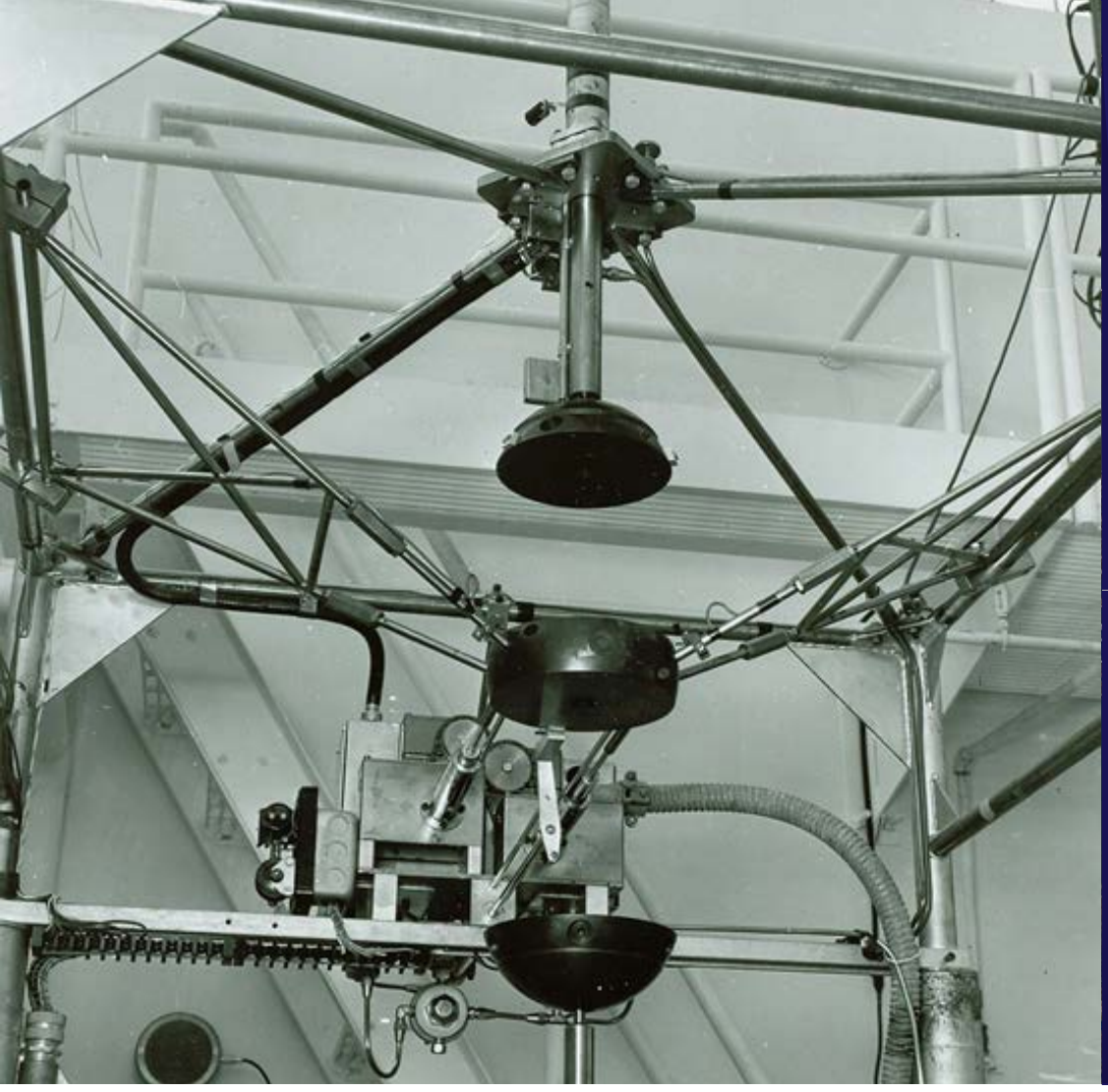


Los Alamos
NATIONAL LABORATORY
EST.1943

NNSA
National Nuclear Security Administration
Operated by Los Alamos National Security, LLC for the U.S. Department of Energy's NNSA

Use of Criticality Eigenvalue Simulations for Subcritical Benchmark Evaluations

J. Hutchinson, T. Cutler

Los Alamos National Laboratory

2016 ANS Winter Meeting

Overview

- Simulations for subcritical multiplication experiments are computationally expensive.
- This work investigates if an alternative method could be used for sensitivity/uncertainty analysis during the design phase of future subcritical benchmark experiments.

Introduction

Subcritical multiplication experiments are important for many applications including:

- nonproliferation
- safeguards
- criticality safety monitoring
- nuclear data validation

These types of measurements have been performed by many organizations in recent years including:

Los Alamos NATIONAL LABORATORY EST. 1943

Sandia National Laboratories

Lawrence Livermore National Laboratory

IAEA International Atomic Energy Agency

Universities

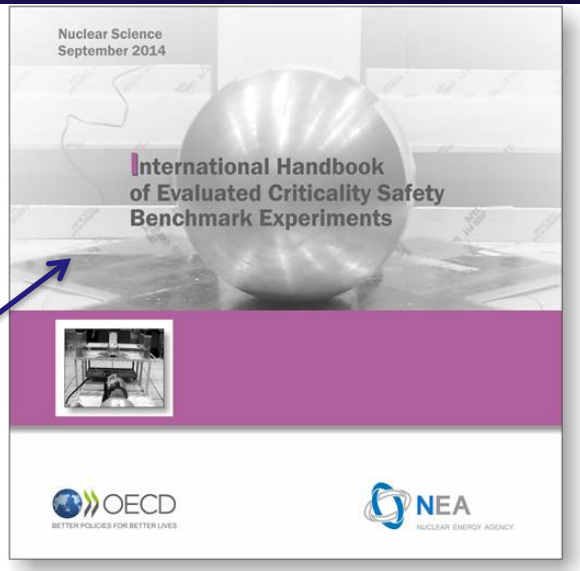
IRSN INSTITUT DE RADIOPROTECTION ET DE SÛRETÉ NUCLÉAIRE

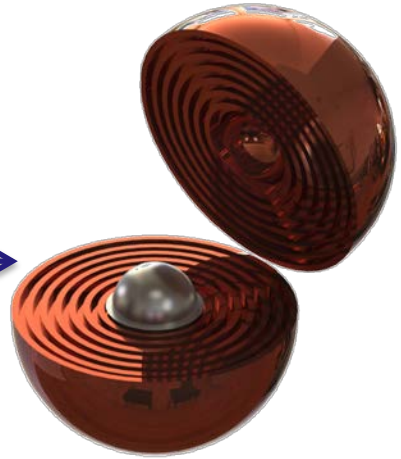
DE LA RECHERCHE À L'INDUSTRIE cea

AWE

Introduction

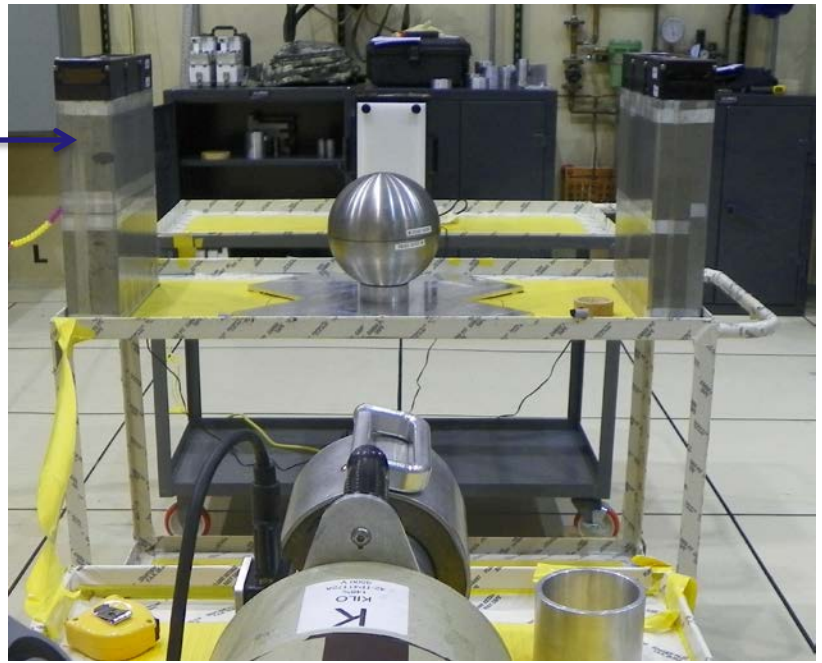
- **Recently, subcritical experiments have been evaluated for inclusion in the International Criticality Safety Benchmark Evaluation Process (ICSBEP).**
  - o BeRP/Ni and BeRP/W experiments.
- **New experiments have also been designed:**
  - o Subcritical Copper-Reflected α-phase Plutonium (SCRαP) Integral Experiment.





Current approach: measurements

- Measure list-mode data (time list of every recorded neutron event) in a neutron detector system (NPOD detector system shown).
- Apply a multiplicity analysis (Hage-Cifarelli formalism of the Feynman Variance-to-Mean method) to measured list-mode data.
- Determine mean and uncertainty of three benchmark parameters:
- R_1 (detector count rate), R_2 (the rate in the detector system in which two neutrons from the same fission chain are detected), and leakage multiplication (M_L).



3 inch-thick tungsten surrounding a ~4.5 kg sphere of WG α-phase Pu.

Current approach: simulations

- **Simulations for subcritical experiments must be run in a specific manner to allow for multiplicity analysis.**
- **MCNP®6 enables the user to directly run fixed-source problems in which the user can obtain list-mode data from the generated PTRAC files.**
- **Scripts to read the PTRAC file to develop list-mode data have recently been developed.**
- **In order to perform multiplicity analysis on the simulated data, the simulations must be run in analog Monte Carlo with no variance reduction.**

Current approach: simulations

- **This results in simulations which are computationally expensive.**

Case	List-mode	KCODE
0	480	0.5
1	480	1.5
2	640	5
3	640	11
4	800	21
5	800	35
6	800	46
7	880	58
Sum	5520	178

For the BeRP/W benchmark:
Time (in hours on a single processor) to achieve same statistical uncertainty.

Scaling used to make this table (list-mode and KCODE simulations used a different number of processors and had different statistical errors) and we do not claim it is very accurate. The bottom line is that list-mode simulations are much slower than criticality eigenvalue simulations.

Current approach: sensitivity

- The sensitivity and uncertainty analysis of experimental parameters is performed in a similar fashion to that of critical experiments in ICSBEP evaluations.
- The sensitivity (**S**) of an experimental parameter **x** (such as fissionable material radius) is given by:

$$S_{k,x} = \frac{k_P - k_R}{P_x}$$

- where **k** is the benchmark parameter (R_1, R_2, or M_L), P_x is a perturbation to the reference model, and the subscripts **P** and **R** are for "perturbed" or "reference".
- **The values for k are typically determined using fixed source list-mode simulations.**

Current approach: uncertainty

- The uncertainty due to the experimental parameter **x** is:

$$\delta k_x = u_x S_{k,x}$$

- where u_x is the uncertainty in experimental parameter **x**.

- This is identical to the approach used in critical experiments, except there k_{eff} is always used for the benchmark parameter k.

- For more info, see the ICSBEP uncertainty guide.

Motivation

- The BeRP/W evaluation has approximately 30 different experimental uncertainty parameters (**x**).
- The computational time required to perform the complete sensitivity/uncertainty analysis for such a benchmark is very large.
- When designing an experiment (subcritical or critical), it is desirable to estimate uncertainties prior to execution of the experiment.
- It can easily be seen, however, that this is currently not practical for subcritical measurements, given the large computational resources required.
- This work presents an alternate approach for experimental design.

Proposed approach

- The leakage multiplication (M_L) parameter is related to the multiplication factor (k_{eff}) of a system using the following basic reactor physics equations:

$$k_{eff} = \frac{k_p}{1-\beta_{eff}} \qquad k_p = 1 - \frac{1}{M_T} \qquad M_T = \frac{M_L \bar{\nu} - 1 - \alpha}{\bar{\nu} - 1 - \alpha}$$

- k_p is the prompt multiplication factor (due to prompt neutrons only)
- β_{eff} is the effective delayed neutron fraction
- M_T is the total neutron multiplication (as opposed to leakage multiplication)
- $\bar{\nu}$ is the average number of neutrons created per fission
- α is the capture cross-section divided by the fission cross-section of the fissile material.

Proposed approach

- β_{eff}, $\overline{\nu}$, and α mostly depend upon the fissionable material (in this case Pu-239) and therefore will not vary much between configurations.
- Given this, one can easily approximate M_L given k_{eff} values.
- It is therefore reasonable to suspect that perhaps one could approximate the uncertainty in leakage multiplication from a criticality eigenvalue calculation.

Hage-Cifarelli formalism

- **If one assumes that no neutrons are produced from (α,n) interactions, the equations reduce to:**

$$R_1 = \varepsilon b_{11} F_S$$

$$R_2 = \varepsilon^2 b_{21} F_S$$

$$with$$

$$b_{11} = M_L \bar{\nu}_{S(1)}$$

$$b_{21} = M_L^2 \left[\bar{\nu}_{S(2)} + \frac{M_L - 1}{\bar{\nu}_{I(1)} - 1} \bar{\nu}_{S(1)} \bar{\nu}_{I(2)} \right]$$

- **ε is the absolute detector efficiency**
- **F_S is the spontaneous fission rate of the system**
- **The terms $\bar{\nu}_{S(1)}$, $\bar{\nu}_{S(2)}$, $\bar{\nu}_{I(1)}$, and $\bar{\nu}_{I(2)}$ are the first and second factorial moments of the P_ν distribution where S refers to the isotope producing spontaneous fission neutrons and I refers to the isotope undergoing induced fission.**

Hage-Cifarelli formalism

- **If one assumes that no neutrons are produced from (α,n) interactions, the equations reduce to:**

$$R_1 = \varepsilon b_{11} F_S$$

$$R_2 = \varepsilon^2 b_{21} F_S$$

with

$$b_{11} = M_L \bar{\nu}_{S(1)}$$

$$b_{21} = M_L^2 \left[\bar{\nu}_{S(2)} + \frac{M_L - 1}{\bar{\nu}_{I(1)} - 1} \bar{\nu}_{S(1)} \bar{\nu}_{I(2)} \right]$$

The spontaneous fission rate is known for the BeRP ball and is constant regardless of the configuration.

All of the ν parameters are fairly constant amongst all configurations with the same fissionable material.

Hage-Cifarelli formalism

- **If one assumes that no neutrons are produced from (α,n) interactions, the equations reduce to:**

$$R_1 = \varepsilon b_{11} F_S$$

$$R_2 = \varepsilon^2 b_{21} F_S$$

with

$$b_{11} = M_L \bar{\nu}_{S(1)}$$

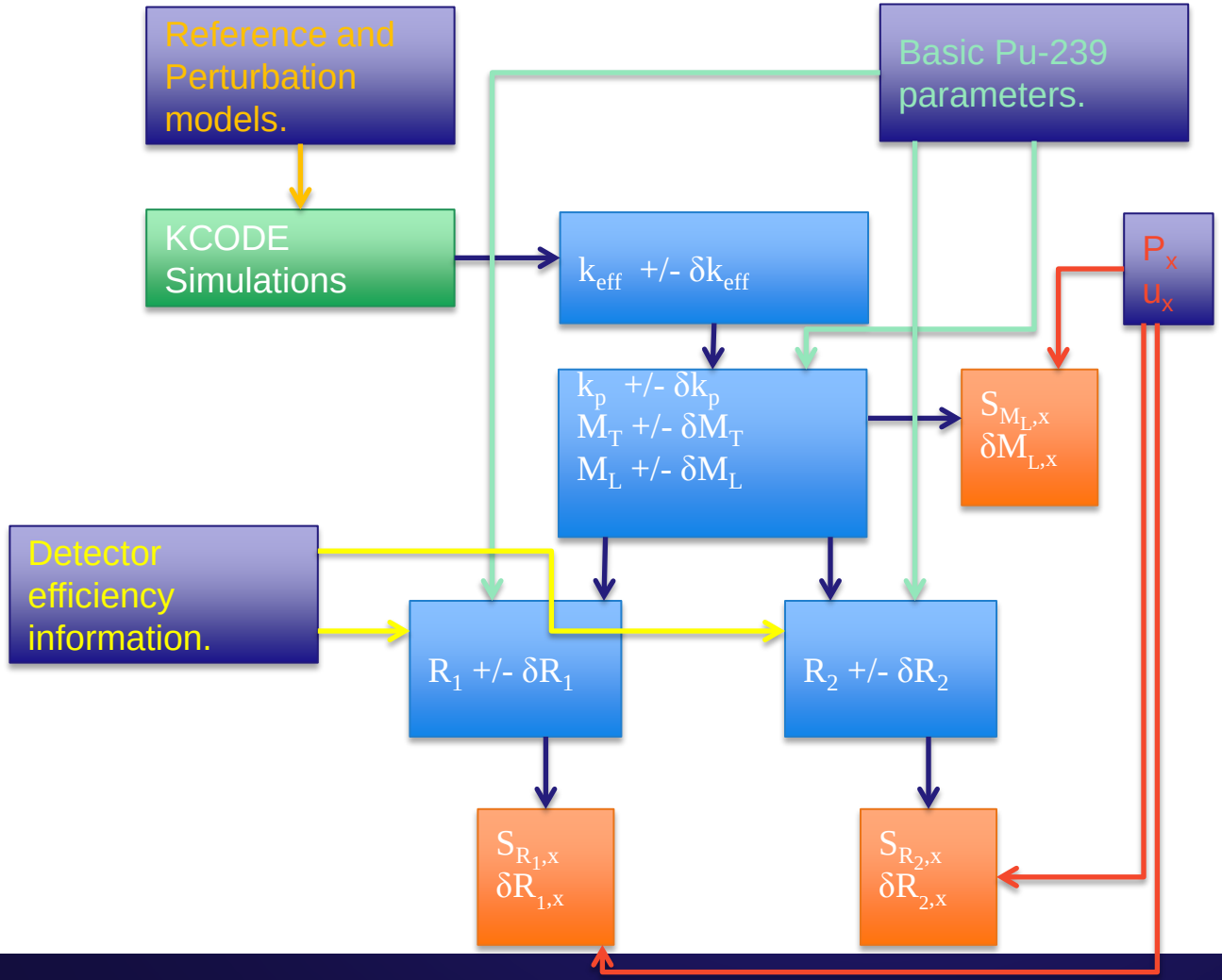
$$b_{21} = M_L^2 \left[\bar{\nu}_{S(2)} + \frac{M_L - 1}{\bar{\nu}_{I(1)} - 1} \bar{\nu}_{S(1)} \bar{\nu}_{I(2)} \right]$$

The detection efficiency has two components: the solid angle and the intrinsic efficiency.

The solid angle is constant amongst all configurations (since they had the same detector setup).

The intrinsic efficiency is configuration dependent (due to the change in the energy of neutrons reaching the He-3 tubes due to absorption in the reflector). This can, however, easily be estimated (a model of the detector is not required to approximate this).

Proposed approach



Inputs.

Simulations.

Outputs.

Desired results.

Proposed approach

Pu-239 parameters.

Detector efficiency information.

Parameter	Value
β_{eff}	0.00202
α	0.033
F_S	130267
$\bar{\nu}_{S(1)}$	2.154 +/- 0.005
$\bar{\nu}_{S(2)}$	1.894 +/- 0.015
$\bar{\nu}_{I(1)}$	3.182 +/- 0.010
$\bar{\nu}_{I(2)}$	4.098 +/- 0.011
ε	0.010 +/- 0.003

Delayed neutron yield per fission for Pu-239 of 642 ± 3.6 %[1] divided by $\bar{\nu}$ (3.182 ± 0.010).

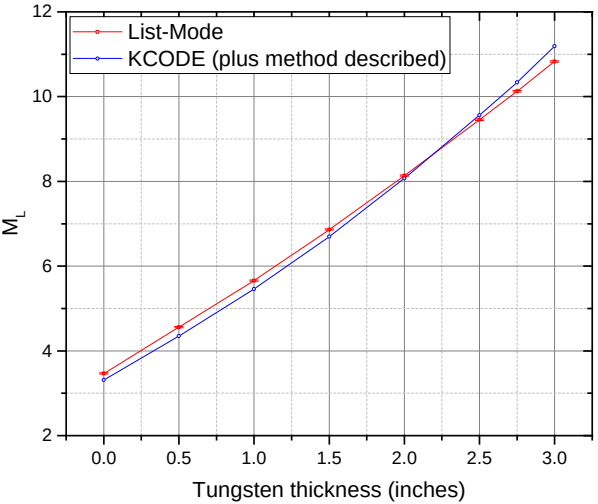
Reported Jezebel capture/fission ratio[2].

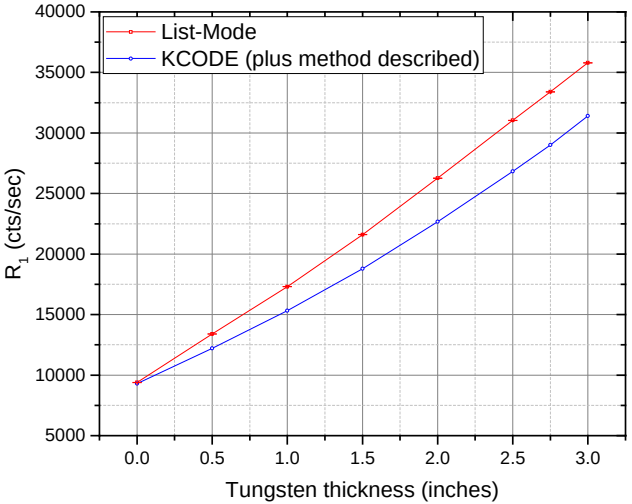
See BeRP/Ni and BeRP/W evaluations.

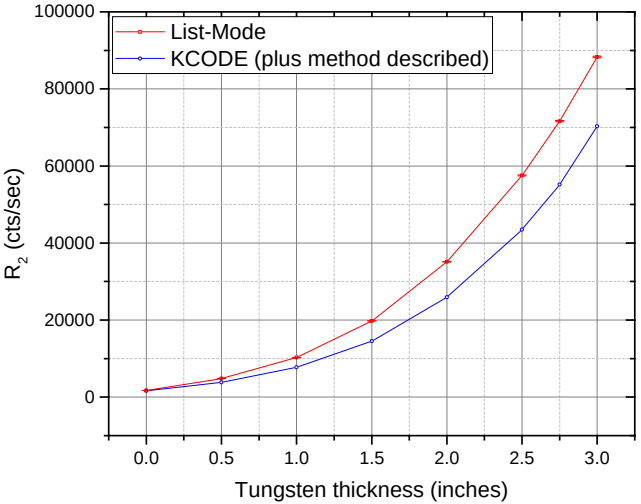
1: G. Rudstam, et. al., “Delayed Neutron Data for the Major Actinides,” NEA Working Party on International Nuclear Data Evaluation Co-operation (WPEC), Volume 6, 2002.
2: M. Chadwick, et. al., “ENDF/B-VII.1 Nuclear Data for Science and Technology: Cross Sections, Covariances, Fission Product Yields and Decay Data,” Nuclear Data Sheets, 112, 2887-2996, 2011.

Results: values

Both curves are simulated data using MCNP6 with ENDF/B-VII.1 cross-sections.







As expected, M_L compares better to the list-mode results than R_1 or R_2 (since R_1 and R_2 calculations involve more assumptions).

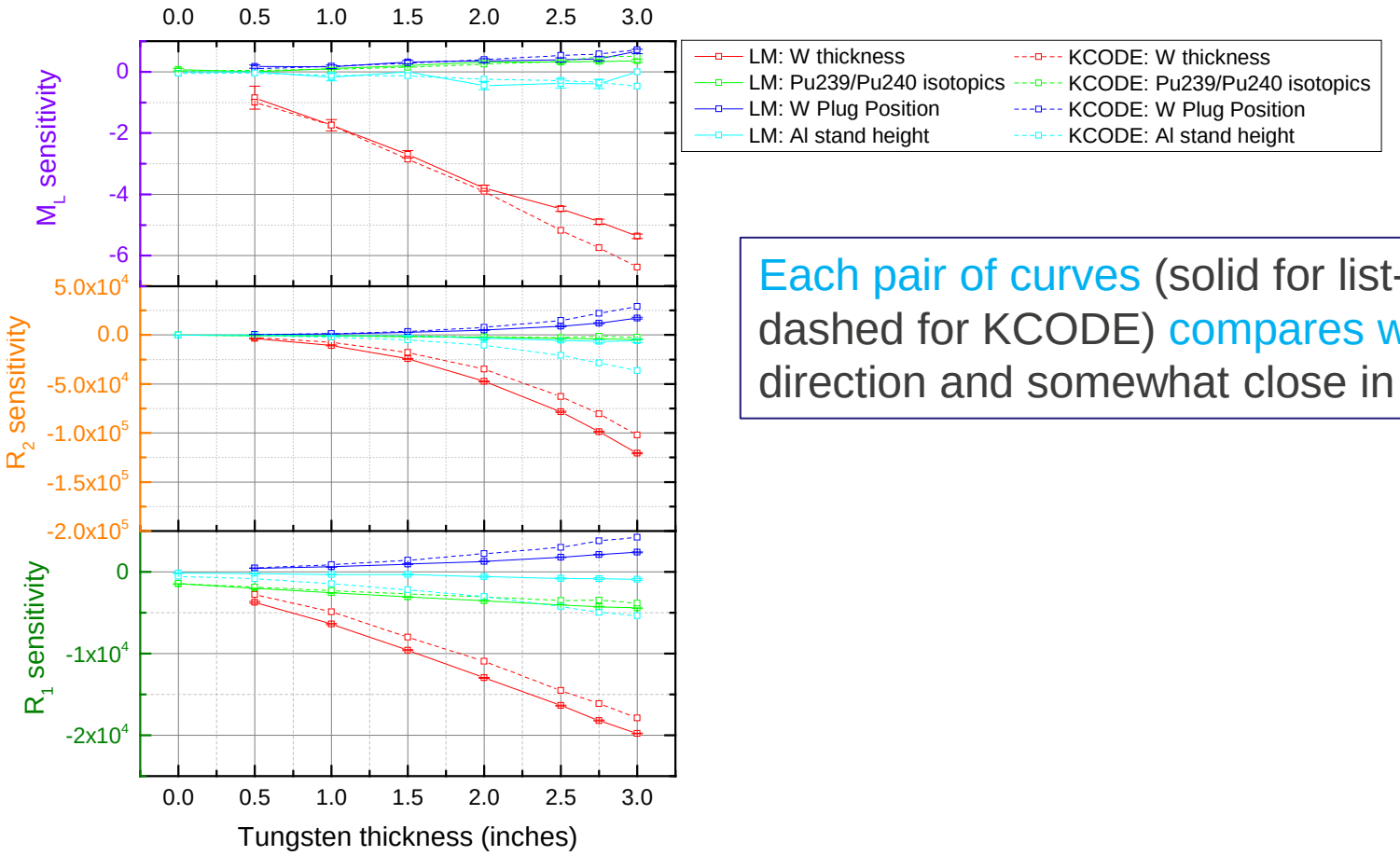
Results: sensitivities and uncertainties

- **Five experimental parameters (x) were used:**
  - Plutonium sphere radius
  - Pu-239/Pu-240 isotopics
  - Tungsten thickness
  - Tungsten plug position
  - Al stand height

Selected because they are known to have non-negligible uncertainties for all 3 parameters.

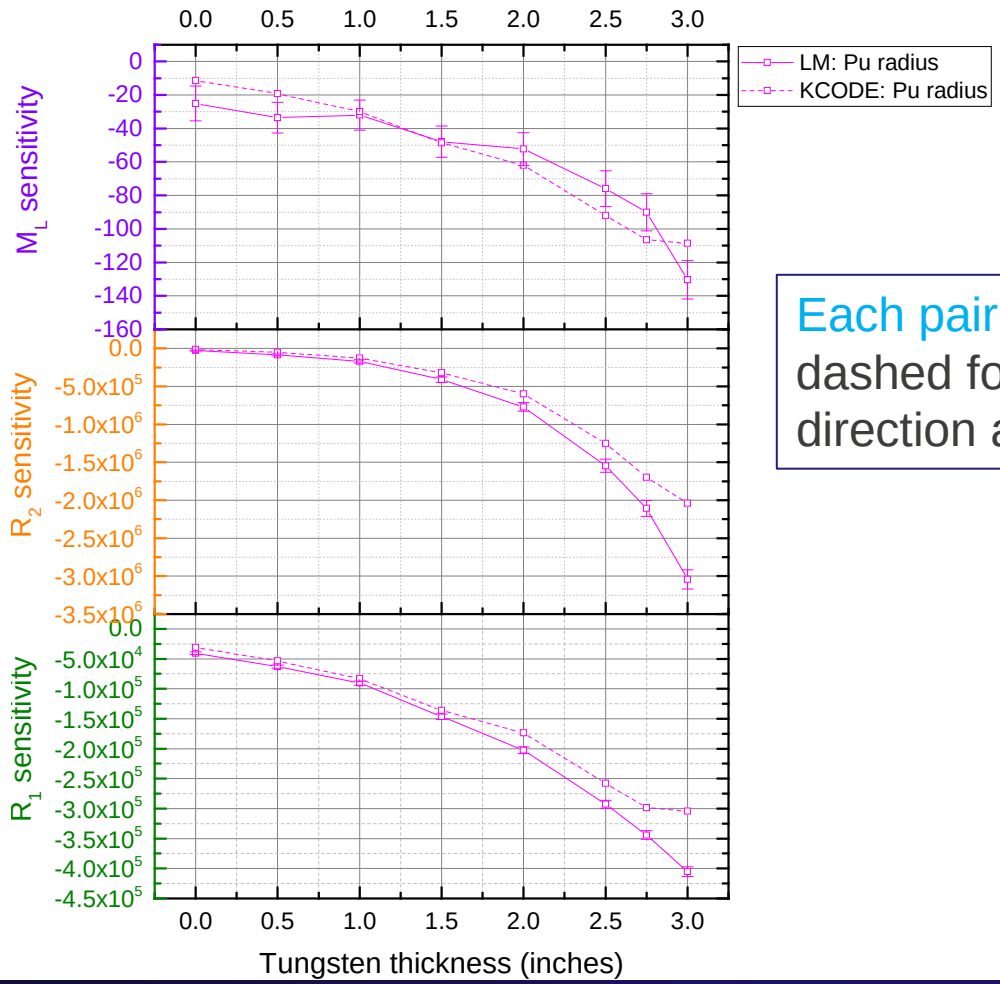
Selected because it is known to have negligible uncertainties for all 3 parameters.

Results: sensitivities



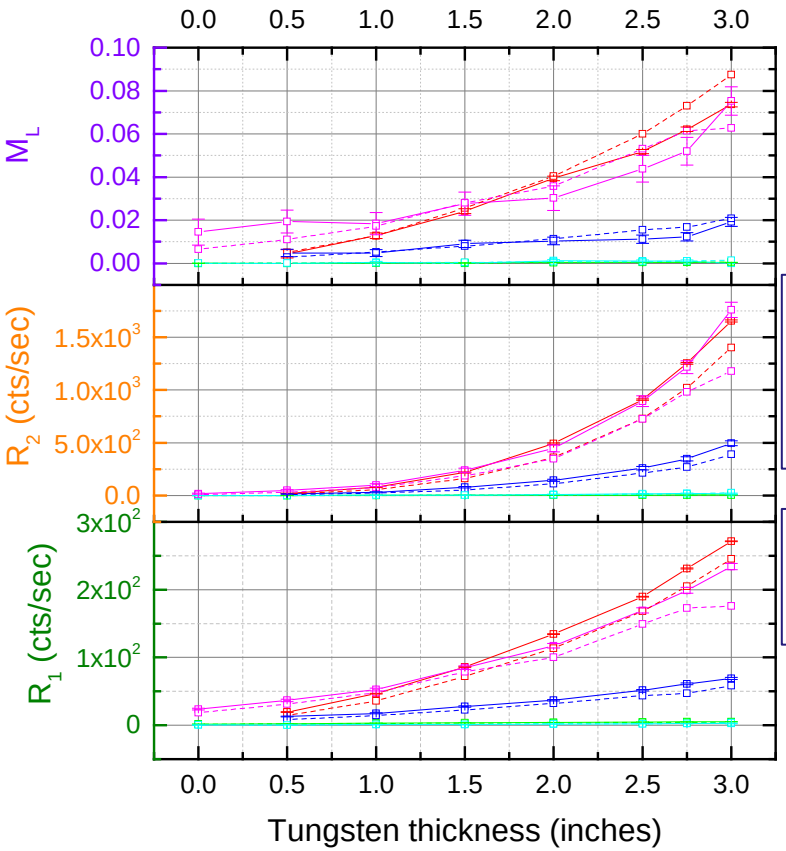
Each pair of curves (solid for list-mode and dashed for KCODE) compares well (same direction and somewhat close in magnitude).

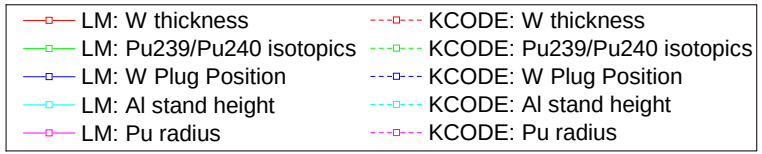
Results: sensitivities



Each pair of curves (solid for list-mode and dashed for KCODE) compares well (same direction and somewhat close in magnitude).

Results: uncertainties





Each pair of curves (solid for list-mode and dashed for KCODE) compares well (same direction and somewhat close in magnitude).

Al stand height uncertainty was negligible for all 3 parameters as expected.

Conclusions

- **It has been shown that one can use criticality eigenvalue calculations to approximate sensitivities and uncertainties for singles count rate (R_1), doubles count rate (R_2), and leakage multiplication (M_L).**
- **The authors are not suggesting that criticality eigenvalue simulations should be used instead of list-mode simulations for benchmark evaluations. We are suggesting, however, that this approach could be very valuable in the experiment design phase.**
  - In addition, this method could also perhaps be used to confirm whether or not certain parameters are negligible.
- **The benefit of using criticality eigenvalue calculations is that they are much faster than list-mode simulations.**

Additional work

- This approach has been used for design of the Subcritical Copper-Reflected α-phase Plutonium (SCRαP) Integral Experiment (see CED-2 report).
- After simulations are performed for the benchmark evaluation they will be compared to the uncertainties predicted by the KCODE simulations.

Acknowledgements

- **This work was supported in part by the DOE Nuclear Criticality Safety Program, funded and managed by the National Nuclear Security Administration for the Department of Energy.**

NCERC
NATIONAL CRITICALITY EXPERIMENTS RESEARCH CENTER