Deciphering the Binning Method Uncertainty in Neutron Multiplicity Measurements LA-UR-14-28688

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Introduction

- Neutron Multiplicity Measurements
 - Important in determining mass, multiplication, and other parameters of fissionable material
 - Requires determination of various rates, which have complex analytical uncertainty
 - Uncertainty dependent on total count time, count rate, how data was sampled, and many others
 - Sampling methods bin the data into histograms and generate fitted parameters
 - Simpler method to estimate uncertainty is via standard deviation from multiple measurements of the same configuration, assuming independent, identically distributed (IID)



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Overview of Approach Using Simpler Method

- Monte Carlo simulations can be performed in lieu of real measurements
 - 100 trials for each method-object combination
- Analyze two separate objects
 - 1) Plutonium: significant neutron output
 - 2) Highly Enriched Uranium: small neutron output
- Use Hage-Cifarelli formalism of the Feynman Varianceto-Mean method
- Data sampled with 4 different methods
 - Results compared to each other, as well as to an analytical expression for uncertainty



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Introduction to Theory: Hage-Cifarelli

- Hage-Cifarelli formalism of the Feynman Variance-to-Mean method
 - Based off listmode data
 - Requires binning collected data into Feynman histograms and calculating reduced factorial moments
 - Feynman histograms bin the data into counts per time-bin of length τ (gate-widths)
 - reduced factorial moments based on chosen gate-width

•
$$\overline{m_{1!}(\tau)} = \frac{\sum_{n} nC_{n}}{\sum_{n} C_{n}}$$
 mean
• $\overline{m_{2!}(\tau)} = \frac{\sum_{n} n(n-1)C_{n}}{2!\sum_{n} C_{n}}$ variance
• $\overline{m_{3!}(\tau)} = \frac{\sum_{n} n(n-1)(n-2)C_{n}}{3!\sum_{n} C_{n}}$.

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Introduction to Theory: Rates

- The singles (R₁), doubles (R₂), and triples (R₃) rates computed from the Hage-Cifarelli method
 - functions of the reduced factorial moments of the individual Feynman histograms

$$R_{1} = \frac{\overline{m_{1!}(\tau)}}{\tau \omega_{1}(\tau)}$$

$$R_{2} = \frac{1}{\tau \omega_{2}(\tau)} \left(\overline{m_{2!}(\tau)} - \frac{1}{2} \left(\overline{m_{1!}(\tau)} \right)^{2} \right)$$

$$R_{3} = \frac{1}{\tau \omega_{3}(\tau)} \left(\overline{m_{3!}(\tau)} - \overline{m_{2!}(\tau)} \overline{m_{1!}(\tau)} + \frac{1}{3} \left(\overline{m_{1!}(\tau)} \right)^{3} \right)$$

$$\omega_{1}(\tau) = 1$$

$$\omega_{2}(\tau) = 1 - \frac{1}{\lambda \tau} (1 - e^{-\lambda \tau})$$

$$\omega_{3}(\tau) = 1 - \frac{1}{2\lambda \tau} \left(3 - 4e^{-\lambda \tau} + e^{-2\lambda \tau} \right)$$







Introduction to Theory: Data Analysis Code

- Momentum code used to analyze the data
 - Allows the data to be binned in a variety of methods
 - Calculates singles, doubles, triples rates
 - Uses a Levenberg-Marquardt minimization routine based on minimizing the sum of squares estimate (SSE) of Y_n
 - asymptotic values of the Y_n's are the respective R₁, R₂, and R₃; w_n(t) functions are the shape of the curves and are used to determine the neutron lifetime of the system

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$$Y_1(\tau) = R_1 \omega_1(\tau) = \frac{\overline{m_{1!}(\tau)}}{\tau}$$

- $Y_2(\tau) = R_2 \omega_2(\tau) = \frac{1}{\tau} \left(m_{2!}(\tau) - \frac{1}{2} \left(m_{1!}(\tau) \right)^2 \right)$
- $Y_3(\tau) = R_3 \omega_3(\tau) = \frac{1}{\tau} \left(\overline{m_{3!}(\tau)} - \overline{m_{2!}(\tau)} \, \overline{m_{1!}(\tau)} + \frac{1}{3} \left(\overline{m_{1!}(\tau)} \right)^3 \right)$



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Introduction to Theory: Data Analysis Code

- – Momentum code
 - Allows the data to be binned in a variety of methods
 - also estimates the uncertainty in the final fit of Y_n for N gate widths using a weighted standard deviation

•
$$(\delta R_n)^2 = \frac{N}{N-1} \frac{\sum_{n=1}^N \omega_n(\lambda) (Y_n - \overline{R_n})^2}{\sum_{n=1}^N \omega_n(\lambda)}$$

- R_n and δR_n dependent on gate widths, number of gate widths, number of samples in each gate width, and binning method used
- Binning methods used to create the Feynman histograms
 - Random, sequential, expanding, incrementing



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Introduction to Theory: Plots, BeRP ball



Feynman Histogram



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Introduction to Theory: Random Binning Method

- Random Binning
 - Binning routine assumed by the Hage-Cifarelli
 - starts at a random time within the measurement for a given gate width
 - The total number of gates evaluated is equal to 90% of the total count time divided by the gate width
 - procedure is then repeated for the next largest gate width until the largest gate width of interest is used





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Introduction to Theory: Sequential Binning Method

- Sequential Binning
 - Data is broken up into equal gate widths which start as soon as the previous gate ends
 - After going through the entire measuring time with one gate width, the next largest gate width is chosen and the process repeats
 - This continues until all gate widths have been cycled





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Introduction to Theory: Expanding Binning Method

- Expanding Binning
 - Starts every gate width at the same time for a given cycle
 - The smallest gate starts at time t_i
 - The next longest gate length is also sampled from the same starting point but ends later in time. This is repeated until the longest gate is reached. This represents one cycle at which time the cycle is repeated, starting at $t_i + t_{cycle}$.
 - This repeats until all time is sampled.



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Introduction to Theory: Incrementing Binning Method

- Incrementing Binning
 - Smallest gate width is started at t_i.
 - At the end of this gate, the next smallest time bin is used.
 - This process continues until the longest gate has been used, at which point, the process repeats with the smallest gate width. Continues until all time has been sampled.
 - Uses each data point once, but also has the smallest number of sample points per gate width





Description of Simulations: Overview

- Neutron Generator, a 1-D monte carlo code method based on Nolen's work [Nolen 2000], was used to create the listmode data based on specified material inputs
- Brute force method used for analysis
 - 100 simulations were run for two different models
- moderately multiplying system with a large intrinsic neutron source strength
 - simulates the Beryllium-Reflected-Plutonium ball
- highly multiplying system with a low intrinsic neutron source strength
 - simulates an HEU system



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Description of Simulations: Neutron Generator Inputs

- Neutron Generator inputs
 - requires input for several parameters for both the material, and the detector
 - Detector characteristics are held constant for both simulations in order to eliminate differences arising from the detector
 - requires the total simulation count time, which was 300 seconds for this work
- 100 trials done for each configuration



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Description of Simulations: Neutron Generator Inputs

	BeRP ball	HEU
Spontaneous Fission Isotope	Pu-240	U-238
Spontaneous Fission Rate (fissions/s)	1.30E+05	16.6
Total Multiplication	4.4	14.0
alpha-n rate (neutrons/s)	0	0
Induced Fission Isotope	Pu-239	U-235
Detector Efficiency	0.012	0.012
Tube Dead Time (µs)	4.0	4.0
Detector Lifetime (µs)	40.0	40.0
Fissile Mass (g)	4,500	40,000

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Description of Simulations: Momentum Inputs

- Once Neutron Generator creates the listmode data, Momentum processes the listmode data into Feynman histograms
 - Then determines the rates, and detector lifetime
 - Momentum requires binning specifications, apply to both HEU and BeRP ball simulations
 - Gate widths used: 4, 8, 12, ... 2048 for a total of 512 separate Feynman histograms

Parameter	Input Value
Shortest Gate Width (µs)	4
Number of Gates	512
Longest Gate (µs)	2048
Binning Method	[varied]
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Results: Momentum Outputs

- Momentum Outputs
 - Gate-width dependent rates and uncertainties [R_n(τ), δR_n(τ)] from the individual Neutron Generator simulations
 - Average of the respective rates $[R_n, \delta R_n]$ for each simulation
 - Neutron lifetime
- Average of the uncertainties is reported as the parametric uncertainty
- The standard deviation of the average rates for the 100 simulations is reported as the statistical uncertainty
- These values are reported for both the BeRP ball and HEU systems, for each of the 4 binning methods



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Results: BeRP ball simulation, R₁

- R_1 value equal to 10846.4 ± 6.2 cps for the **random** binning
 - Considered to be the standard value for comparison
 - For all binning methods, R₁ falls within the statistical uncertainties of each other
 - Parametric uncertainties, however, vary by nearly 2 orders of magnitude; and their respective standard deviations vary by nearly 3 order of magnitude
 - Parametric and Statistical Unc. nearly match for sequential and random binning

	Binning Method	R ₁	Statistical Uncertainty	Parametric Uncertainty
	Sequential	10846.4	6.2	7.3 ± 0.2
	Expanding	10845.1	8.7	13.9 ± 4.4
	Incrementing	10849.2	11.6	237.6 ± 133.5
mo	Random	10846.4	6.2	7.3 ± 0.2
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Results: BeRP ball simulation, R₂

- R_2 value equal to 2045.0 ± 18.7 for the random binning
 - For all binning methods, except incrementing, R₂ falls within the statistical uncertainties of the other
 - statistical uncertainties for R₂ reasonably good agreement to one another
 - parametric uncertainty for both expanding and incrementing binning methods is much greater than for random binning
 - the **sequential** binning method nearly matches random

	Binning Method	R ₂	Statistical Uncertainty	Parametric Uncertainty	
	Sequential	2044.9	18.7	23.2 ±1.1	
	Expanding	2050.6	24.1	61.6 ±3.7	
	Incrementing	2006.3	25.9	524.4 ±19.7	
	Random	2045.0	18.7	23.2 ±1.1	
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Results: BeRP ball simulation, R₃

- R_3 value equal to 673.4 ± 49.3 for the random binning
 - R₃ values have surprisingly little variation
 - statistical uncertainties for expanding and incremental binning methods have significant deviations from the random and sequential binning methods
 - The parametric uncertainties exhibit very similar trends for R_3 as they do for R_1 and R_2

Binning Method	R ₃	Statistical Uncertainty	Parametric Uncertainty
Sequential	673.6	49.6	67.0 ± 5.2
Expanding	673.6	73.6	51.0 ± 17.7
Incrementing	633.6	72.9	1123.8 ± 57.3
 Random	673.4	49.3	66.4 ± 5.3
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Results: BeRP ball simulation, Lifetime

- Lifetime equal to $39.9 \pm 1.1 \ \mu s$ for the random binning
 - Detector lifetimes for the BeRP ball agree well with one another, with the exception of the incrementing binning
 - Statistical uncertainty for the expanding and incrementing binning methods are more than double the statistical uncertainty for the sequential and random methods
 - still within one order of magnitude

Binning Method	Lifetime (µs)	Statistical Uncertainty
Sequential	39.9	1.1
Expanding	41.3	2.4
Incrementing	30.2	7.2
Random	39.9	1.1



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Results: HEU simulation, R₁

- Yields significantly lower R_1 values than for the BeRP ball system
 - R_1 value equal to 3.4 ± 0.3 cps for the random binning
 - Nearly 4 orders of magnitude lower than for BeRP ball system
 - The statistical uncertainty in R₁ is correspondingly much lower
 - the parametric uncertainty for the sequential method is two orders of magnitude less than both the random binning method statistical and parametric uncertainty

Binning Method	R ₁	Statistical Uncertainty	Parametric Uncertainty	
Sequential	3.4	0.2	0.002 ± 0.003	
Expanding	3.4	0.3	0.267±0.119	
Incrementing	9.4	2.6	17.0 ± 22.5	
Random	3.4	0.3	0.207 ± 0.486	
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Results: HEU simulation, R₂

- R_2 value equal to 4.6 ± 1.42 for the random binning
 - all in close proximity to one another, with the exception of the incrementing binning
 - The statistical uncertainty of the incrementing yields a statistical uncertainty nearly 10x that of the random binning
 - parametric uncertainty of R₂ for each binning method varies greatly from the others for the HEU system
 - sequential is approximately 10 % of random, while incrementing is nearly 40 times more than random

	Binning Method	R ₂	Statistical Uncertainty	Parametric Uncertainty
	Sequential	4.7	1.18	0.13 ± 0.05
	Expanding	5.0	5.01	1.00 ± 0.59
	Incrementing	13.2	8.0	40.9 ± 59.2
2000	Random	4.6	1.42	0.96 ± 0.49
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Results: HEU simulation, R₃

- The R_3 values for the HEU system show increased deviation from one another, as compared to the R_3 values for the BeRP ball system
 - statistical uncertainties for the expanding and incremental binning methods differ significantly from the random
 - parametric uncertainties show extreme deviation from one another, similar to that of R_1 and R_2 for the HEU system
 - Sequential binning shows the lowest parametric uncertainty and parametric uncertainty-standard deviation

Binning Method	R ₃	Statistical Uncertainty	Parametric Uncertainty	
Sequential	12.3	7.5	1.1 ± 1.0	
Expanding	15.7	13.0	6.6 ± 9.6	
Incrementing	57.4	155.7	129.4 ± 325.4	
Random	12.1	9.1	5.1 ± 5.9	
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Results: HEU simulation, Lifetime

- Lifetimes have more variability for the HEU system than for the BeRP ball system
- Lifetime equal to $40.4 \pm 4.2 \ \mu s$ for random binning
 - the expanding method has a lifetime more than 2x that of the other methods
 - statistical uncertainties of the expanding and incrementing methods are greater than the respective lifetimes

Binning Method	Lifetime	Statistical
Binning Motiloa	(μs)	Uncertainty
Sequential	39.8	2.7
Expanding	100.5	127.3
Incrementing	35.4	215.2
Random	40.4	4.2



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Summary of Work and Methods

- Four methods for binning neutron multiplicity data were compared
 - sequential, expanding, incrementing, and random
 - Random identified as base-case for comparison
- Parameters examined: singles, doubles, triples rate
 - Standard deviations from brute force approach compared against standard deviation reported by Momentum
- Large and low intrinsic neutron source strength system were analyzed



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Summary of Results

- Large intrinsic neutron source strength system (BeRP ball)
 - R₁ is nearly constant for all binning methods but the statistical and parametric uncertainty showed statistically significant variations
 - sequential method replicated the variance seen by random
- Low intrinsic neutron source strength system (HEU)
 - yielded very similar results for all parameters and systematic uncertainty
 - parametric uncertainties did not match for the sequential method



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Conclusions and Future Work

- the best alternate method to the random binning method is the sequential binning method which yields nearly identical results for high count rate systems
- the proposed parametric uncertainty formula in Momentum is in good agreement with the statistical uncertainty for the random sampling method
- Further work needs to be completed on the parametric uncertainty formula for low multiplying systems before it may be used with binning methods other than random



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Specific Application

- Subcritical ICSBEP benchmark, accepted September 2014
 - BeRP ball reflected by Nickel

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Plots: BeRP ball, Sequential



Feynman Histogram



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Plots: BeRP ball, Expanding



Feynman Histogram



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Plots: BeRP ball, Incrementing





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Plots: HEU, Random



Feynman Histogram



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Plots: HEU, Sequential



Feynman Histogram



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Plots: HEU, Expanding



Feynman Histogram



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Plots: HEU, Incrementing



Feynman Histogram



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