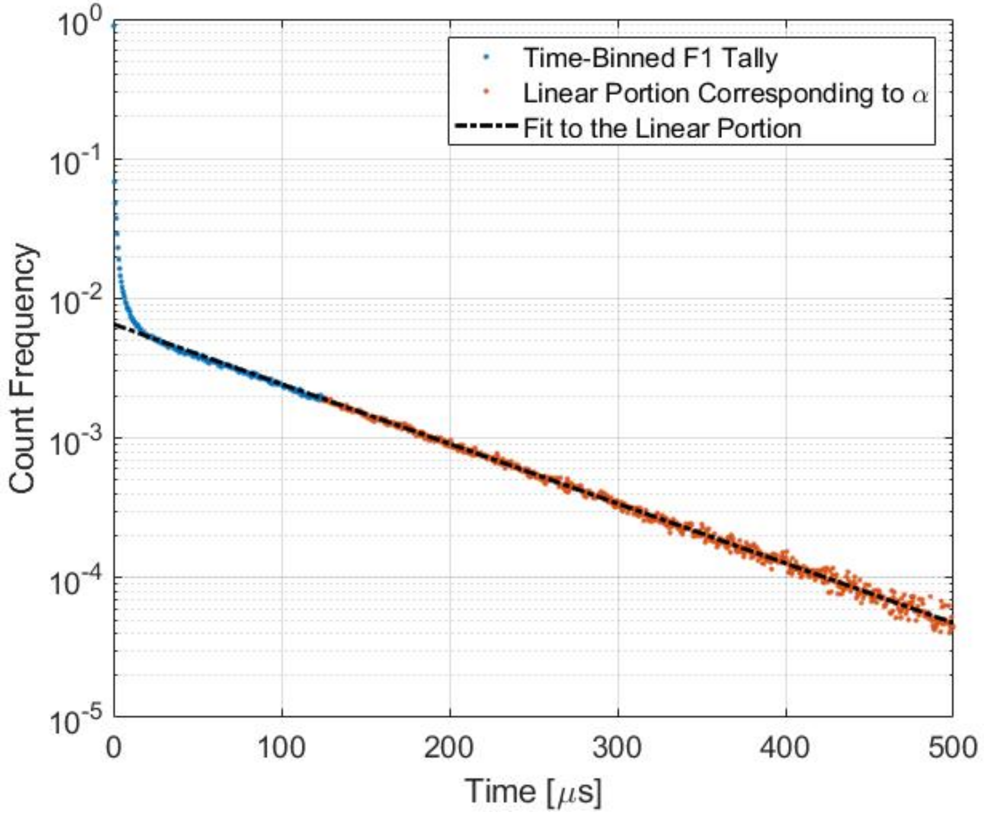




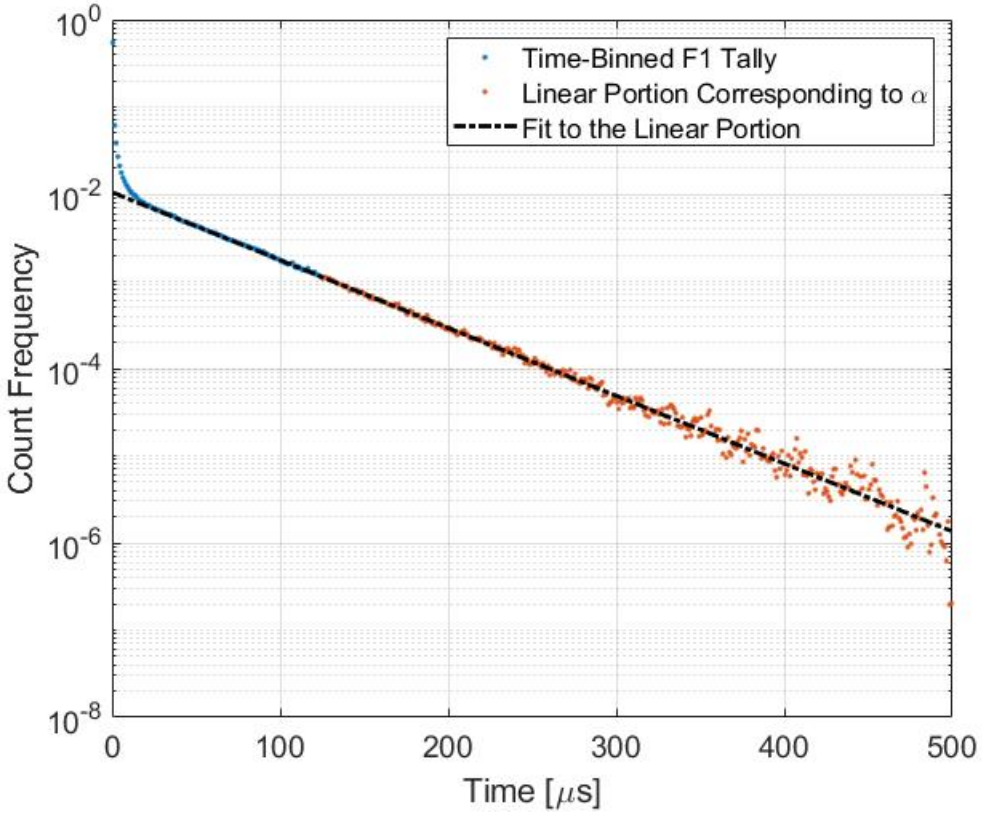
Simulation of the prompt neutron decay constant α.

Fit to the last 75% of data.

- Case 12: $\alpha^{-1} = 102.20 \pm 0.49\ \ \mu s$



- Case 13: $\alpha^{-1} = 55.83 \pm 0.37\ \ \mu s$



Comparison of simulated and measured values.

- Uncertainty:
  - From fit uncertainty for measured data and time-bin tail-fit.
- Good agreement between measurement and time-bin tail-fit estimates of α^{-1}.
- Measured 1-exp α^{-1} < Measured 2-exp α^{-1}
  - 1-exp is skewed lower since it does not deconvolve the shorter, known slowing-down-time (35-40 μs).
- Disagreement between measured two-exp and time-bin tail-fit could be due to the presence of more than two regions.
  - Core, reflector, "shield"

α estimates	Case 12 $[\mu s]$	Case 13 $[\mu s]$
Measured 2-exp	95.32 ± 1.09	64.04 ± 0.35
Time-Bin Tail-Fit	102.20 ± 0.49	55.83 ± 0.37
Measured 1-exp	88.26 ± 0.23	61.70 ± 0.08

Case 12 error: -6.7%
Case 13 error: 12.8%

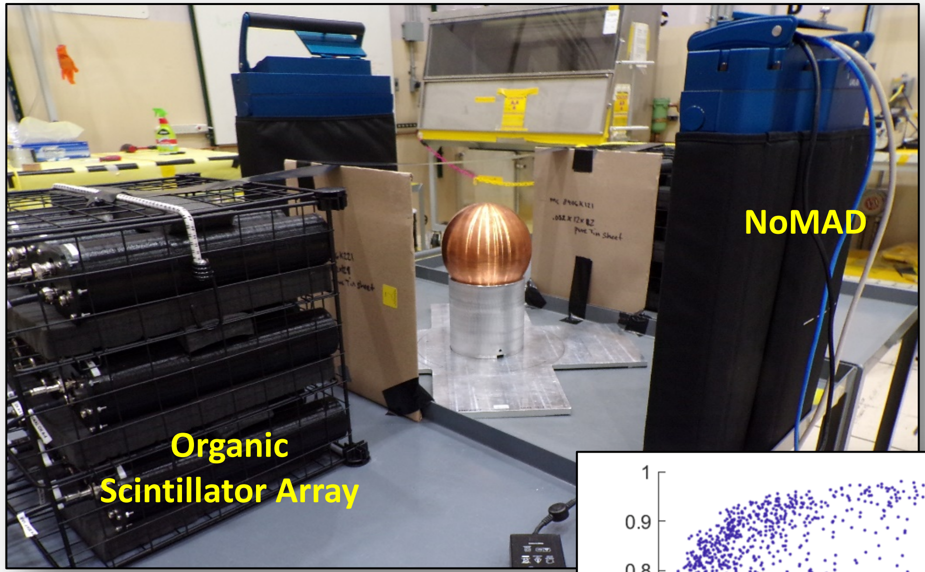
Error is on the order of uncertainty we see in other, similar measurements with copper.

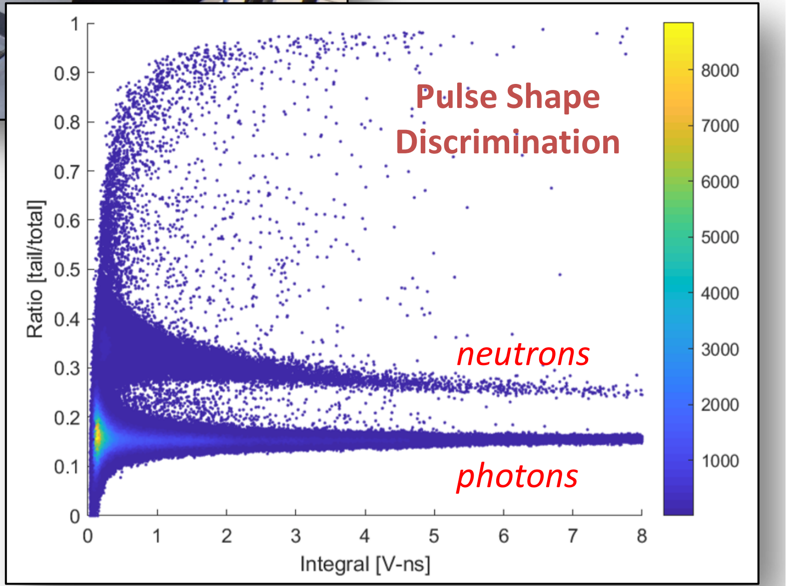
Inherent limitation of ^{3}He ⇒ non-moderated detectors such as organic scintillators.

- Fitting sums of exponentials (e.g. two-exp) is a mathematically ill-posed problem.

- ^{3}He-capture-based detector systems typically moderate neutrons prior to detection to increase intrinsic efficiency.
- Moderating material on the detector may add an additional regional (exponential).

- Organic scintillation detectors detect neutrons via scattering and do not require moderating material.
- Simultaneous measurement of copper-reflected Pu with organic scintillators and ^{3}He for preliminary comparison.



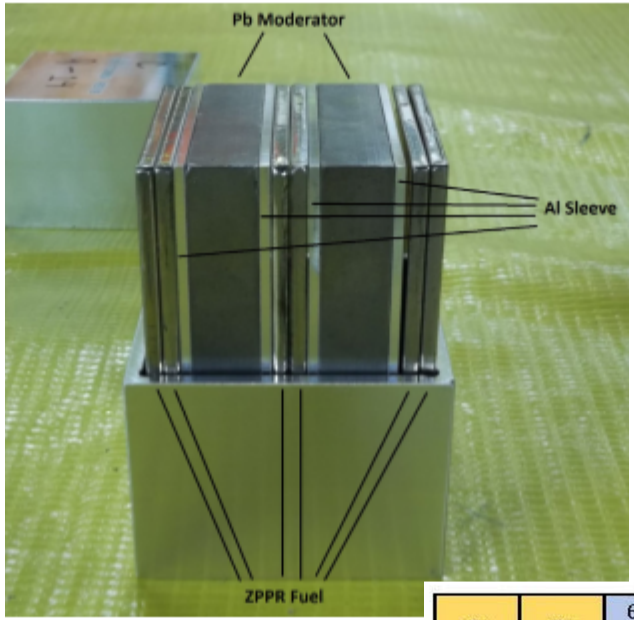


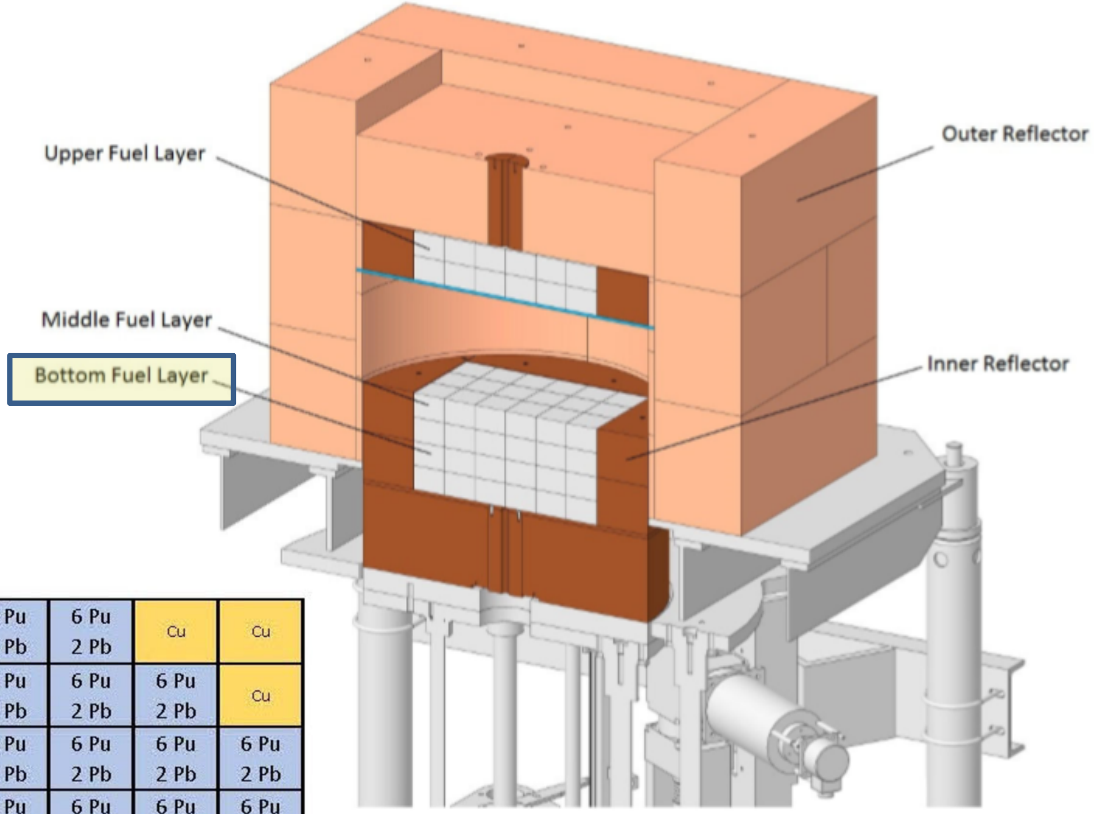
Experimental setup of the organic scintillator and ^{3}He simultaneous measurement of copper-reflected plutonium.

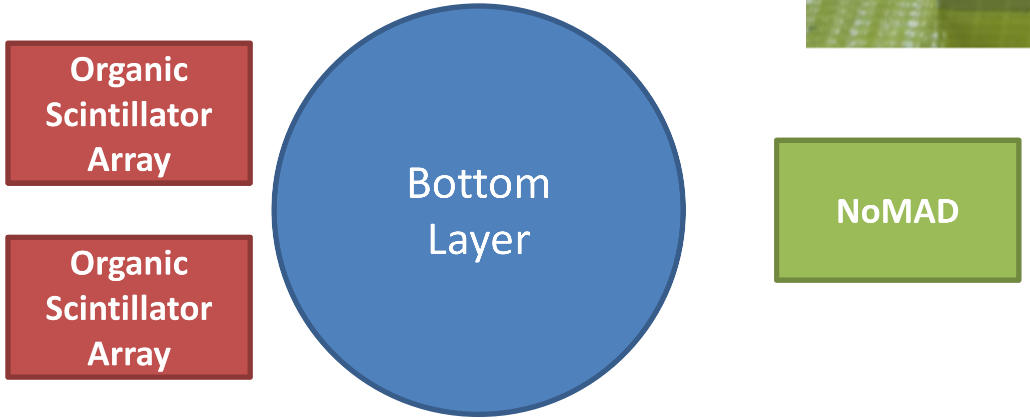
$k_{\text{eff}} \approx$ **0.624**
15 kg Pu (93% ^{239}Pu)
Lead moderated
Copper reflected

Simultaneous measurement.

Fast, metal assembly…
$\alpha = 52.3 \pm 2.5$ ns (time-bin tail-fit).







Cu	Cu	6 Pu 2 Pb	6 Pu 2 Pb	Cu	Cu
Cu	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	Cu
6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb
6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb
Cu	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	6 Pu 2 Pb	Cu
Cu	Cu	6 Pu 2 Pb	6 Pu 2 Pb	Cu	Cu

Bottom Layer

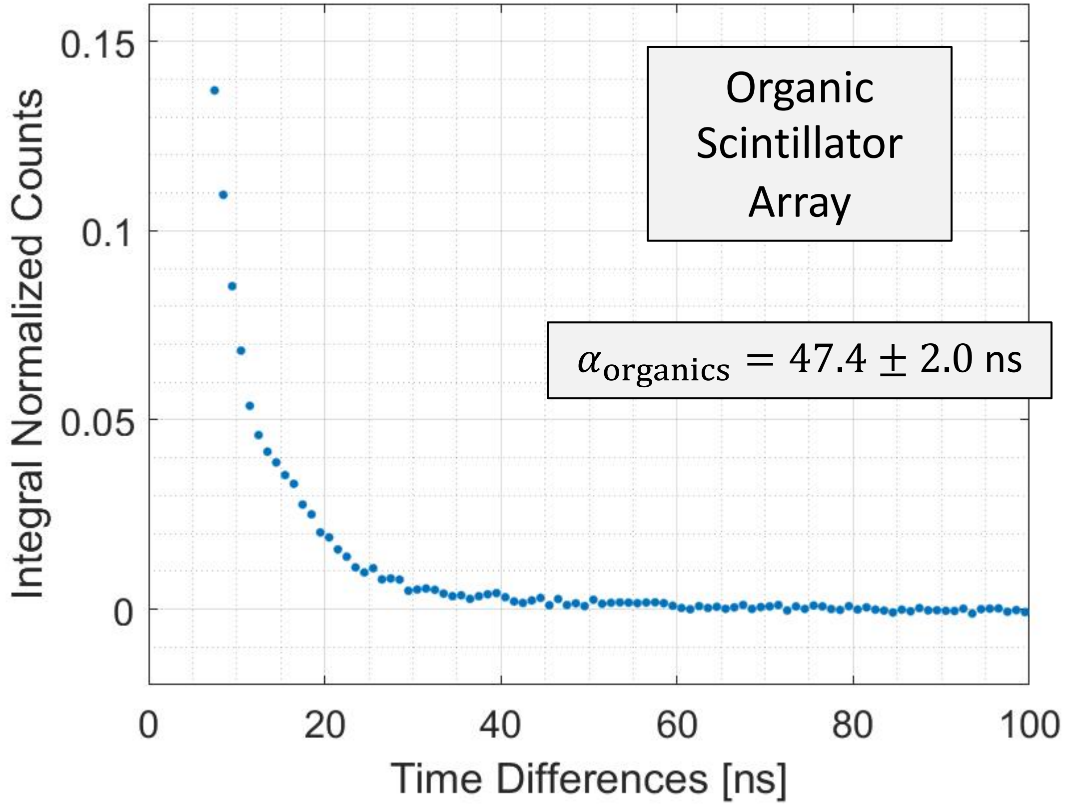
Comparison of Rossi-alpha histograms after 7 hours, 20 minutes.

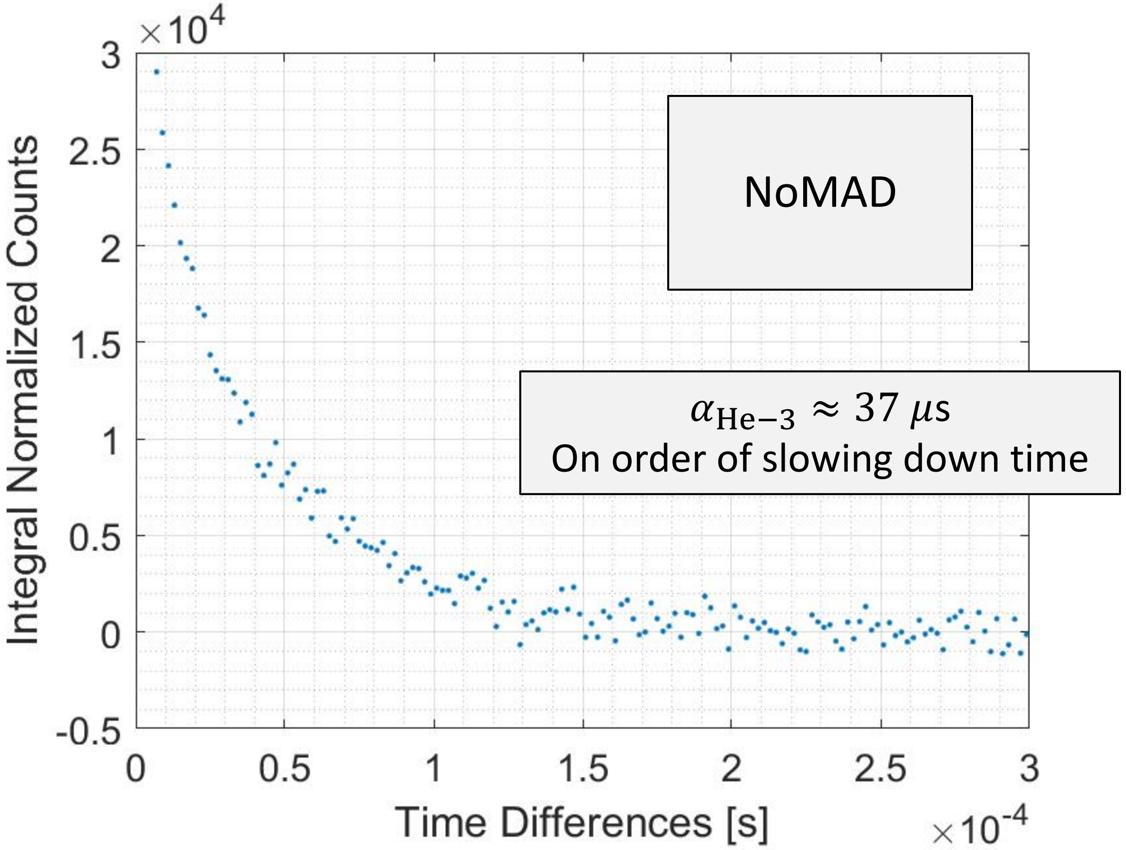
$\alpha_{\text{sim}} = 52.3 \pm 2.5$ ns (time-bin tail-fit).

Constant-subtracted, integral-normalized, small-bin-truncated

Nanoseconds vs. tens-to-hundreds of microseconds.

Note: the entire organic scintillator window is the same length as the NoMAD clock tick length.





Summary, Conclusion, and Future Work.

- For most SCRαP configurations, the prompt neutron decay is too fast for the NoMADs to see.
  - NoMAD known slowing down time: 35-40 μs
  - Two largest $-1/\alpha$ (as predicted by KCODE):
    - Case 12: 55 μs
    - Case 13: 26 μs (greater competition with slowing down time)
- We use the time-bin tail-fit method to obtain a baseline estimate of α since KOPTS performs poorly for deeply subcritical, reflected systems.
  - Case 12: -6.7% sim-measured error
  - Case 13: 12.8% sim-measured error
- Preliminary analysis shows organic scintillators may be sensitive to the α of fast assemblies.

- Future work:
  - Investigate organic scintillators:
    - Fast-neutron Rossi-alpha
    - Gamma-ray Rossi-alpha
    - Energy dependence
    - Angular correlation
    - Neutron cross talk (erroneous double counting)
  - Comparison between ^{3}He-based systems and organic scintillators.
  - Comparison between KOPTS and time-bin tail-fit as a function of k_{eff} and reflector thickness.

Acknowledgements

This work was supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1256260, by the Consortium for Verification Technology under Department of Energy National Nuclear Security Administration award number DENA0002534, by the DOE Nuclear Criticality Safety Program, funded and managed by the National Nuclear Security Administration for the Department of Energy, and by the DOE Office of Material Management and Minimization, funded and managed by the National Nuclear Security Administration for the Department of Energy.

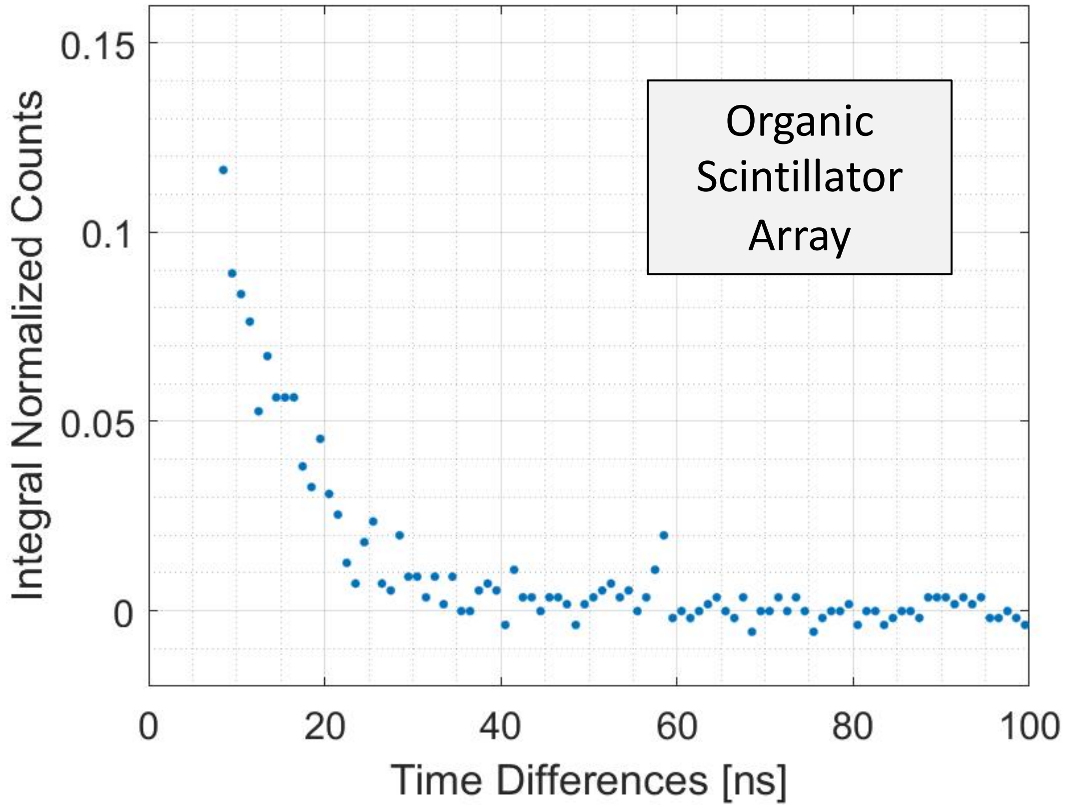
Any opinion, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation or other funding organizations.

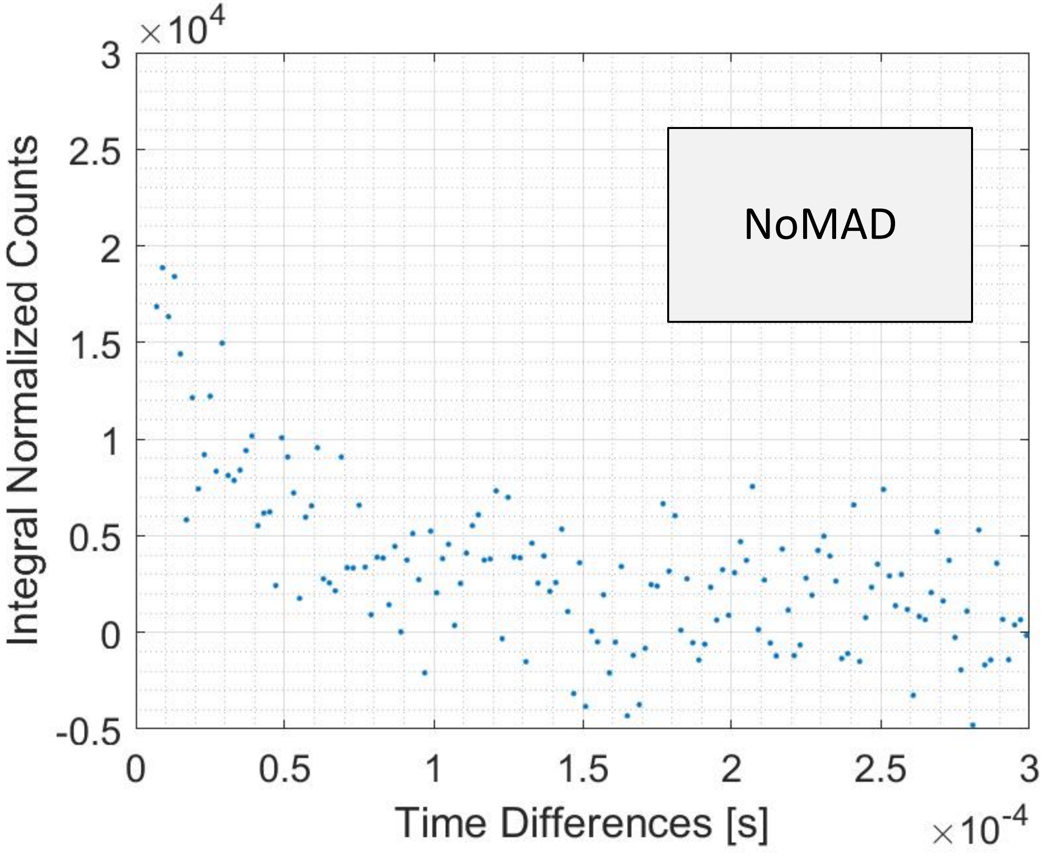
Extra Slides

Comparison of Rossi-alpha histograms after 10 minutes.

Constant-subtracted, integral-normalized, small-bin-truncated

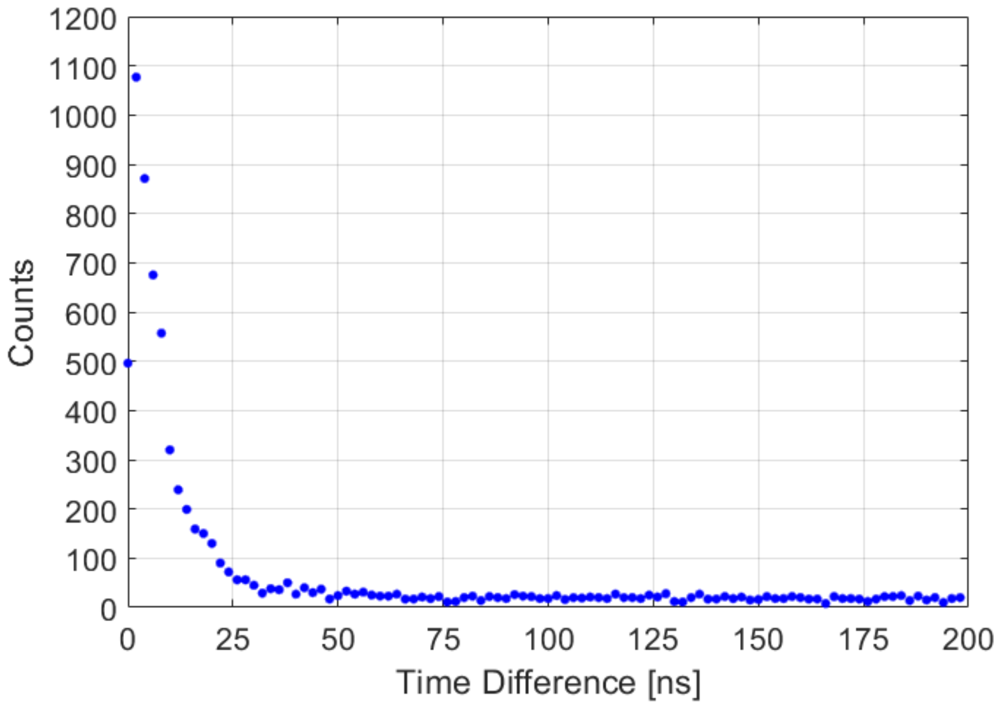
We could rebin both histograms (larger bins for less fluctuation); however, the NoMAD is already binned at 2 $\mu s \Rightarrow$ all α information is lost in the first bin.



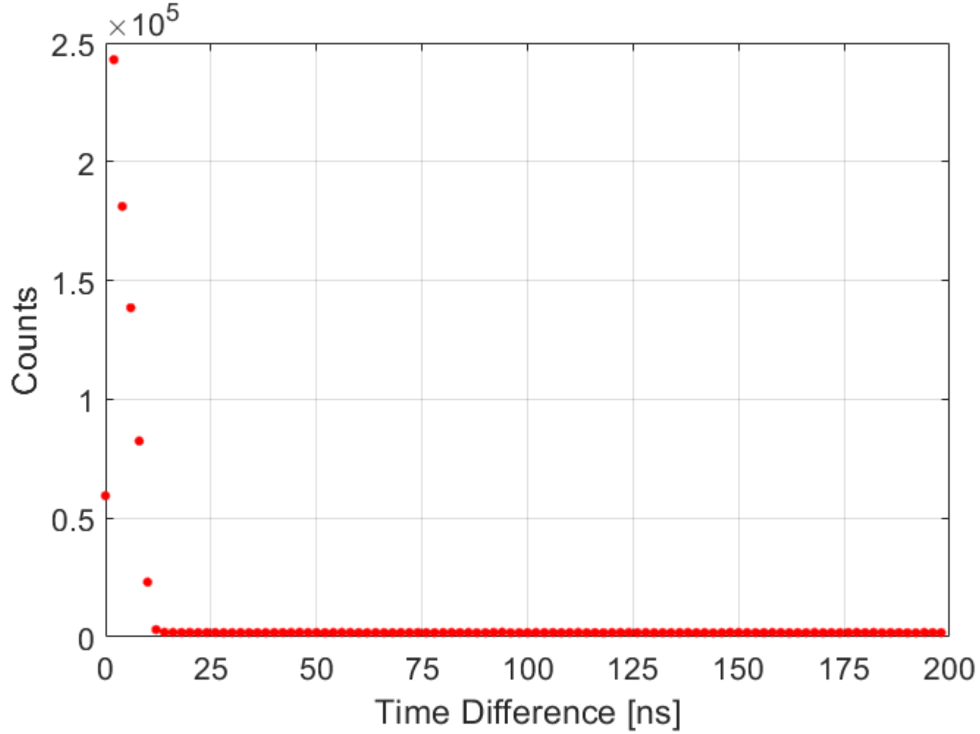


Neutron and gamma-ray Rossi-alpha.

- Neutrons

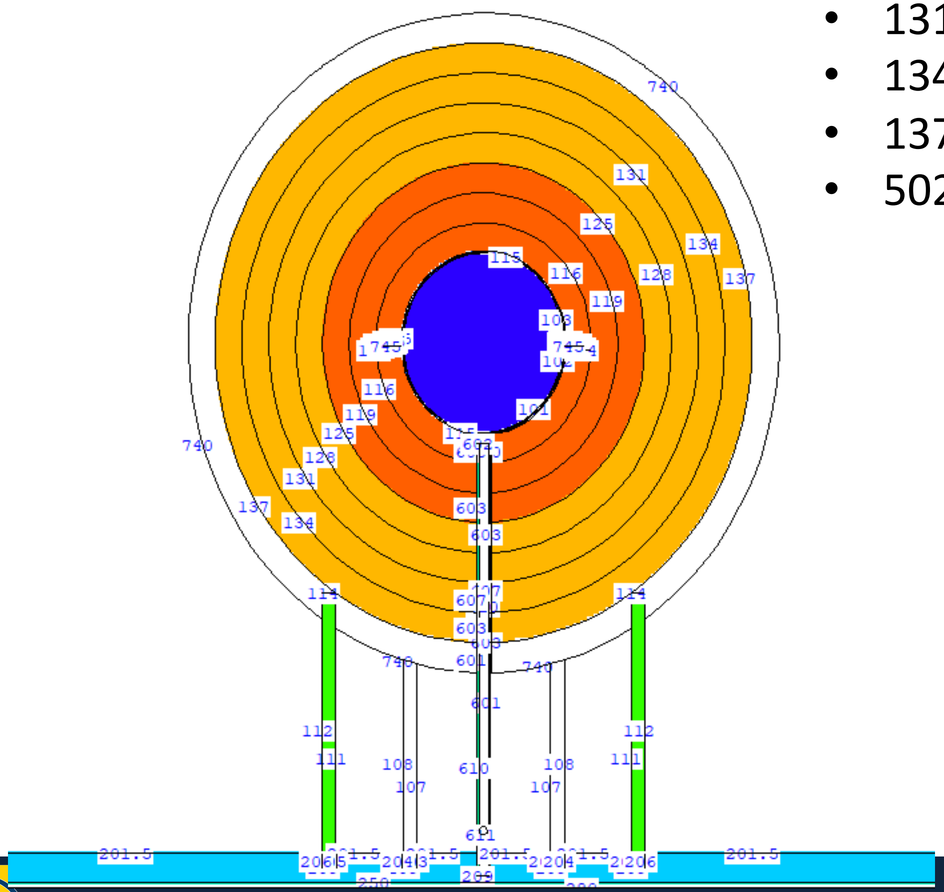


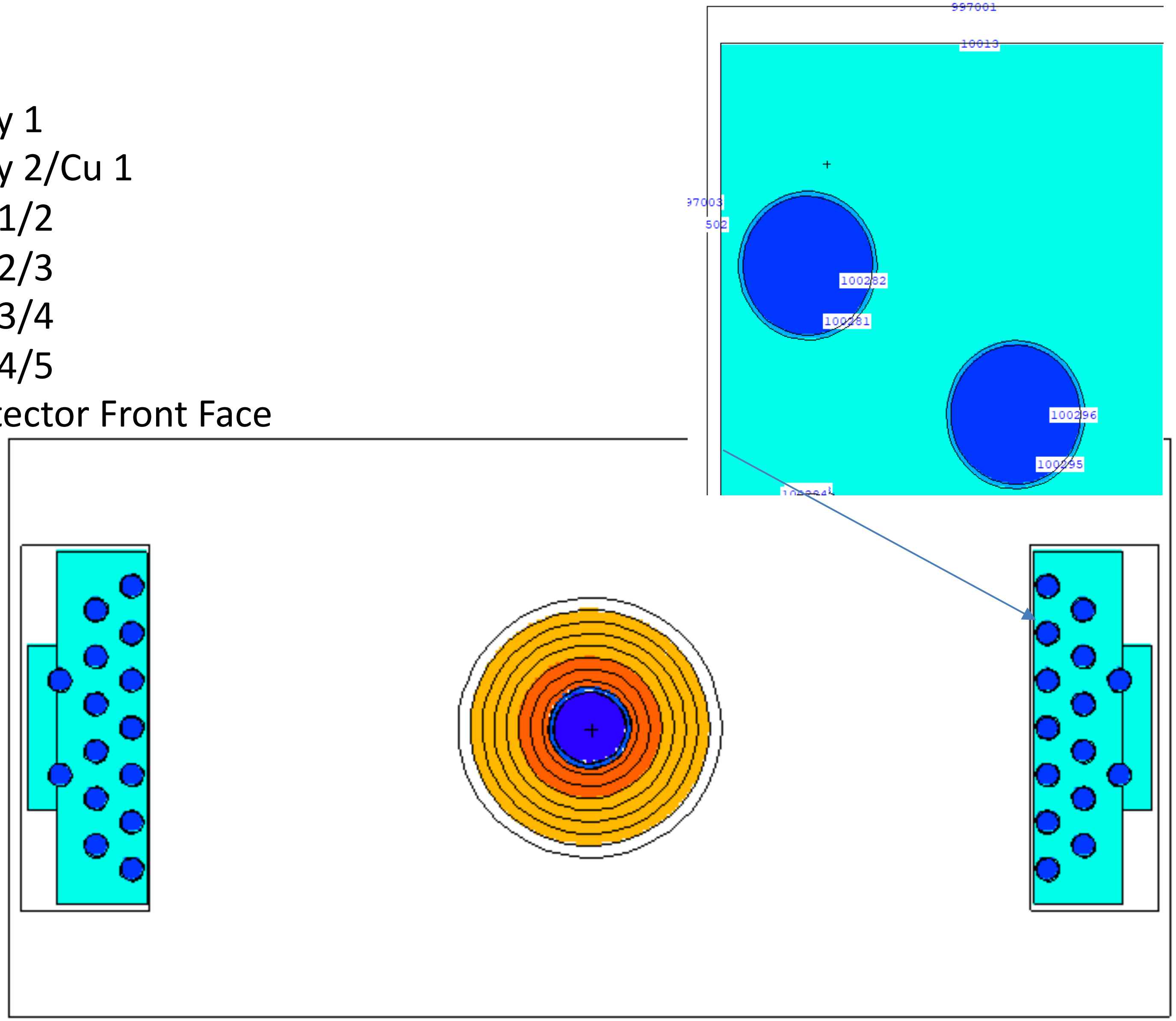
- Gamma Rays



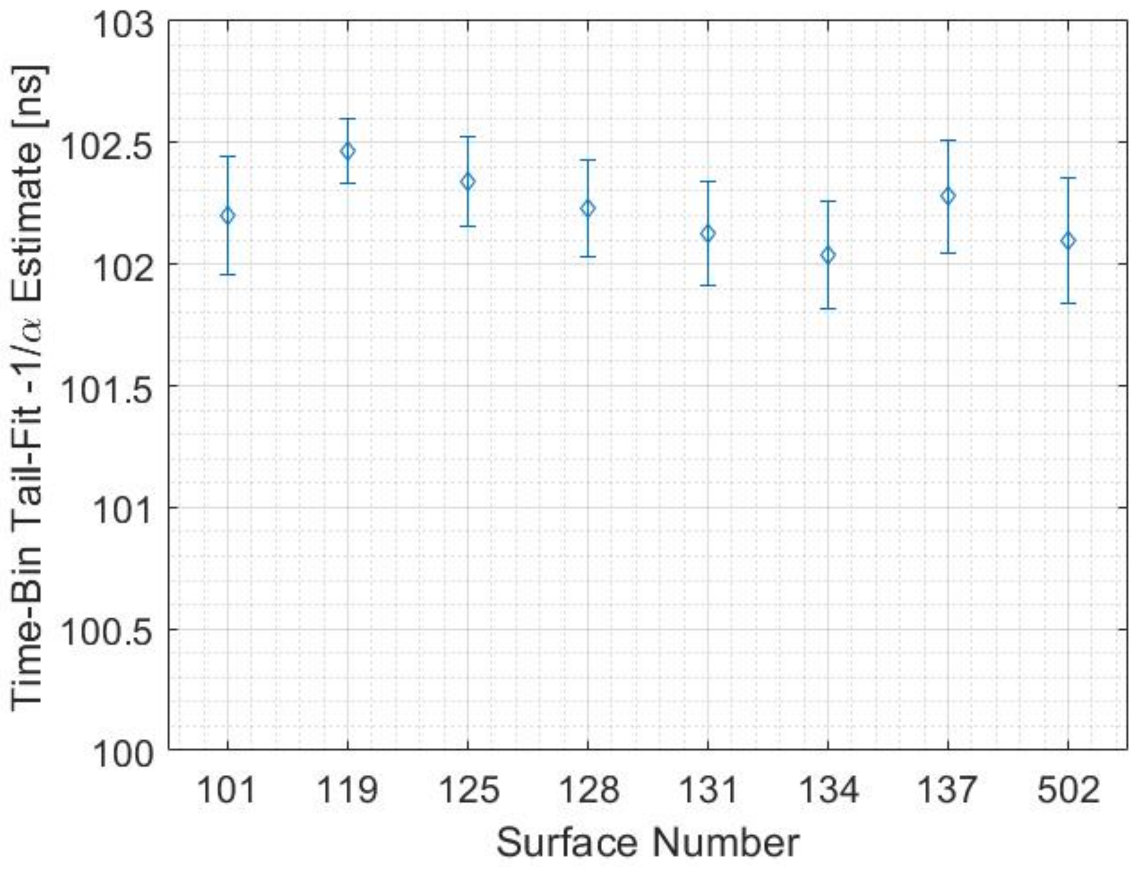
Additional Tally Surfaces

- 119 – Poly 1
- 125 – Poly 2/Cu 1
- 128 – Cu 1/2
- 131 – Cu 2/3
- 134 – Cu 3/4
- 137 – Cu 4/5
- 502 – Detector Front Face





$-1/\alpha$ Estimate from Time-Bin Tail-Fit Method is Invariant of F1 Tally Position.



- 119 – Poly 1
- 125 – Poly 2/Cu 1
- 128 – Cu 1/2
- 131 – Cu 2/3
- 134 – Cu 3/4
- 137 – Cu 4/5
- 502 – Detector Front Face

- Uncertainties are from fit uncertainty only.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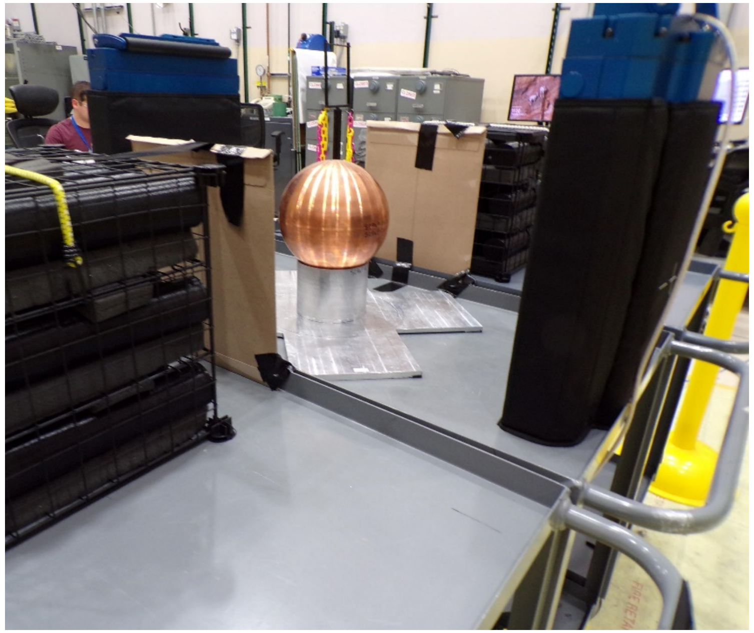
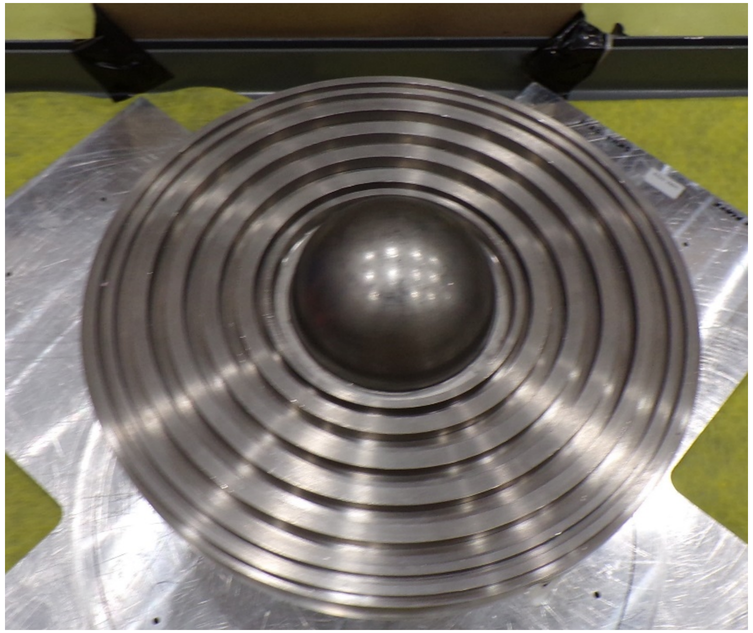
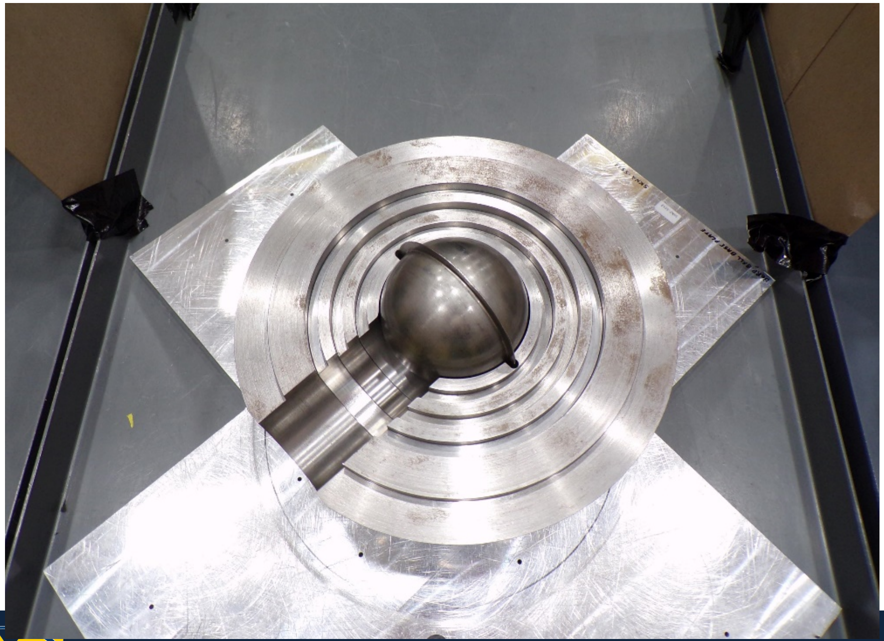
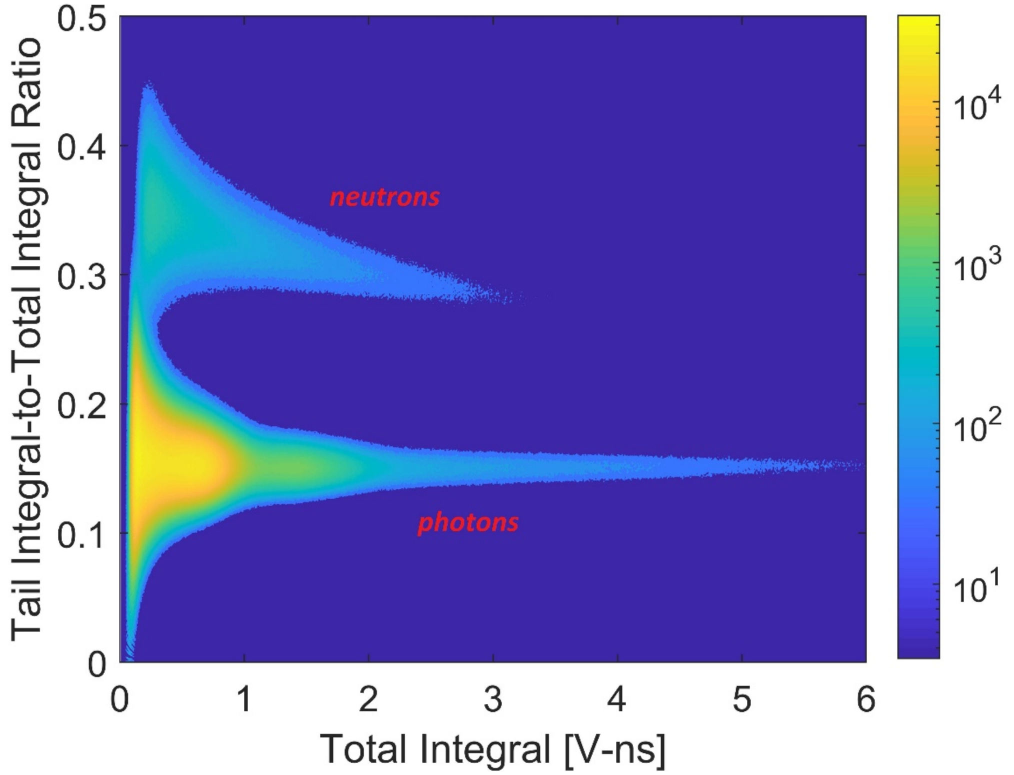


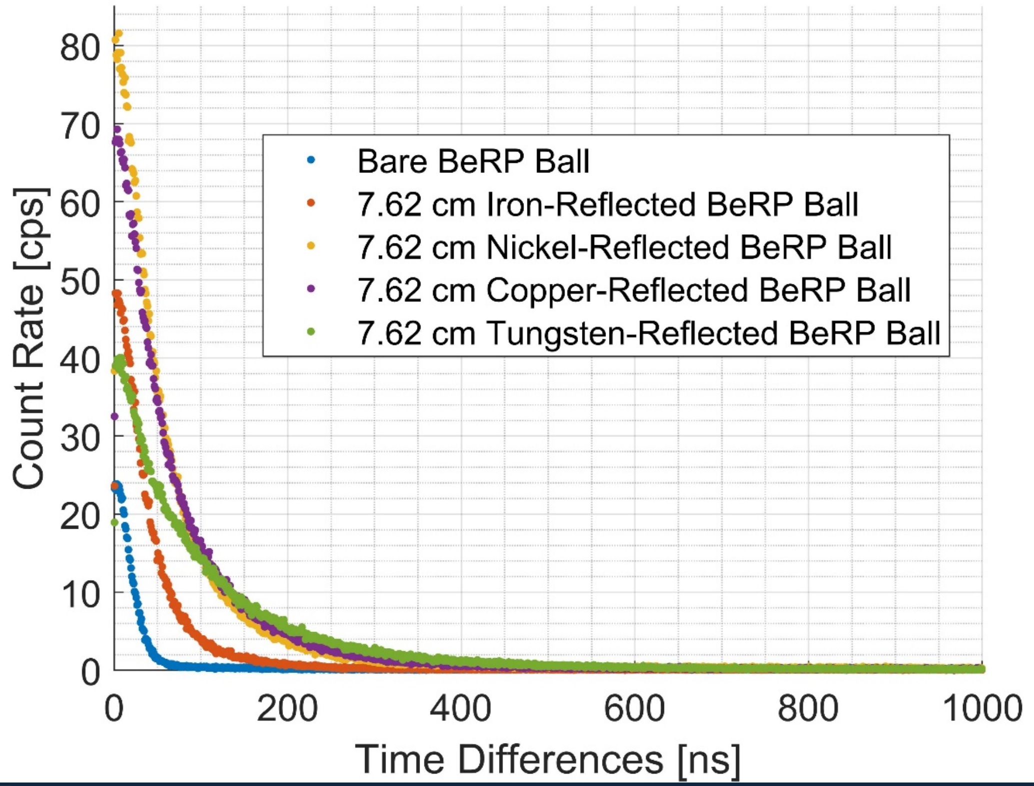


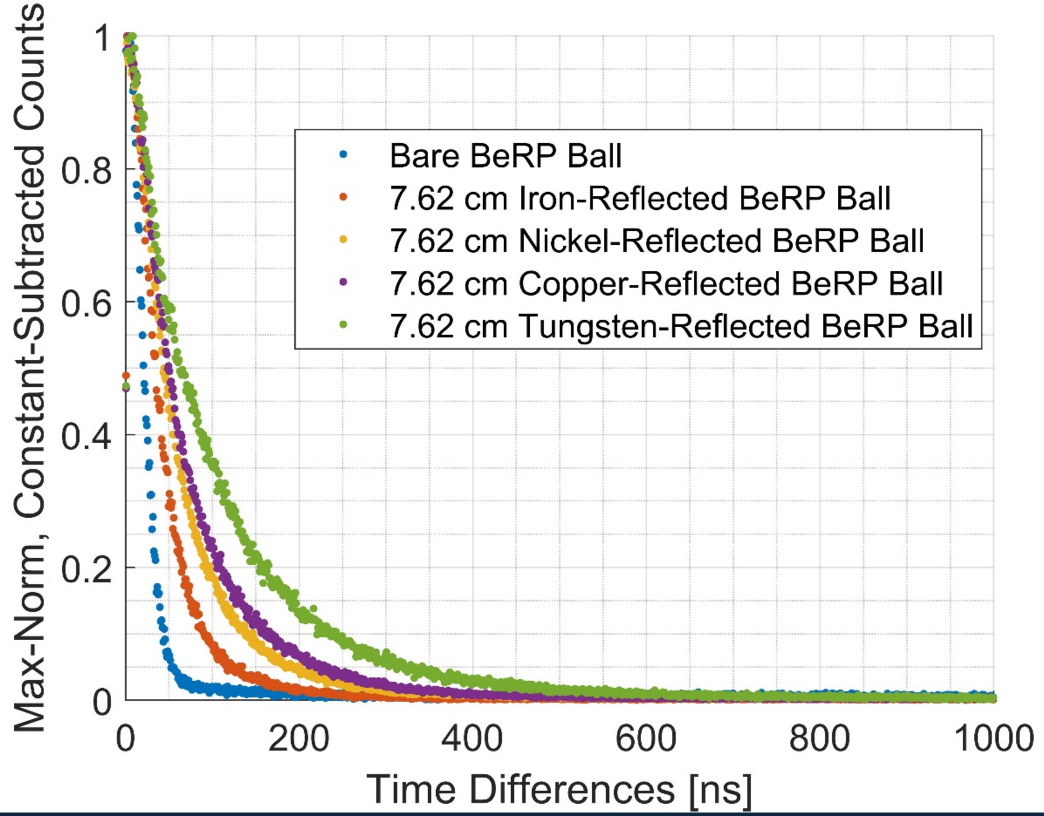


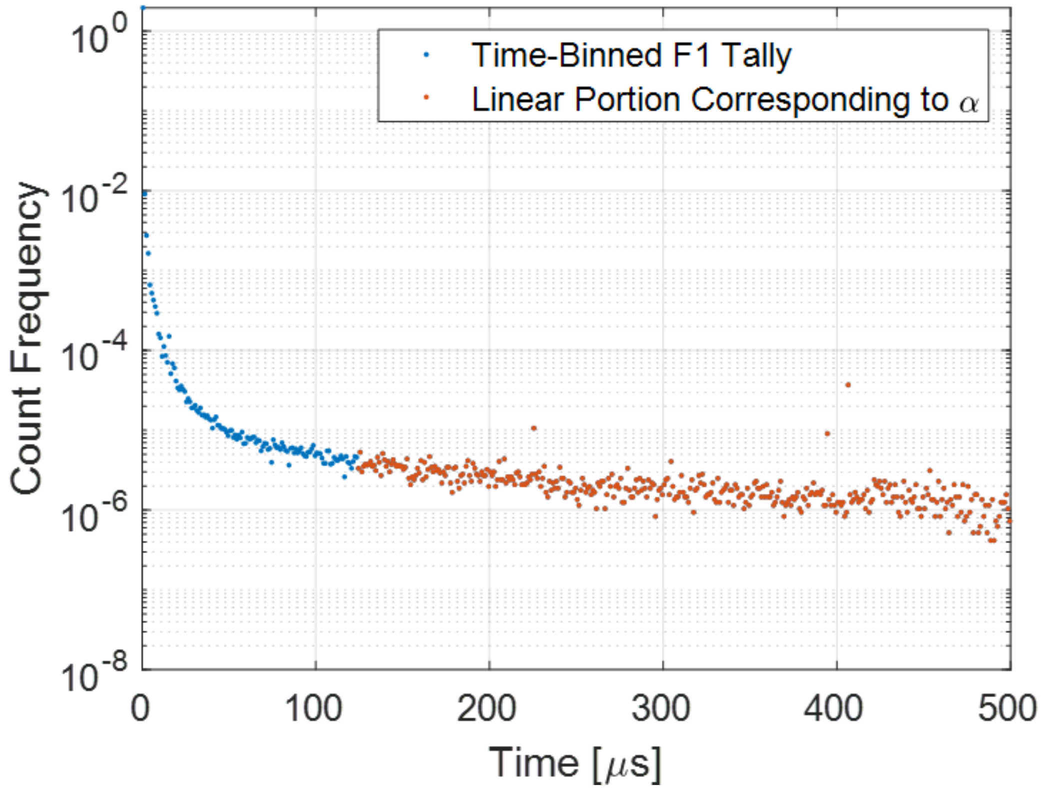
Measurement System and PSD

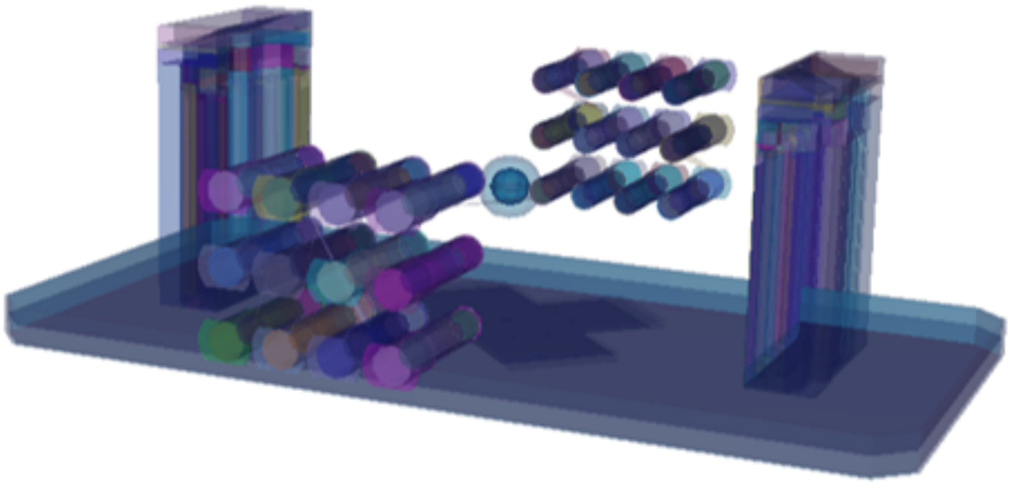
Bare BeRP Ball



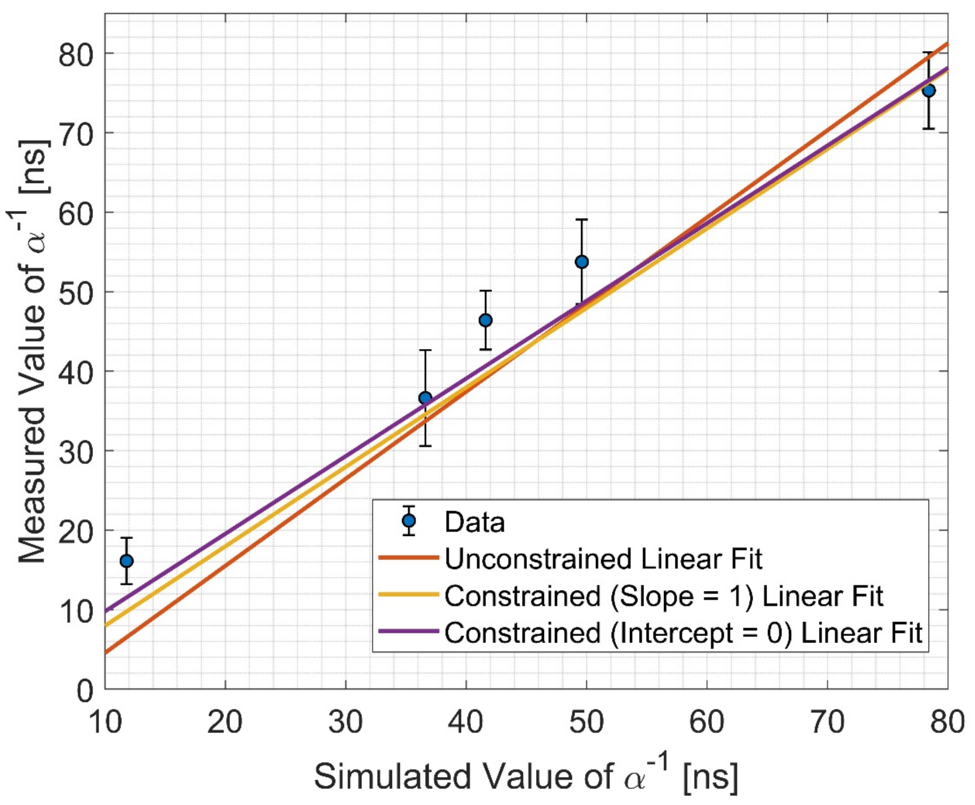

Count Rate [cps]
Time Differences [ns]
Bare BeRP Ball
7.62 cm Iron-Reflected BeRP Ball
7.62 cm Nickel-Reflected BeRP Ball
7.62 cm Copper-Reflected BeRP Ball
7.62 cm Tungsten-Reflected BeRP Ball

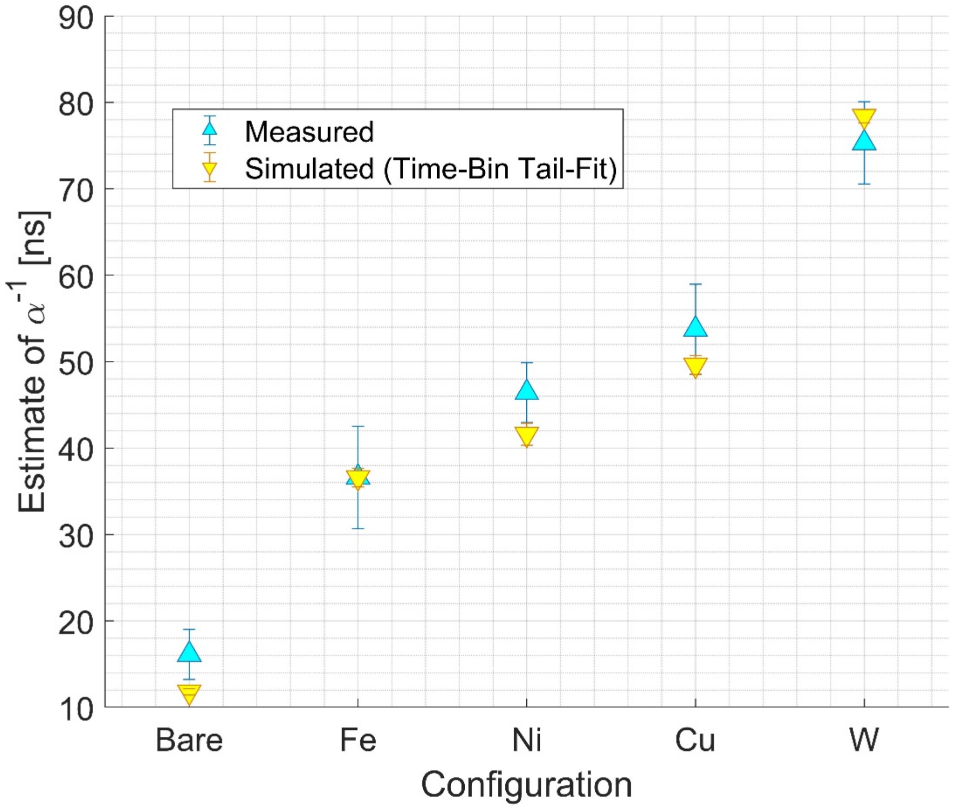

Max-Norm, Constant-Subtracted Counts
Time Differences [ns]
Bare BeRP Ball
7.62 cm Iron-Reflected BeRP Ball
7.62 cm Nickel-Reflected BeRP Ball
7.62 cm Copper-Reflected BeRP Ball
7.62 cm Tungsten-Reflected BeRP Ball


Time-Binned F1 Tally
Linear Portion Corresponding to α
Count Frequency
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
100
200
300
400
500
Time [μs]



Method One: Direct Simulation (Time-Bin Tail-Fit) and Measurement of α

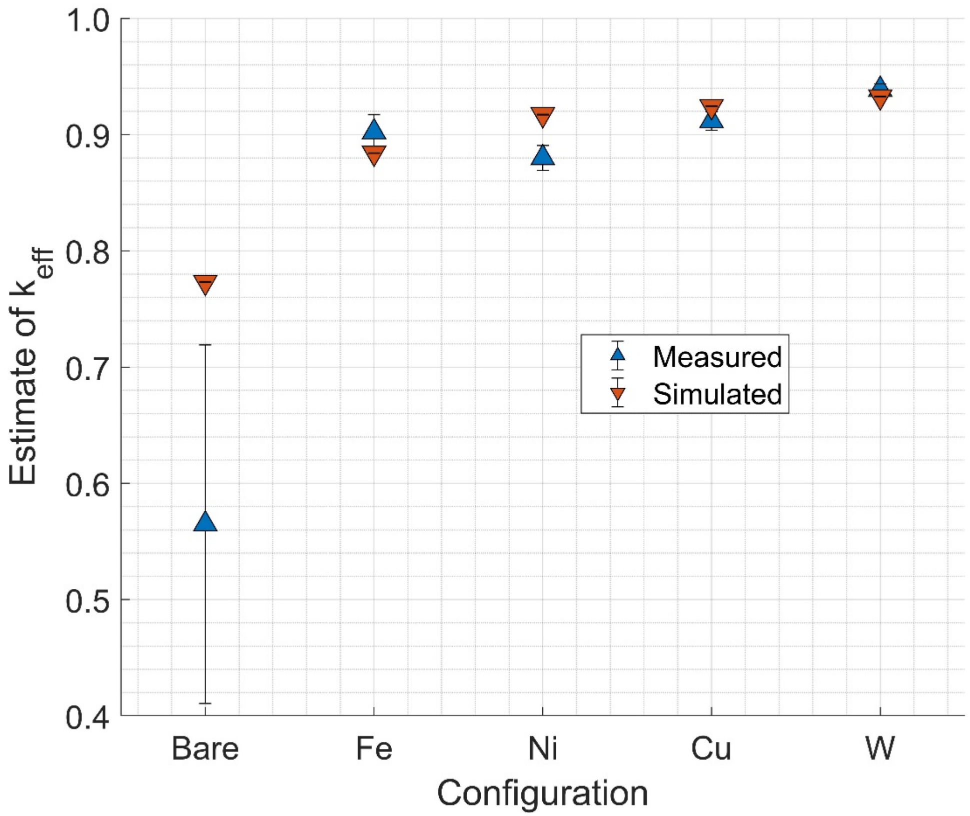


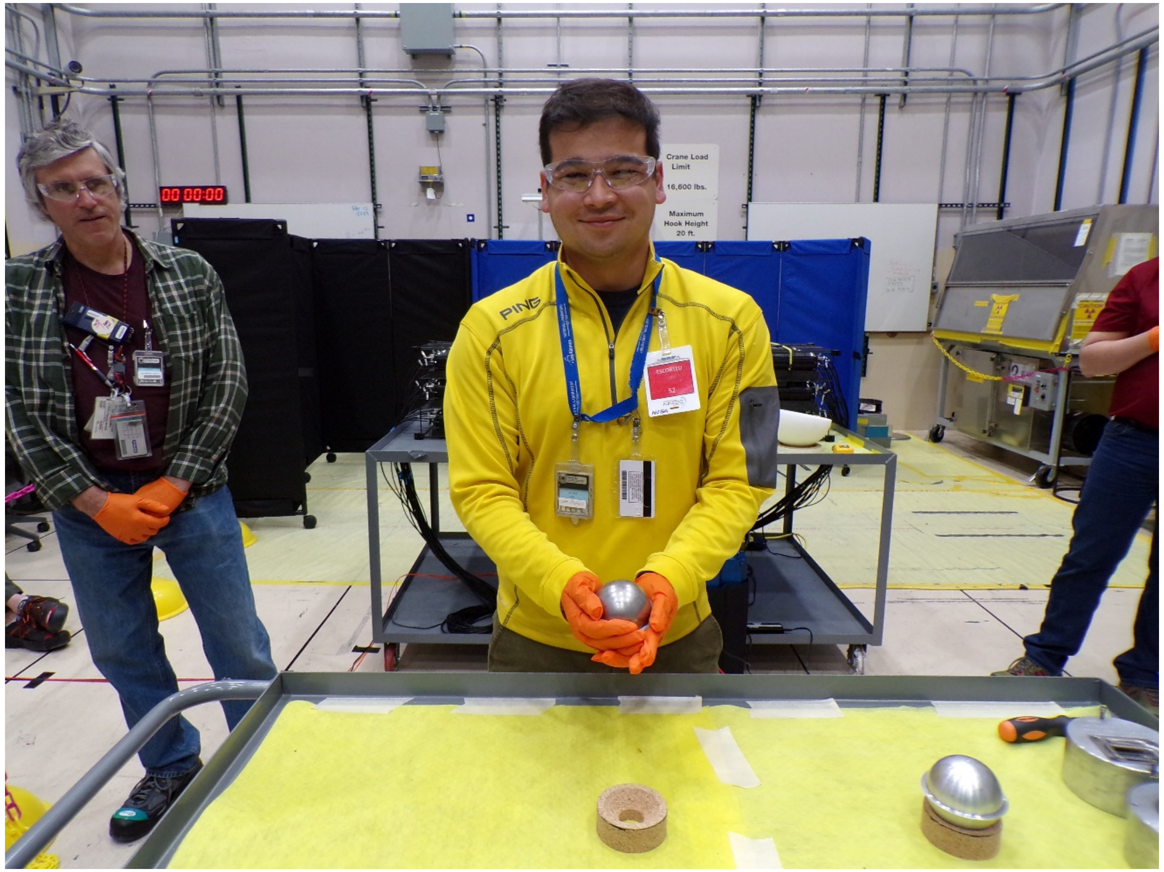


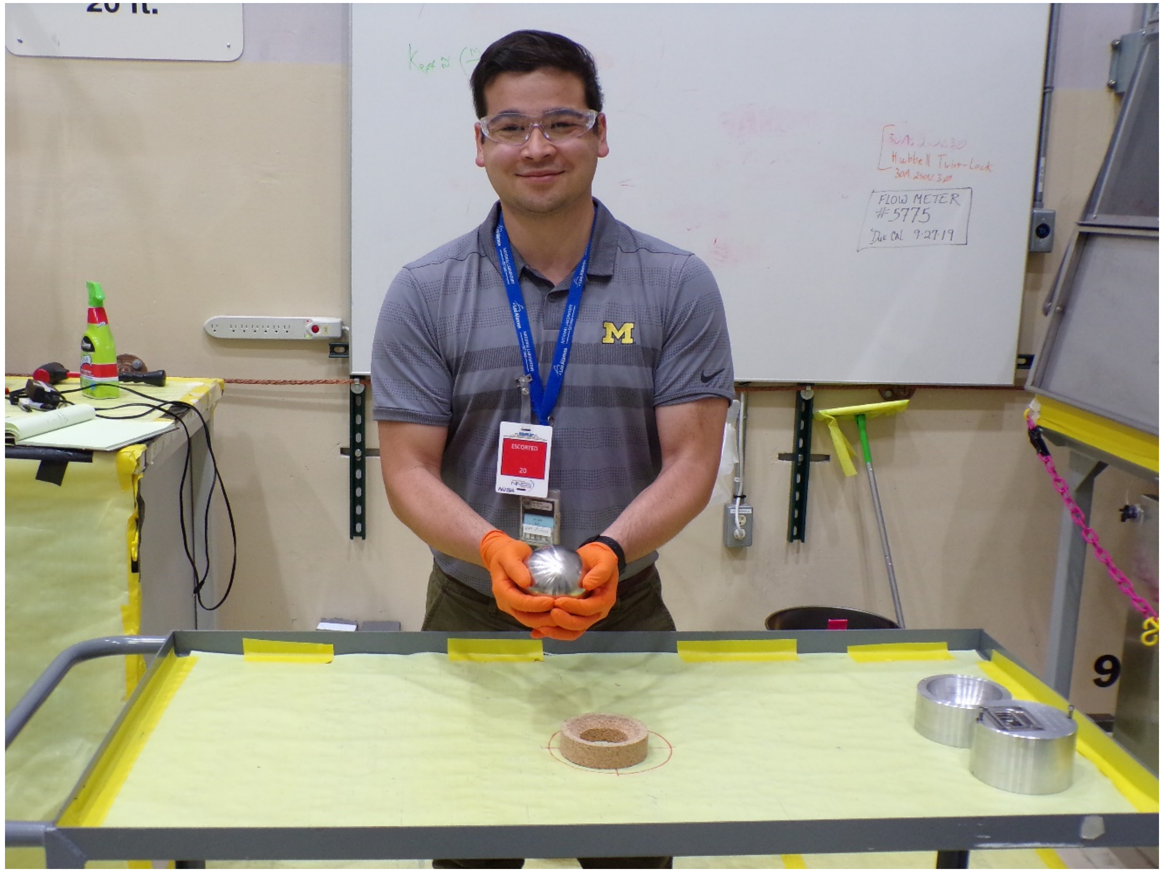
Method Two: k-Eigenvalue, Point Kinetics

- KCODE Output: k_{eff}
- KOPTS Output: β_{eff} and Λ
- Measurement: α
- Sim-Meas Inferred k_{eff}:

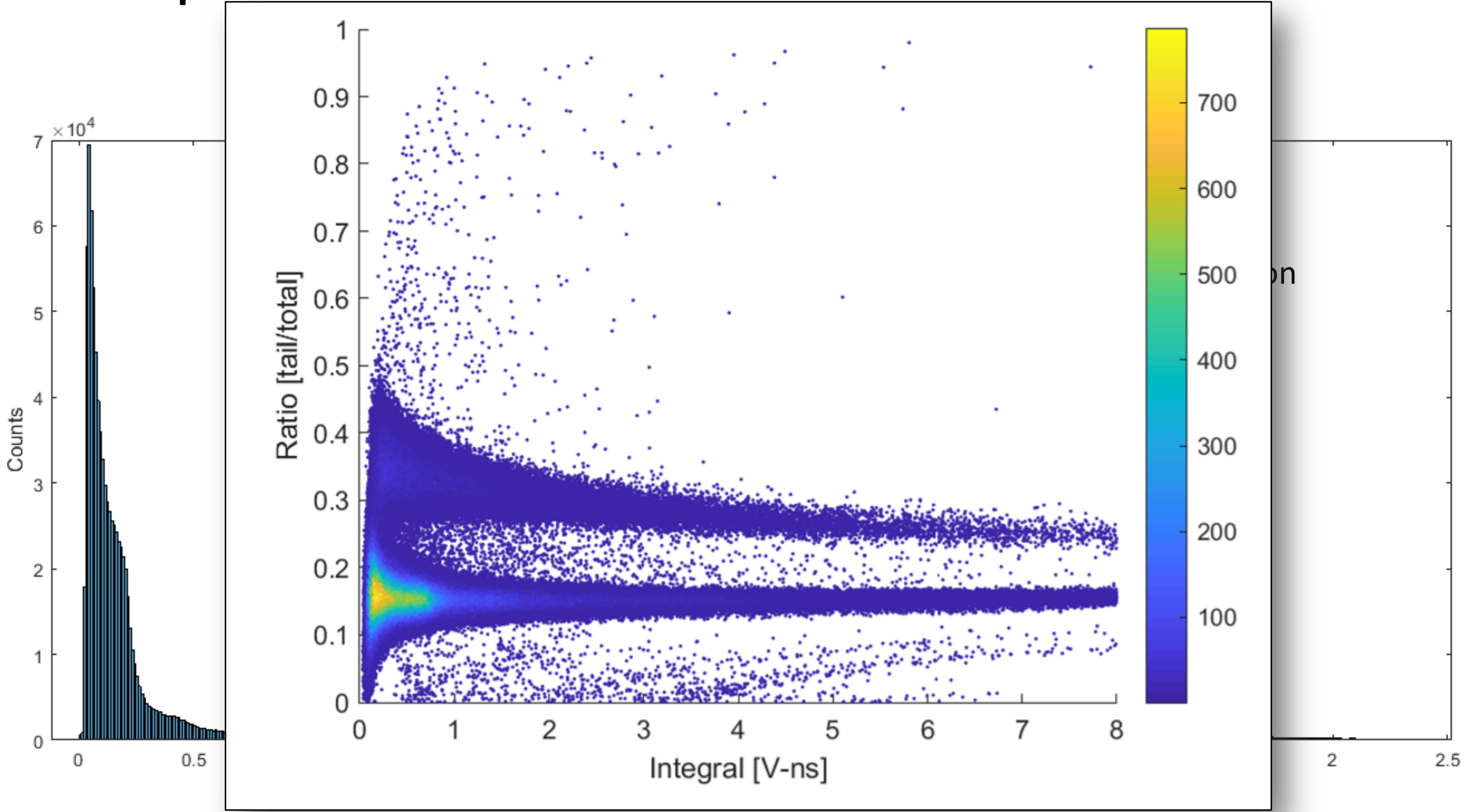
$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = \beta_{\text{eff}} + \alpha\Lambda$$




Crane Load Limit
16,600 lbs.
Maximum Hook Height
20 ft.
00:00:00
PING

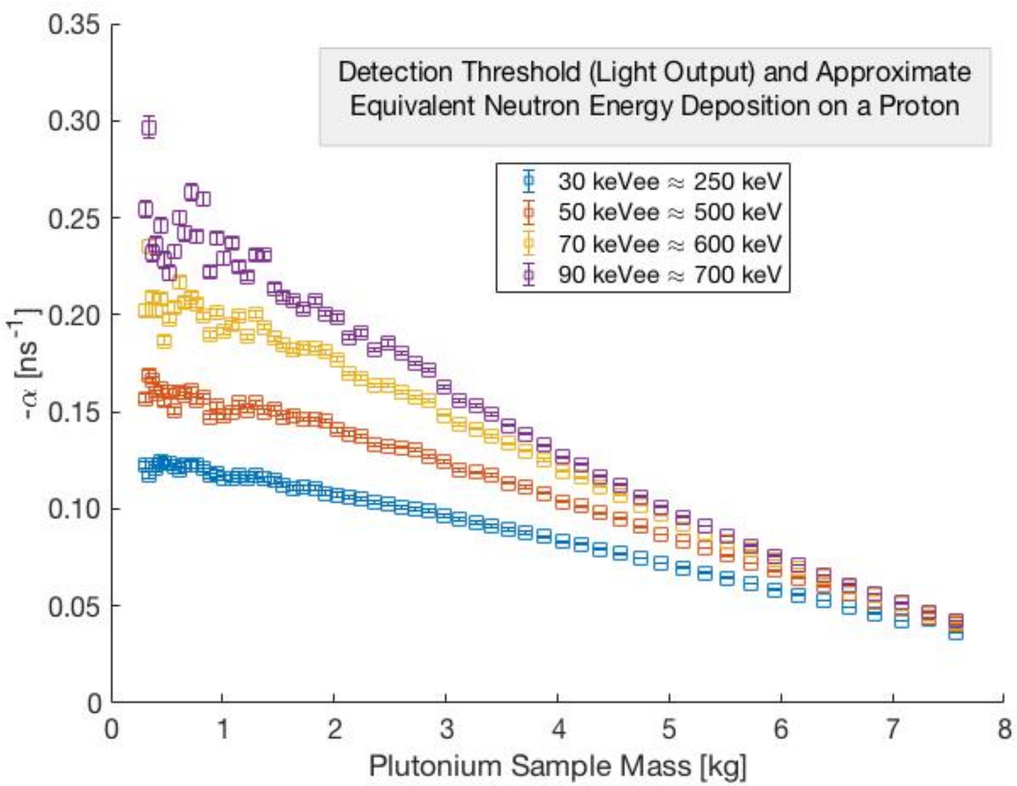

FLOW METER
#5775
M

Pulse Shape Discrimination

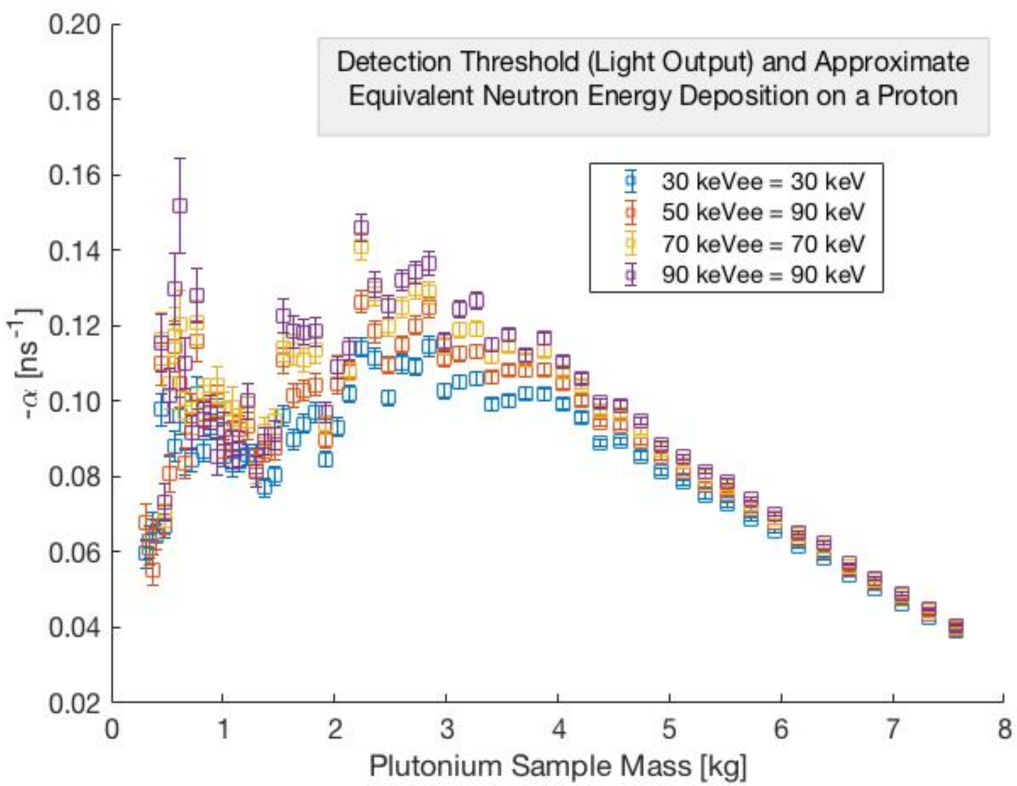


Light output thresholds affect fast-neutron Rossi-alpha more than gamma-ray Rossi-alpha.

Fast-Neutron

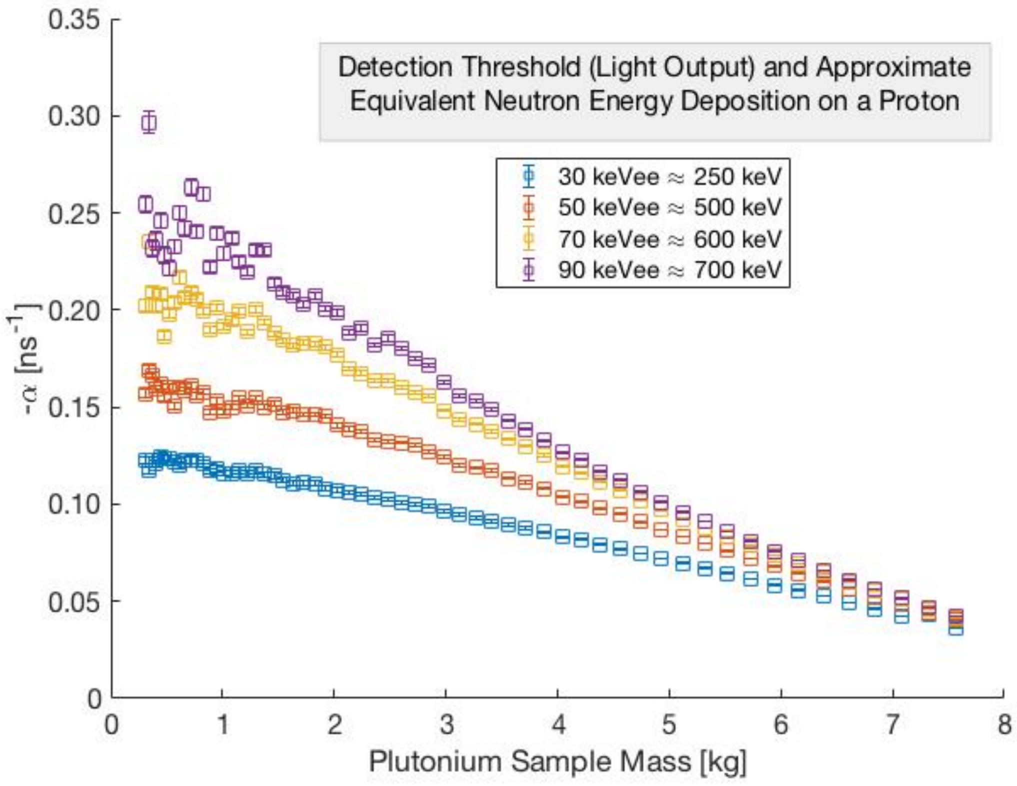


Gamma-Ray

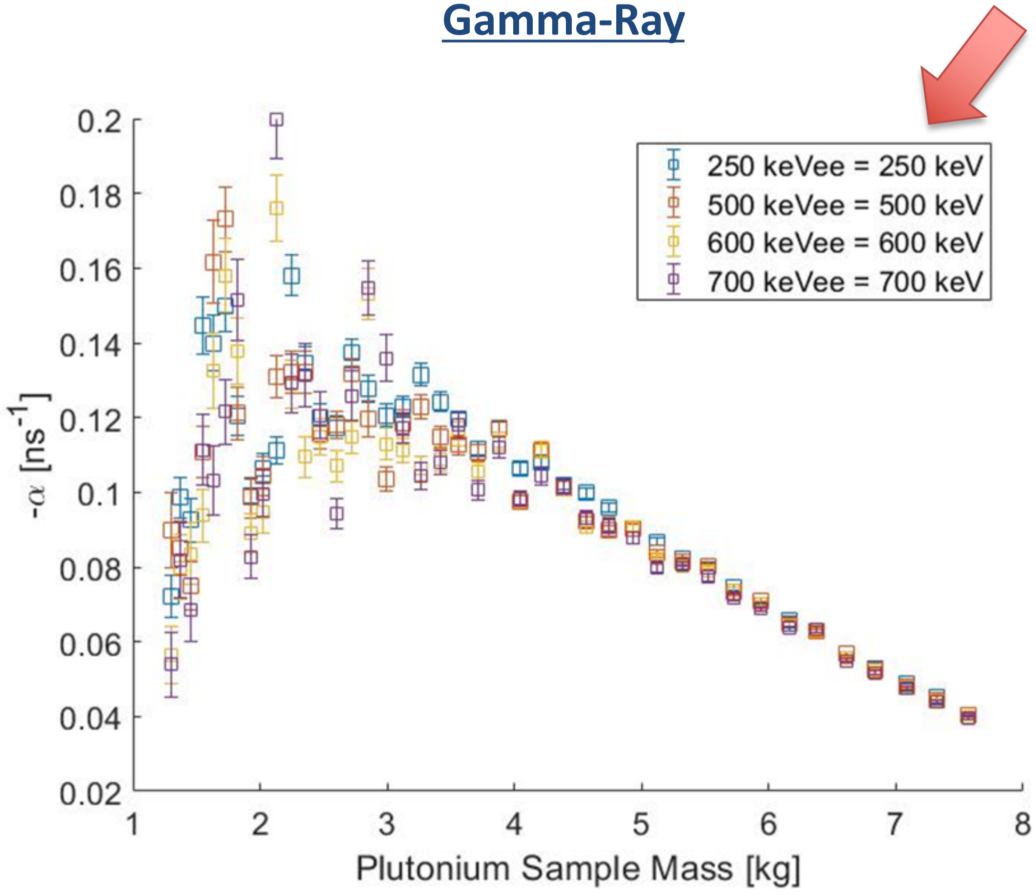


Applied thresholds have little effect on gamma-ray Rossi-alpha estimates.

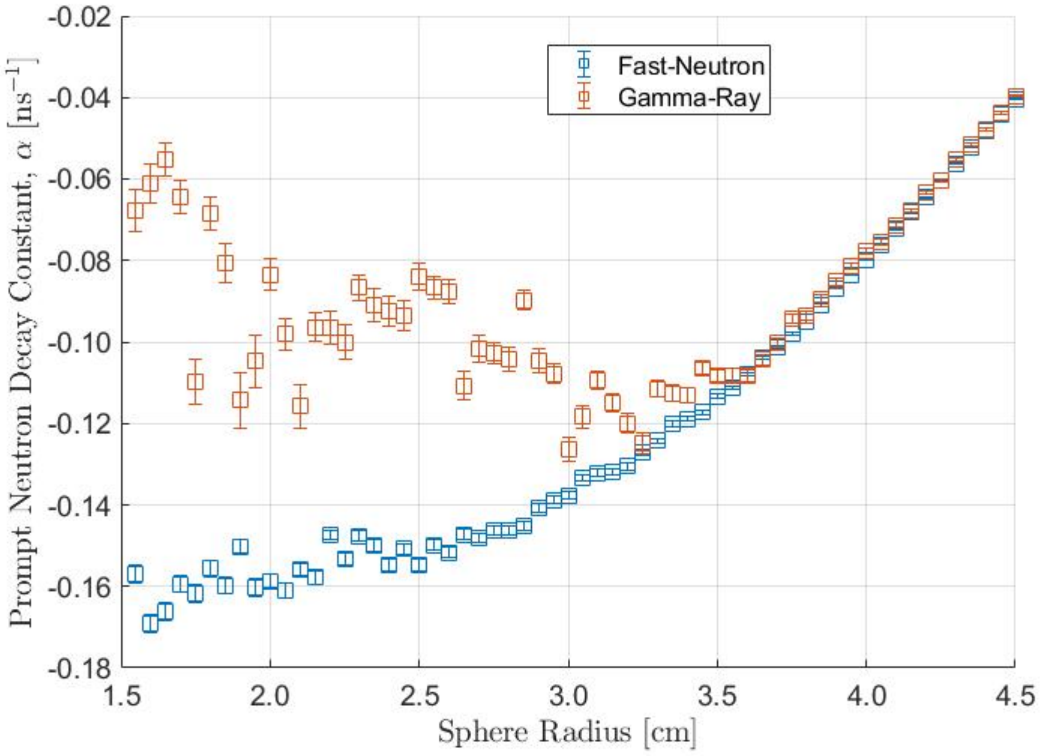
Fast-Neutron

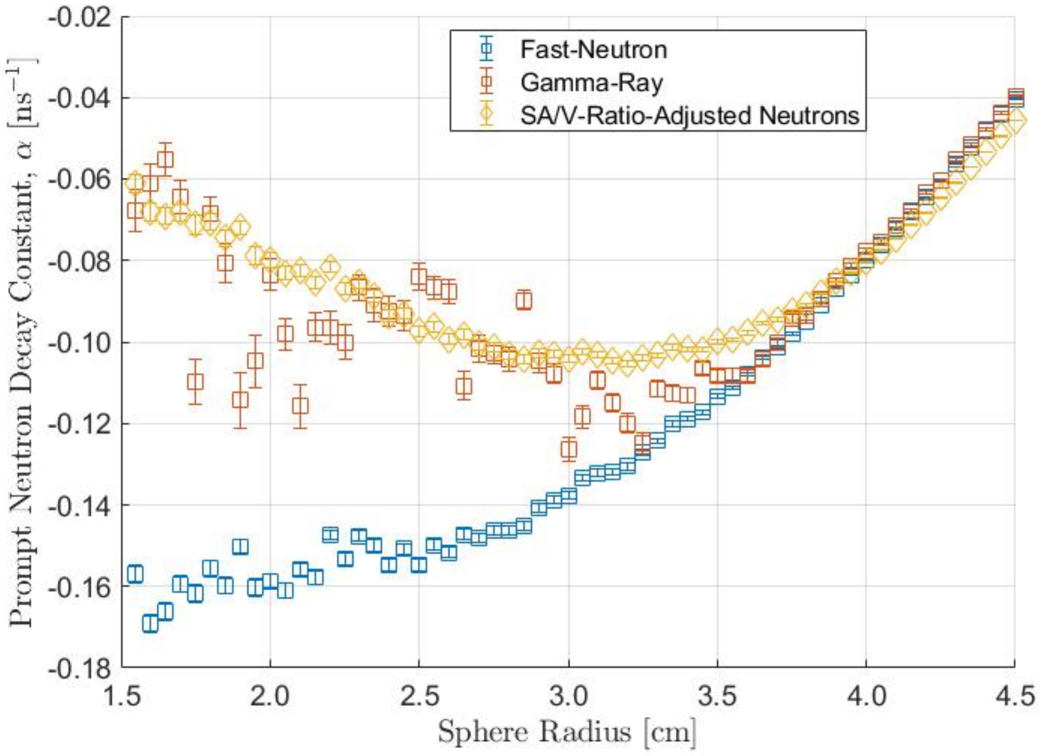


Gamma-Ray



[Spheres] Gamma rays are affected by leakage (surface area-to-volume ratio).





$$\frac{SA}{V} = \frac{4\pi r^2}{4/_3\, \pi r^3} = \frac{3}{r}$$

Stilbene Light Output from Shin et al.

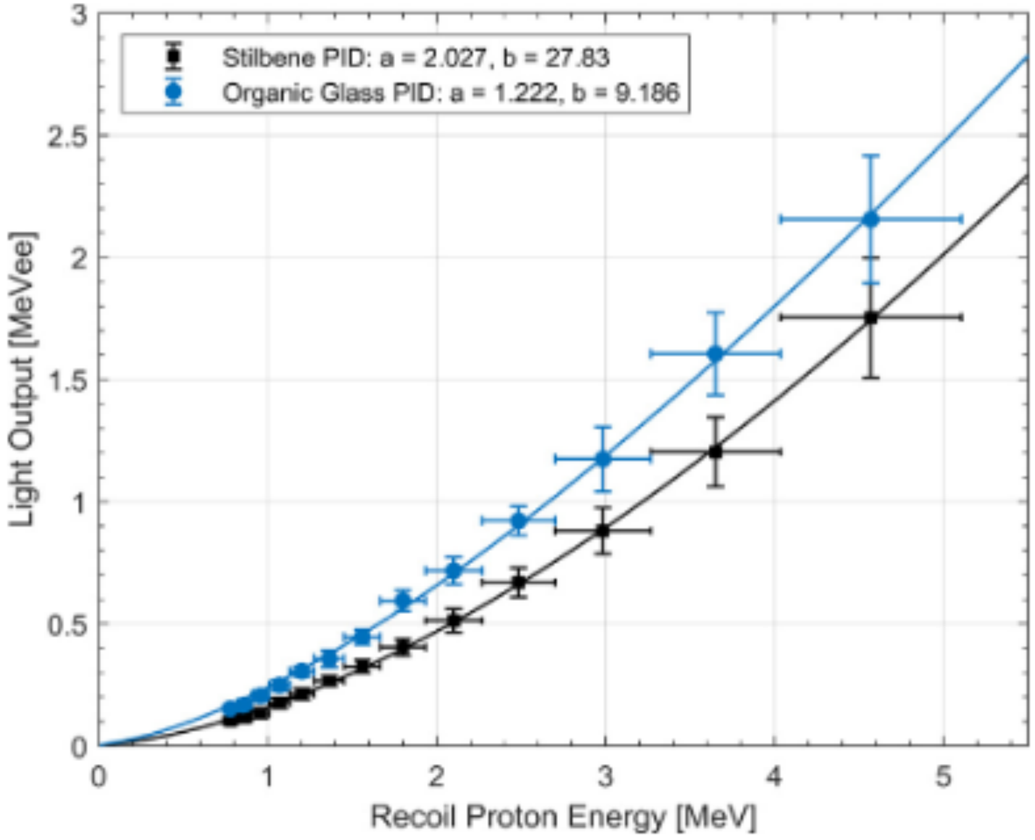


Fig. 18. Light output response using PIDs for neutron energies ranging from 0.79 ± 0.04 MeV to 4.57 ± 0.54 MeV for the stilbene and organic glass detectors, along with the semi-empirical fit function.

Tony H. Shin, Patrick L. Feng, Joseph S. Carlson, Shaun D. Clarke, Sara A. Pozzi, “Measured neutron light-output response for trans-stilbene and small-molecule organic glass scintillators,” *NIM:A*, **939**, 2019.

Prior Work

- Original derivation:

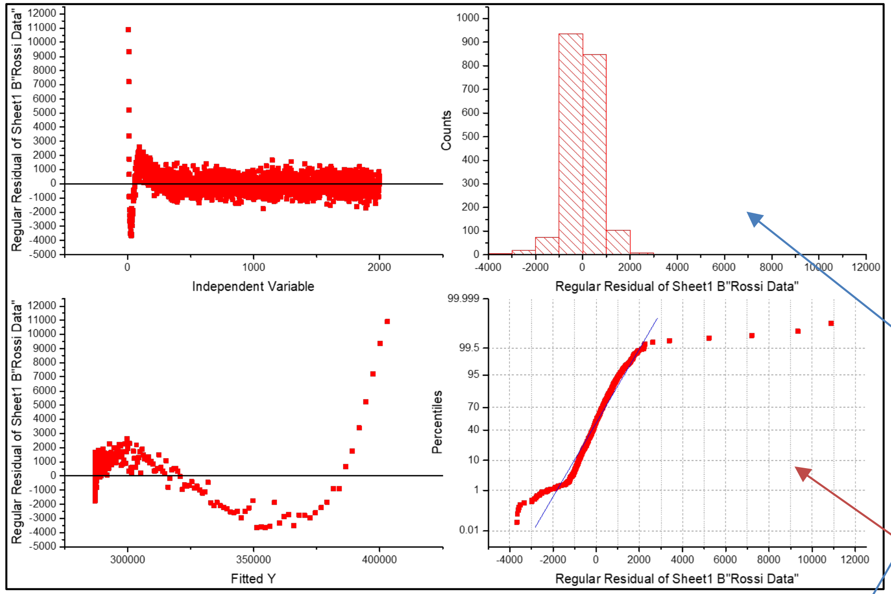
$$p(t) = A + Be^{-\alpha t}$$

- Many authors show that a two-exponential (2-exp) fit is better.

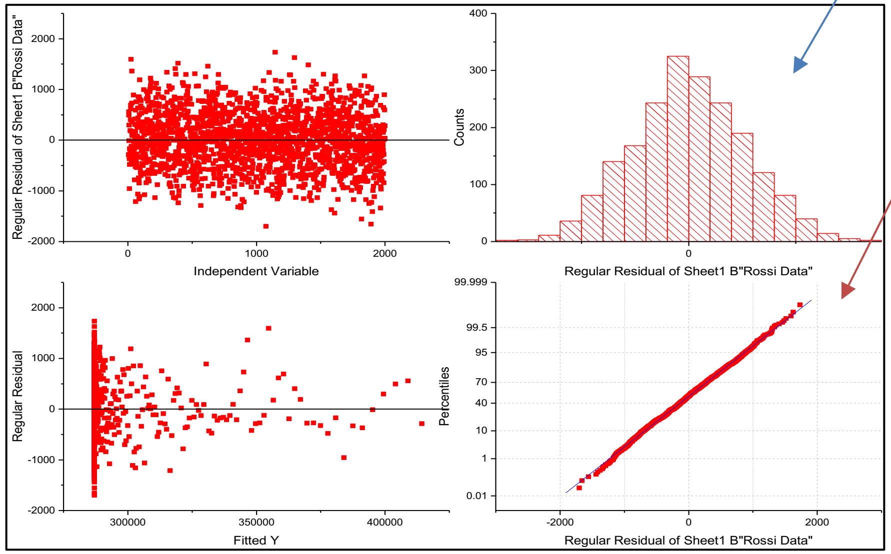
$$p(t) = A + B_1 e^{-r_1 t} + B_2 e^{-r_2 t}$$

- The second exponential is needed when there is significant amounts of reflector.

- Residual plots on the right demonstrate the superiority of a two-exponential fit.



Residual plots from one-exponential fit to Rossi-alpha histogram.



Residual plots from two-exponential fit to Rossi-alpha histogram.

Plots on left should be uniformly distributed (vertically) about 0

Should be normally distributed about 0.

Should be linear.

Plots from Hutchinson et al., ANS Winter Meeting 2017