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Countdown to 2030
Framing the Nuclear Technology Agenda for the Next 10 Years

Application of Bootstrap Method to Estimated Criticality Lower-Limit Multiplication Factor Considering Nuclear Data-Induced Uncertainty

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Highlights

- Estimated Criticality Lower-Limit Multiplication Factor (ECLLMF) is developed for an arbitrary distribution of k_{eff} using the bootstrap method
- The proposed method is applied to the consideration of
  - nuclear data-induced uncertainty of k_{eff}
- The ECLLMF was evaluated for UO_2 and concrete system

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- The ECLLMF was comparable with an upper subcritical limit without the margin of subcriticality of software

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Motivations, Background, Purposes

2. Theory

Multi-modal distribution, Bootstrap method

3. Numerical analysis

Mixture of UO_2 and concrete,

Nuclear data-induced uncertainty,

Comparison of ECLLMF and USL

4. Conclusion

Summary, Future works, Q&A

Motivations of the present study

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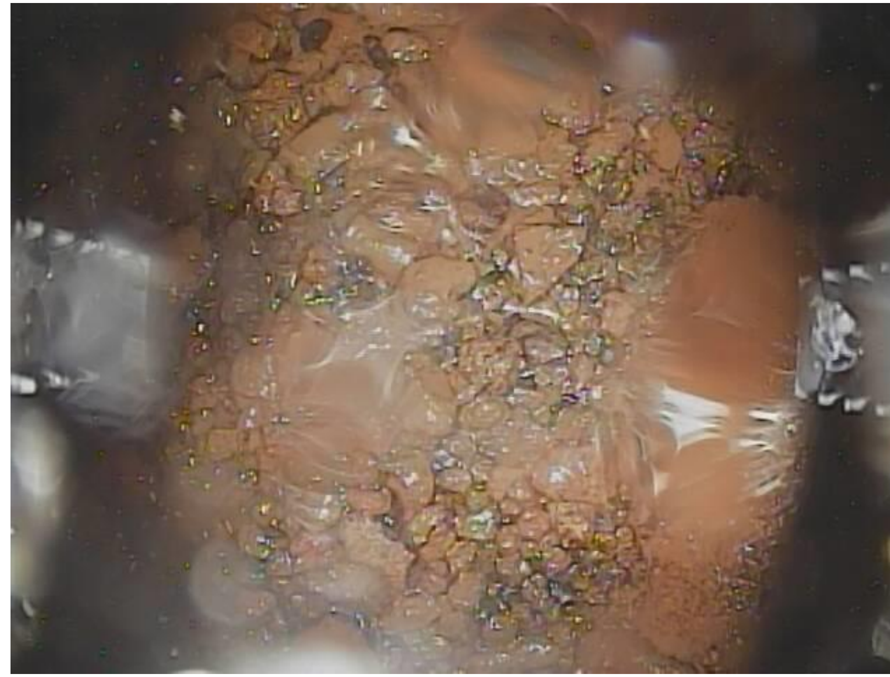
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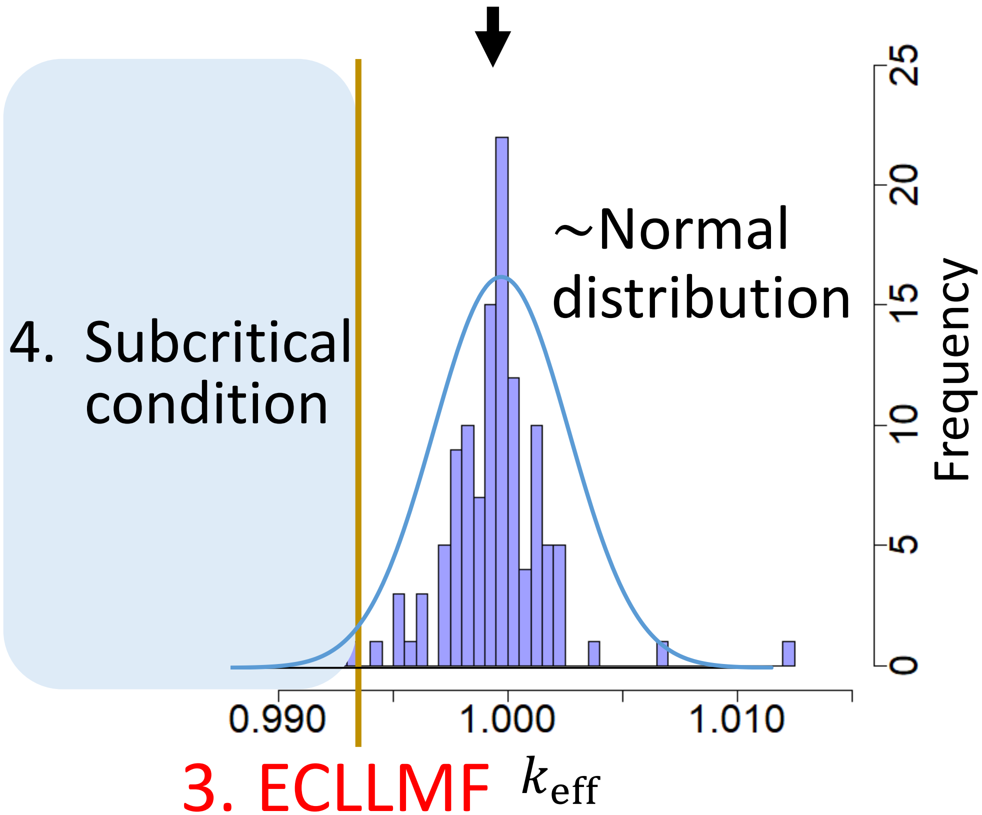
Criticality safety

- Concrete contains light elements (*e.g.*, H, C, Si,…)
- It is necessary to investigate criticality of MCCI products
- Critical or subcritical condition?

Judgment of subcritical condition

1. Modeling of target system
2. Criticality calculation of benchmark experiments



ECLLMF: Estimated Criticality Lower-Limit Multiplication Factor

- The assumption of the normal distribution is necessary for the conventional ECLLMF[1]

[1] H.Okuno, J. Nucl. Sci. Technol, 44(2), pp. 137-146 (2007).

Purposes of the present study

- Develop an ECLLMF for an arbitrary distribution of k_{eff}[2]
- Consider:
  - neutronic similarity of benchmark experiments[3]
  - experimental and statistical uncertainties of k_{eff}[3]
  - nuclear data-induced uncertainty of k_{eff}
- Evaluate the ECLLMF for the mixture of UO_2 and concrete
- Compare the ECLLMF with USL evaluated by Whisper

[2] T. Hayashi et al., "Calculation Method of Estimated Criticality Lower-Limit Multiplication Factor Using the Bootstrap Method," *Proc. M&C 2019* (2019).
[3] T. Hayashi et al., "Estimated Criticality Lower-Limit Multiplication Factor Considering Neutronic Similarity and Uncertainties of Effective Multiplication Using the Bootstrap Method," *Proc. RPHA19* (2019).

Calculation flow

Phase 1: Multi-modal distribution

- Standard deviation of k_{eff}
- Weighting factor of benchmark experiment
- Mean of k_{eff}
- Define multi-modal distribution as k_{eff}-uncertainty

Phase 2: Bootstrap method

- Random sampling with replacement
- Determine margin of subcriticality from k_{eff}-uncertainty
- Calculate ECLLMF

Phase 1: Multi-modal distribution

1. Synthesize experimental, statistical, and cross-section uncertainties for i-th benchmark:

$$\sigma_i = \sqrt{\sigma_{\mathrm{exp},i}^2 + \sigma_{\mathrm{calc},i}^2 + \sigma_{\mathrm{XS},i}^2}$$

   - $\sigma_{\mathrm{exp},i}$: experimental uncertainty of benchmark data
   - $\sigma_{\mathrm{calc},i}$: statistical uncertainty of a continuous energy Monte Carlo
   - $\sigma_{\mathrm{XS},i}$: cross-section uncertainty after applying the GLLS method[4]

2. Calculate weighting factor w_i for i-th benchmark:

$$w_i = \frac{c_{k,i} - c_{k,\mathrm{acc}}}{c_{k,\mathrm{max}} - c_{k,\mathrm{acc}}}$$

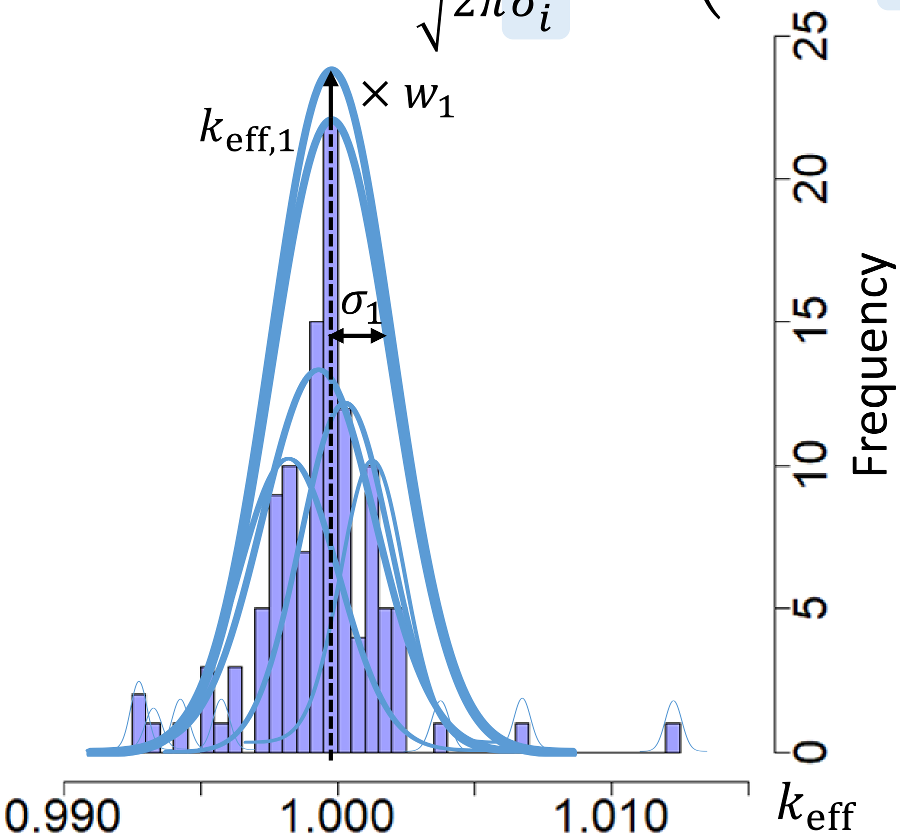
   - $c_{k,i}$: correlation coefficient of sensitivity for nuclear data
   - $c_{k,\mathrm{acc}}$: acceptable value of c_k
   - $c_{k,\mathrm{max}}$: maximum value of c_k

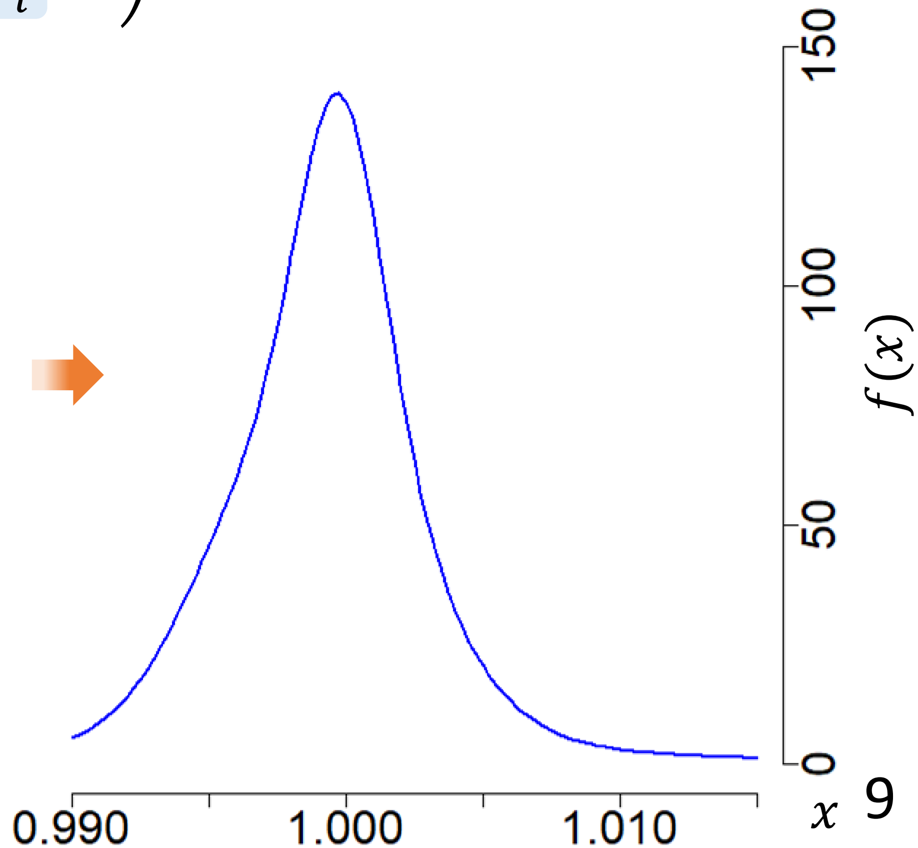
[4] Sensitivity and Uncertainty Analyses Applied to Criticality Safety Validation, NUREG/CR-6655, NRC (1999).

Phase 1: Multi-modal distribution

3. Calculate $k_{\mathrm{eff},i} = k_{\mathrm{calc},i} - (k_{\mathrm{exp},i} - 1)$ for i-th benchmark
4. Define a multi-modal distribution as:

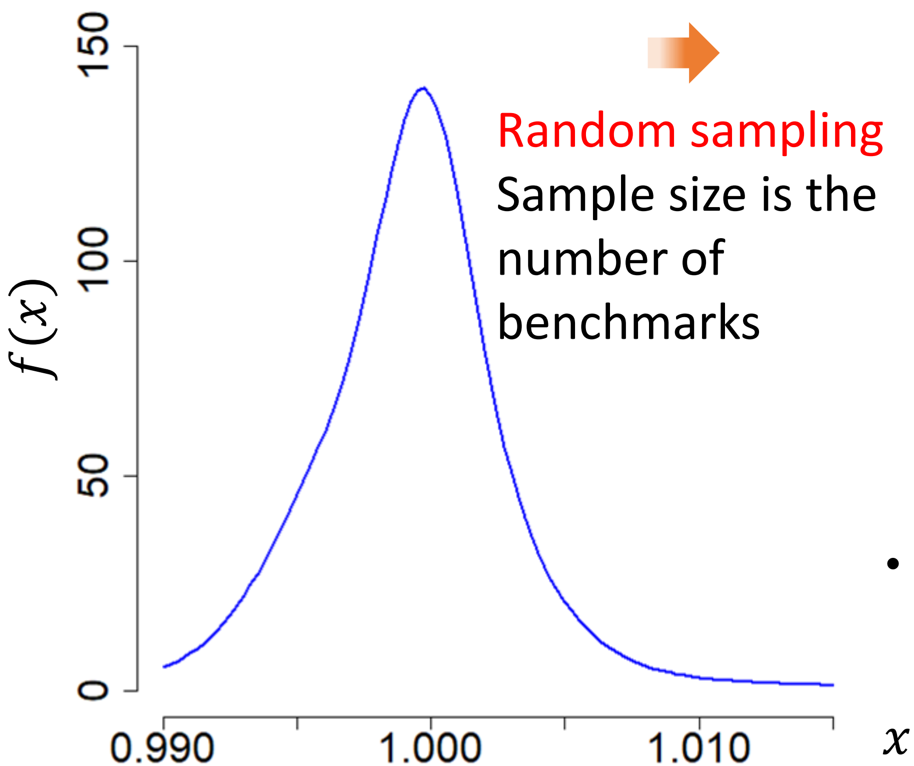
$$f(x) = \sum_{i=1}^{n} \frac{w_i}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(x - k_{\mathrm{eff},i})^2}{2\sigma_i^2}\right)$$



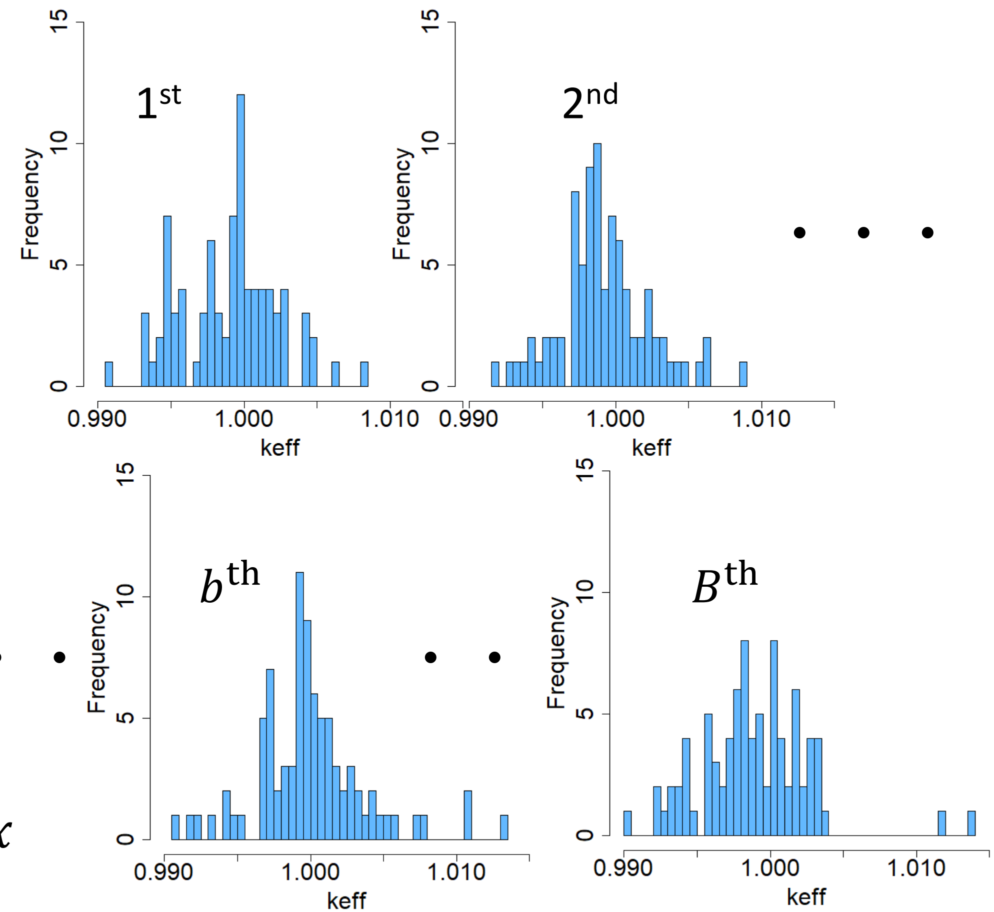


Phase 2: Bootstrap method

5. Generate B resamples from the multi-modal distribution



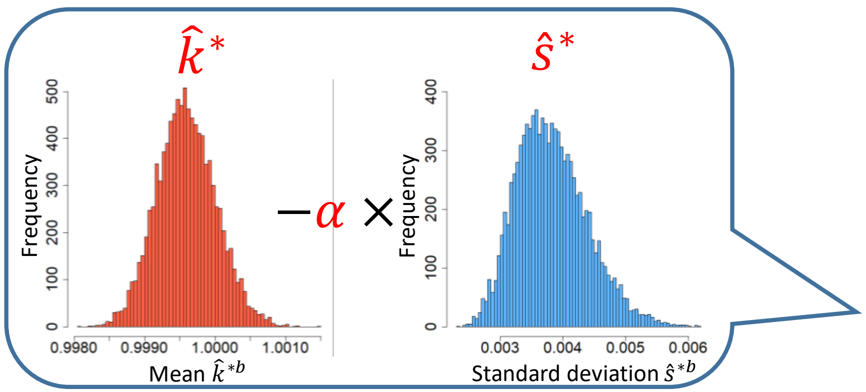
Random sampling
Sample size is the number of benchmarks

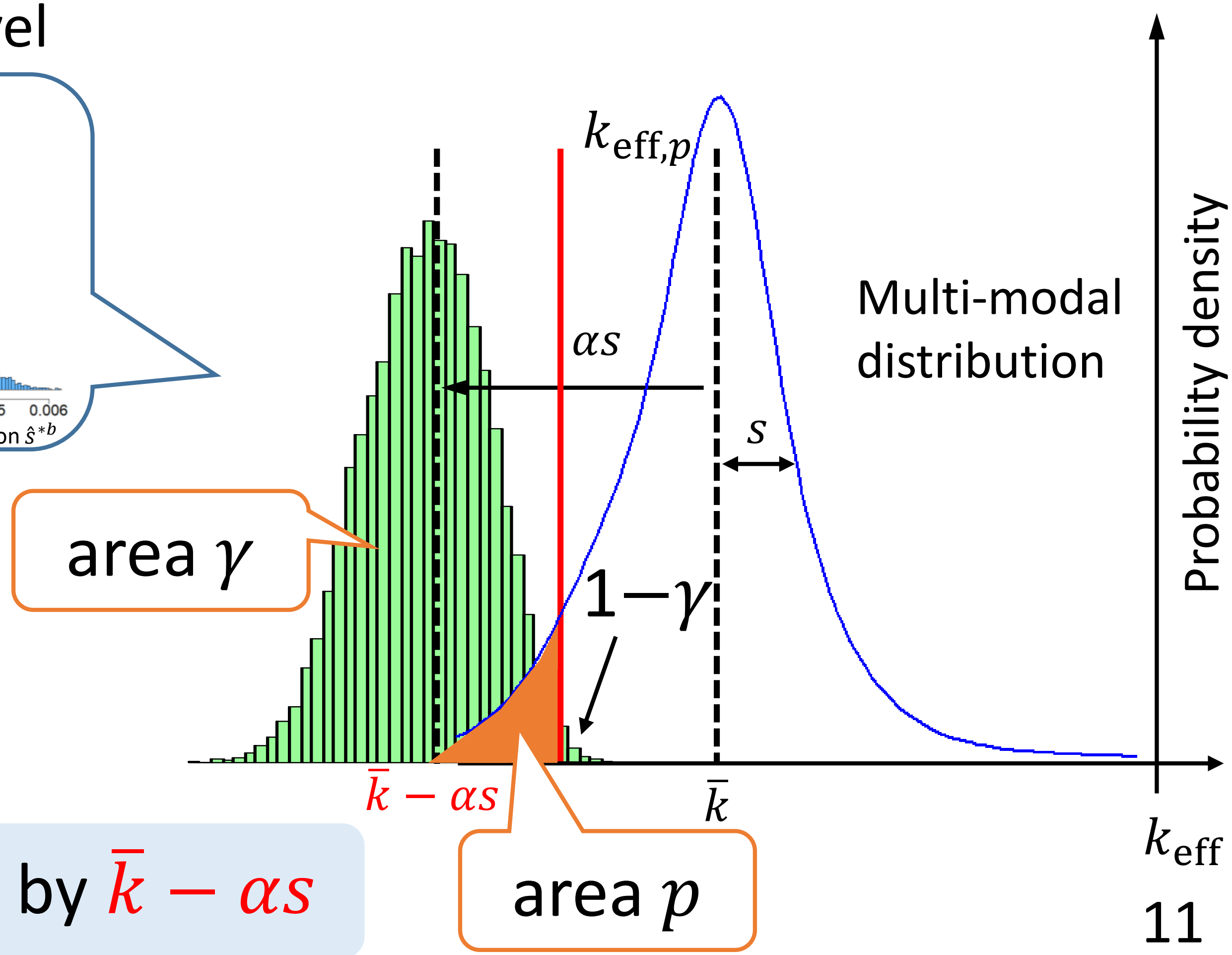


6. Calculate mean $\hat{k}^{*b}$ and standard deviation $\hat{s}^{*b}$ of bootstrap samples $(1 \leq b \leq B)$

Phase 2: Bootstrap method

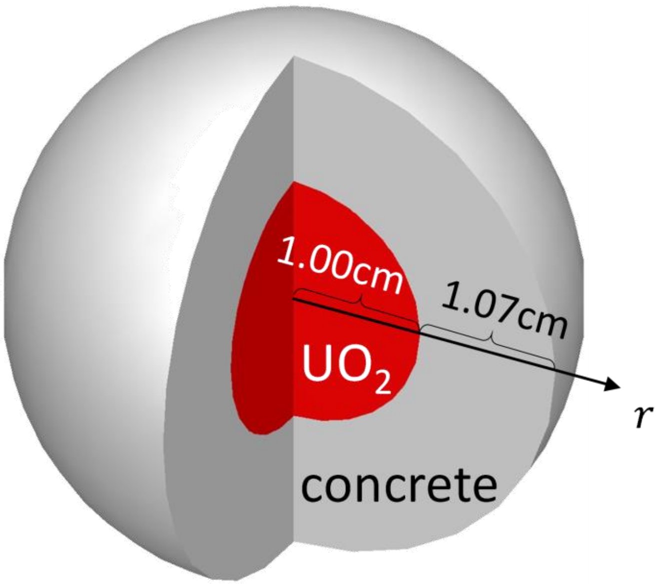
7. Determine margin of subcriticality α
   - p: probability of excess critical condition
   - γ: confidence level





8. Evaluate ECLLMF by $\bar{k}-\alpha s$

Calculation conditions



Geometry

- Modeling of MCCI products
- White boundary
- Optimum moderation
- 293K
- $p = 1\%, \gamma = 99\%$ for ECLLMF

Composition[5]

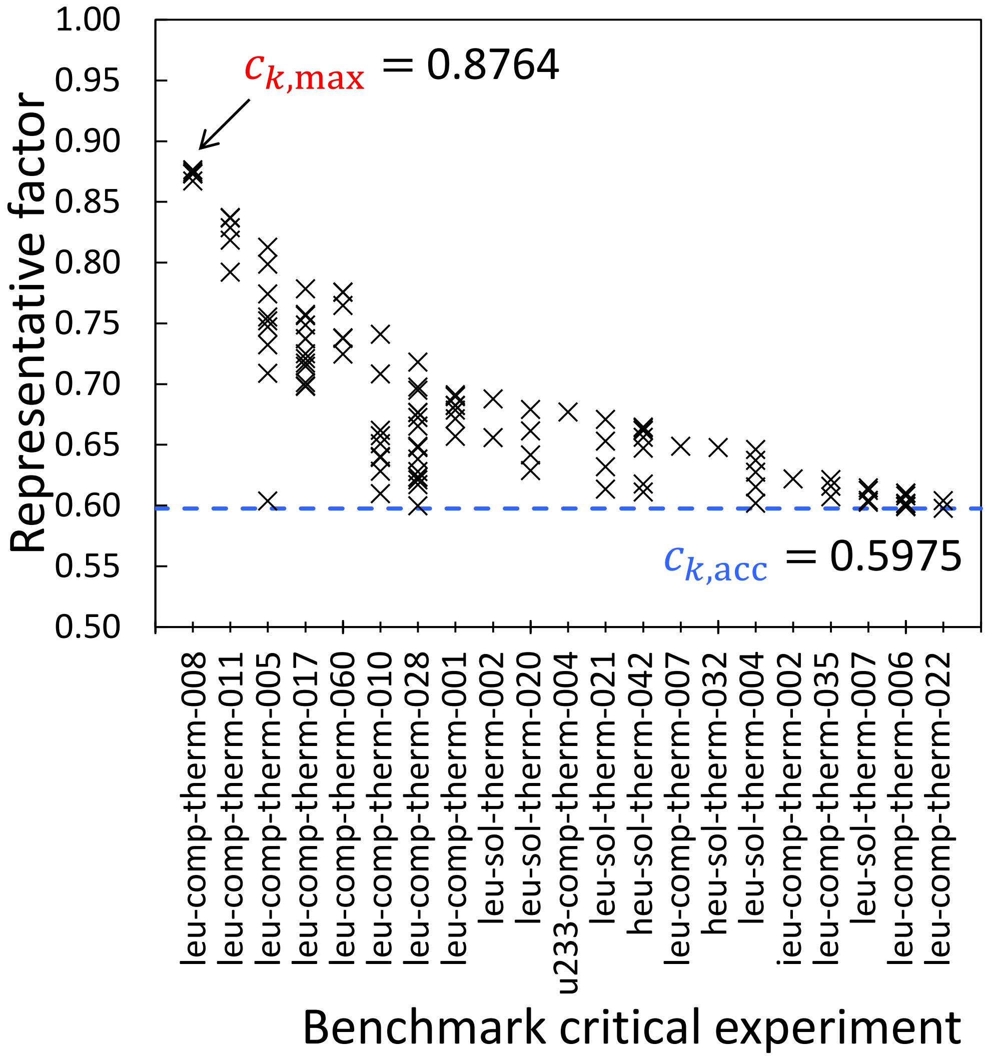
	Nuclide	Atomic number density (atoms/barn-cm)
UO_2 (5 wt.%)	U-235	1.1757e-3
	U-238	2.2057e-2
	O	4.6465e-2
Concrete (2.3 g/cm^3)	H	1.3742e-2
	O	4.5921e-2
	C	1.1532e-4
	Na	9.6397e-4
	Mg	1.2389e-4
	Al	1.7409e-3
	Si	1.6617e-2
	K	4.6054e-4
	Ca	1.5026e-3
	Fe	3.4507e-4

[5] K. Izawa et al., J. Nucl. Sci. Technol., **49**(11), pp.1043–1047 (2012).

Benchmark critical experiments

- Whisper code[6]:
  - evaluates representative factor c_k between the target system and critical experiments
  - 118 benchmark critical experiments
  - The $c_{k,\mathrm{acc}}$ value was satisfied:

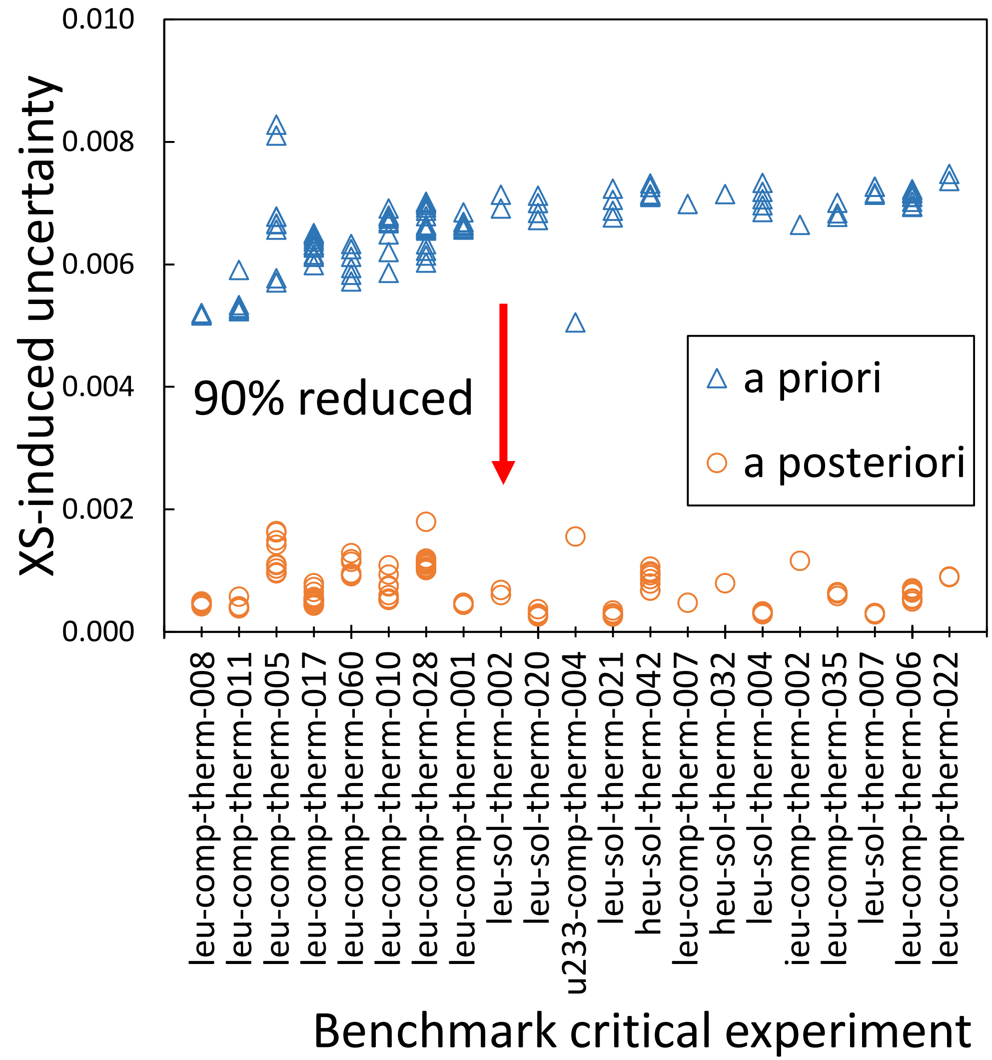
$$\sum_{i=1}^{n} w_i \geq \left(25 + 100\left(1 - c_{k,\max}\right)\right)$$



[6] B. C. Kiedrowski *et al.*, *Nucl. Sci. Eng.*, **181**(1), pp.17–47 (2015).

Nuclear data-induced uncertainty

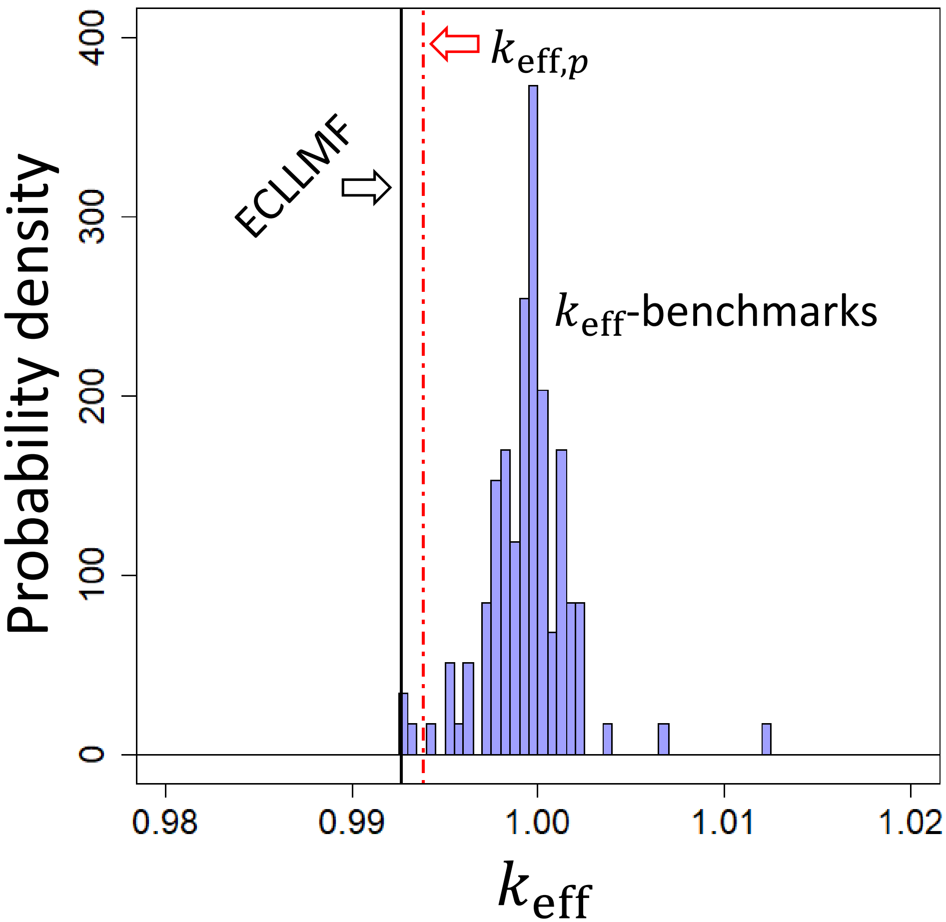
- SCALE6.2.3/TSURFER[7]:
  - The sdf-formatted k_{eff}-sensitivity files were obtained by converting the MCNP6.2 output files
  - TSURFER evaluated a posteriori nuclear data-induced uncertainty



[7] B. T. Rearden, M. A. Jessee, ORNL/TM-2005/39, ORNL (2018).

Results of conventional ECLLMF

☑ Assumption of the normal distribution



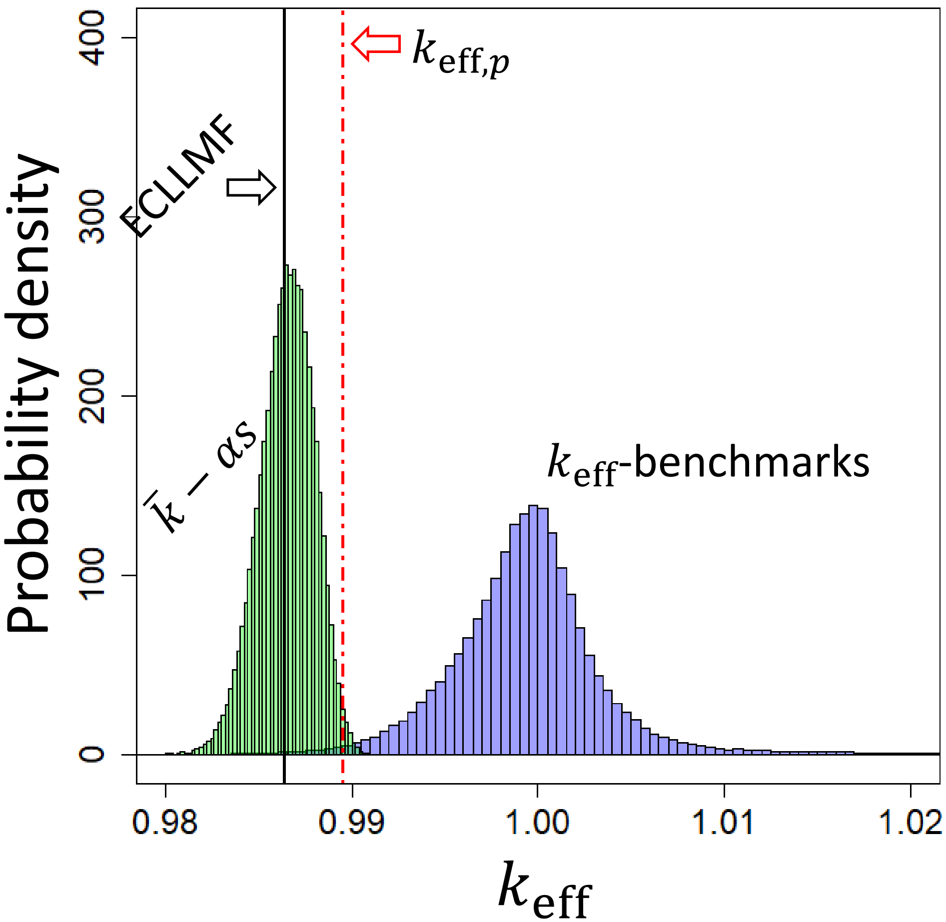
	Conventional method
$\bar{k}$	0.99939
α	2.8015
s	0.00239
ECLLMF	0.99270

- P-value of normality test was 8.9e-8 (< 5.0e-2)
- k_{eff}-benchmarks did not obey the normal distribution

Results of proposed ECLLMF

☑ Neutronic similarity of benchmark experiments

☑ Total uncertainties of k_{eff}



	Conventional method	Proposed method
$\bar{k}$	0.99939	0.99937
α	2.8015	3.1848 ↑
s	0.00239	0.00407 ↑
ECLLMF	0.99270	0.98641 ↓

- The α and s values increased, consequently, ECLLMF decreased

- The proposed ECLLMF was smaller than the conventional ECLLMF by 0.00634 [dk/k]

Comparison of ECLLMF and USL

- $\mathrm{USL} = 1 - \beta - \sigma_\beta - 2.6\mathrm{MOS}_{\mathrm{data}} - \mathrm{MOS}_{\mathrm{software}} - \mathrm{MOS}_{\mathrm{application}}$
  - β: bias of a calculated k_{eff} from the expected value
  - σ_β: experimental and statistical uncertainties of k_{eff}
  - $\mathrm{MOS}_{\mathrm{data}}$: margin of subcriticality (MOS) for nuclear data unc.
  - $\mathrm{MOS}_{\mathrm{software}}$: MOS for undetected errors in software
  - $\mathrm{MOS}_{\mathrm{application}}$: MOS left to the criticality safety analyst

Whisper code	
β	0.00944
σ_β	0.00821
$\mathrm{MOS}_{\mathrm{data}}$	0.00183
$\mathrm{MOS}_{\mathrm{software}}$	0.00500
$\mathrm{MOS}_{\mathrm{application}}$	0.00000
$1 - \beta - \sigma_\beta - 2.6\mathrm{MOS}_{\mathrm{data}}$	0.97759
USL	0.97260

Proposed method	
$\bar{k}$	0.99937
α	3.1848
s	0.00407
ECLLMF	0.98641

Summary and future works

- ECLLMF for an arbitrary distribution of k_{eff} was developed using the bootstrap method
- The proposed ECLLMF was evaluated by random sampling based on the multi-modal distribution
- The ECLLMF of UO_2 and concrete system was comparable with the USL without $\mathrm{MOS}_{\mathrm{software}}$

- Future works
  - Consider actual situations of fuel debris (*e.g.*, submerged in water, composition of concrete)
  - Validation for judgment of subcritical condition using critical approach experiments at KUCA-C core