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Verification Suite for the Application of the Limiting Surface Density Method to Arrays of 9975 Shipping Packages

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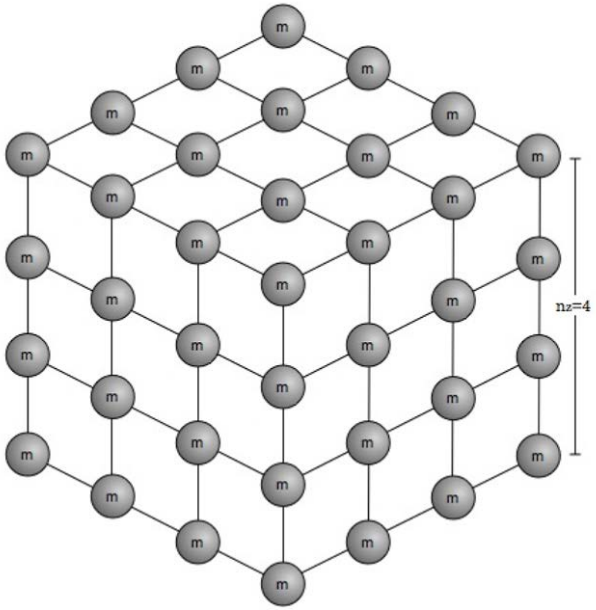
What is this all about?

- Extension of Joe Thomas' Limiting Surface Density (LSD) Method, which was originally developed for air spaced arrays



- LA-14244-M (Hand Calculation Primer) has an overview and several example applications

Basic Concept

- Buckling relationships can be used to relate one critical array to another, using empirically derived constants.
- First, assume there is a critical array of identical fissile items, specified by its isotopics, mass/unit, spacing, array shape, etc.
- Changes in one parameter (e.g., mass or spacing) may be compensated by changes in another parameter so that the resulting array is also critical.

Genesis of This Work

- K Area Complex (KAC) at Savannah River Site stores Plutonium metal and oxide in 9975 shipping packages
- Large arrays, varying shapes & arrangements
- Much work put into Monte Carlo analyses

Wouldn't it be better if we could simply…?

- Get results with hand calculations? Or spreadsheets?

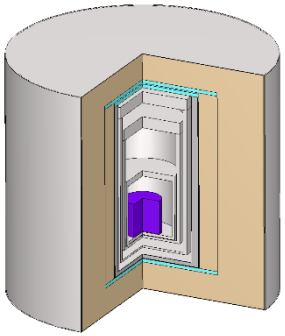
	A	B	C	D	E	F	G	H	I	J
8					Shape	Array		Constants	Solver	LSD
9	n_x	n_y	n_z	N	Factor	Pitch	a_n	Multiplier	Equation	m_c
10	4	4	4	64	1.000	46.5986	23.3	6.3E+18	-1.31E-08	7.832
11	10	10	10	1000	1.000	150	75	2.4E+19	-8.54E-07	8.080
12	2	20	1	40	1.767	46.5986	23.3	7.6E+18	-3.35E-09	7.867
13	5	5	3	75	1.031	46.5986	23.3	5.93E+18	8.483E-08	7.821

Can LSD work for shipping package arrays?

- Thomas' LSD method is very good for air-spaced arrays of solid items (see Hand Calculation Primer Sec. 7)
  - Caveat 1: Derivation uses cubic arrays of cubic units
  - Caveat 2: Each unit may be surrounded by ≤ ½ inch of steel
- Problems and Challenges:
  - 9975s are not cubic; nor are the arrays
  - 9975s have several nested layers of packaging material (steel, lead, Celotex™)
  - Some packaging varies among 9975s
  - Unclear how to derive the necessary constants

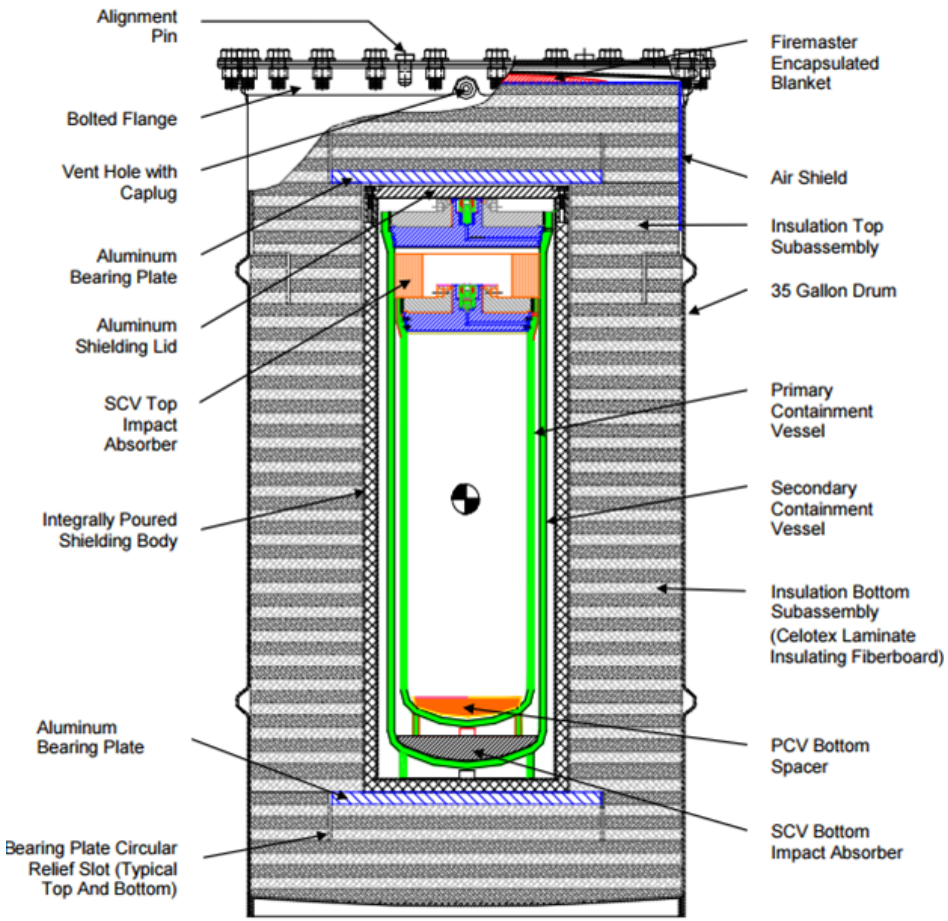
Preliminary Work

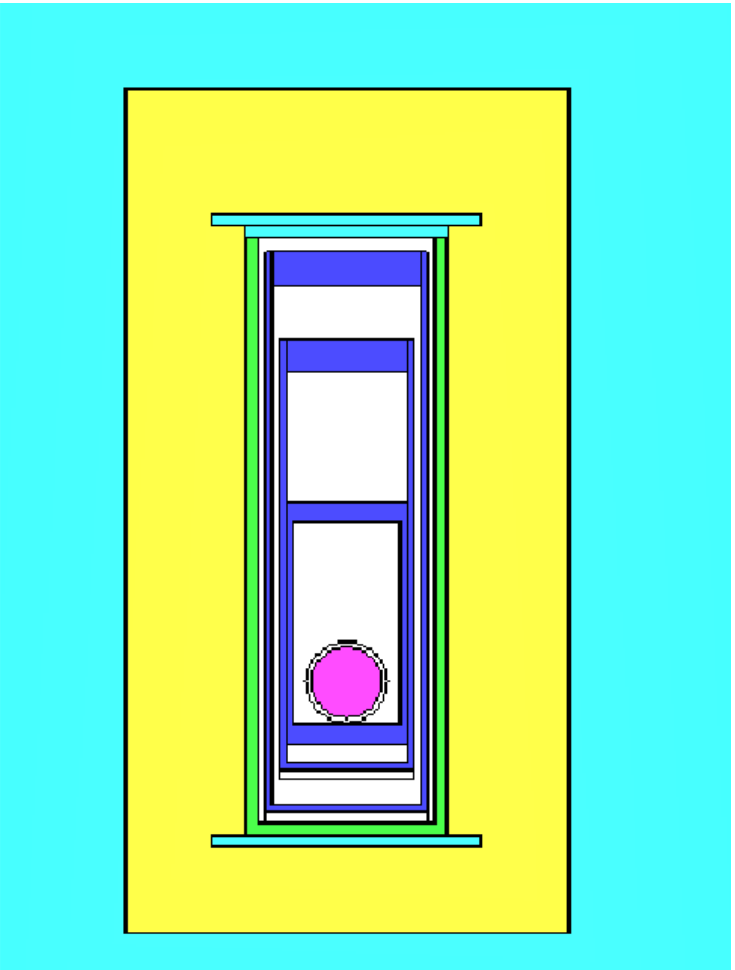
- Method was tested using hypothetical cubic shipping packages



  – See paper in ANS Transactions Vol. 114 (Stover, et al)
  – Initial testing showed promise for success
- Create a simple, accurate model for the 9975
  – Composite model for the product container (e.g., 3013)
  – Ignored unimportant geometric complexities
  – Fissile unit is a sphere or compact cylinder (H/D=1) of Pu metal

Derivation of Simplified Model

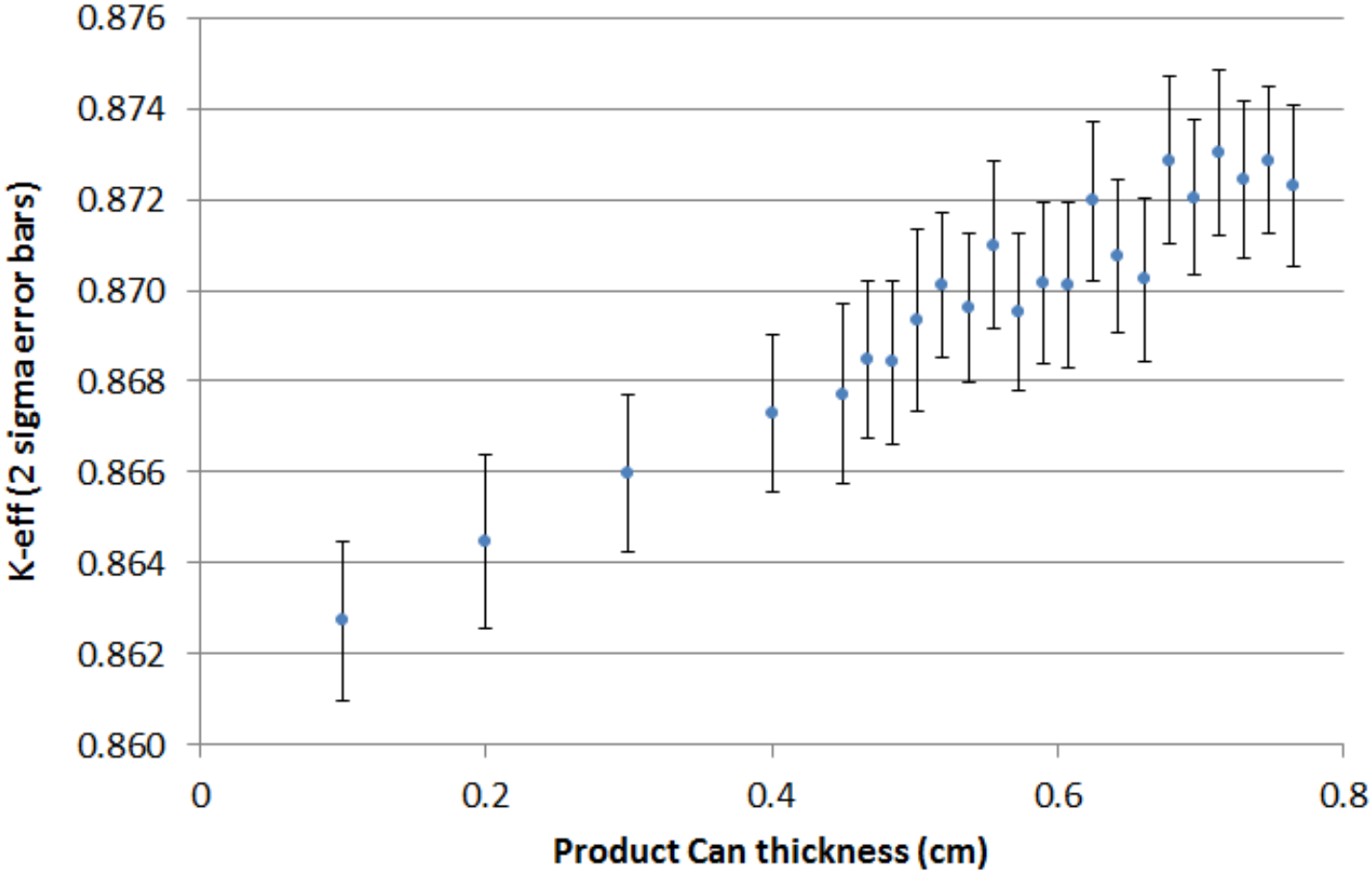




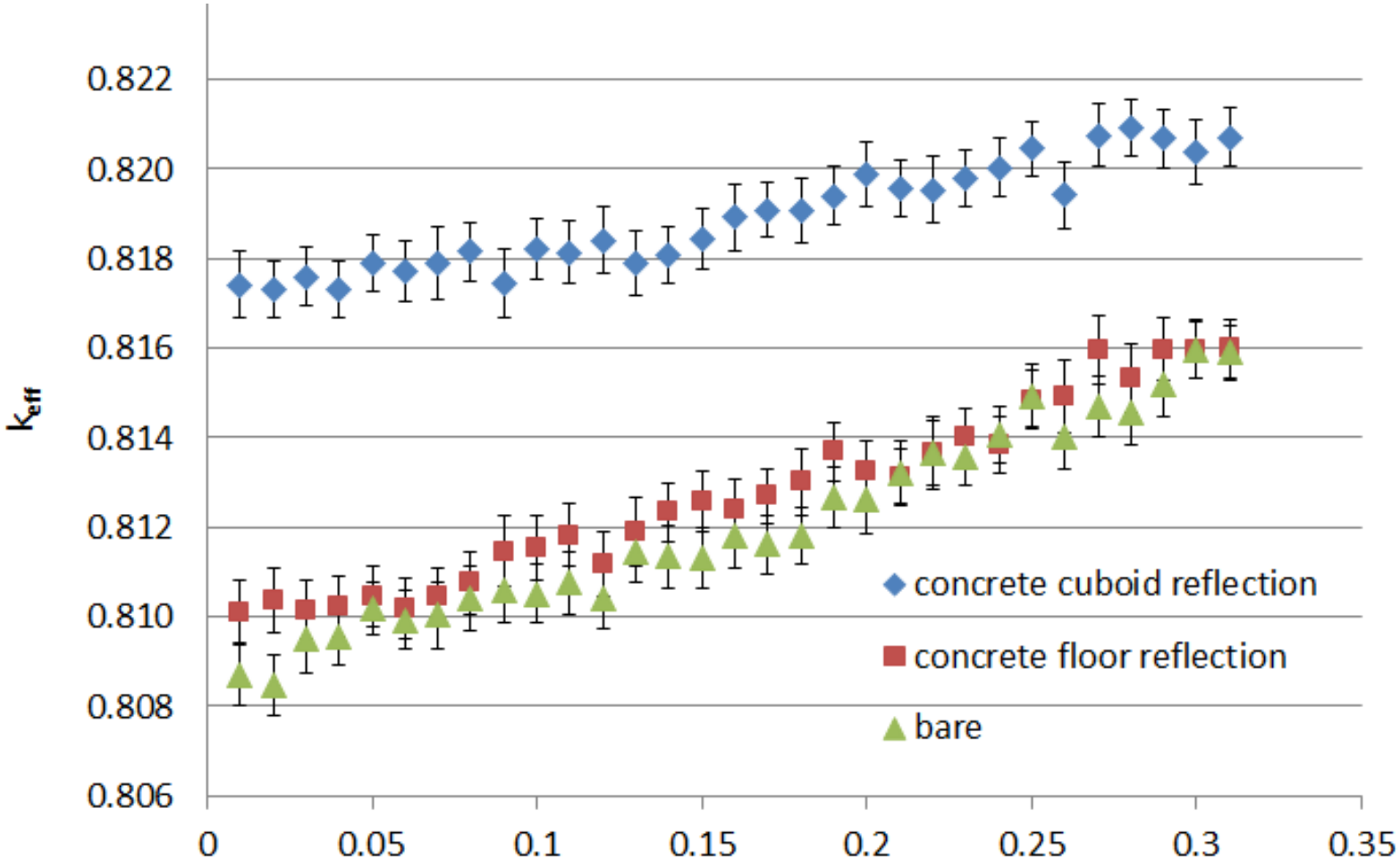
Packaging Variations Were Also Addressed

- The Product Cans (e.g., 3013s) vary in type, thickness
  - Calculations done for infinite planar array of 9975s, minimum spacing
  - For thicknesses from 0.4 cm to ~0.76 cm, $\Delta k < 0.01$
- Celotex™ varies in density around a nominal 0.22 g/cm^3
  - Affected by aging and other abnormal conditions
  - These calculations assume 0.31 g/cm^3
  - Small reactivity effect for modest density changes

k_{eff} vs. Fissile Material Container Wall Thickness



Effect of Celotex™ Density on Multiplication Factor



Full Derivation of LSD Relationships

- Start with basic reactor physics relationships:

$$B_g^2 = \frac{\pi^2}{(d_x + 2\lambda_x)^2} + \frac{\pi^2}{(d_y + 2\lambda_y)^2} + \frac{\pi^2}{(d_z + 2\lambda_z)^2}$$

- After 7 pages of algebra you have:

$$\frac{\mathrm{m_c n}}{(2\mathrm{a_n})^2}\left(1 - \frac{\mathrm{c}}{\sqrt{\mathrm{N}}}\right)^2 = \mathrm{c_2}(\mathrm{m_c} - \mathrm{m_o})$$

- See derivation in excruciating detail in our Journal paper (soon to be published)
- Primer has additional info on graphical solutions in its examples, but does not present the same derivation

Deriving New Constants

- From Thomas' classic (original) method derivation:

$$\frac{m_c n}{(2a_n)^2}\left(1-\frac{c}{\sqrt{N}}\right)^2 = c_2(m_c - m_o)$$

  – where:

$$c = \sqrt{\frac{4\lambda_{array}{}^2 N B_N^2}{3\pi^2}}$$

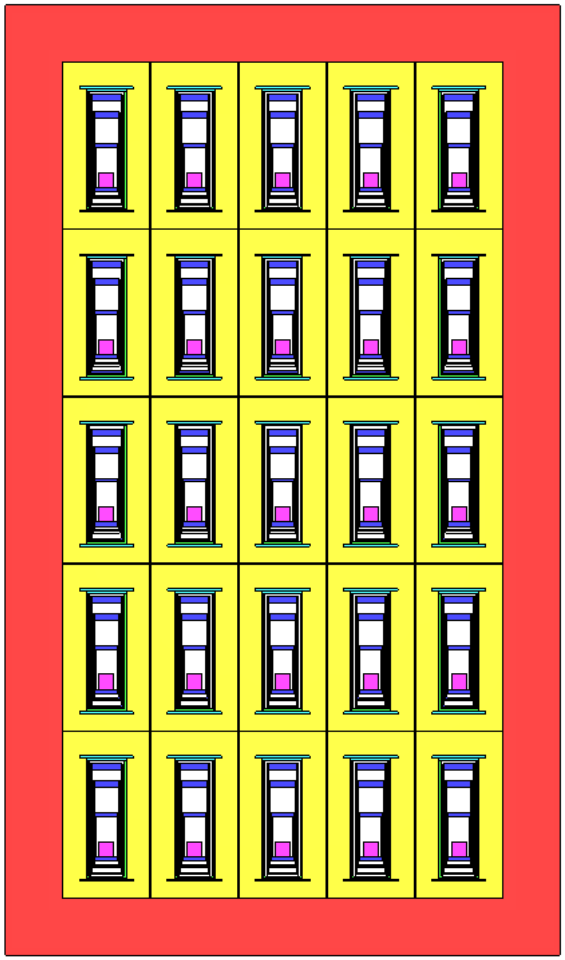
  – c and c_2 are empirically determined constants

- How to derive c??
  – Clues given in Thomas' paper Y-CDC-10, Appendix B

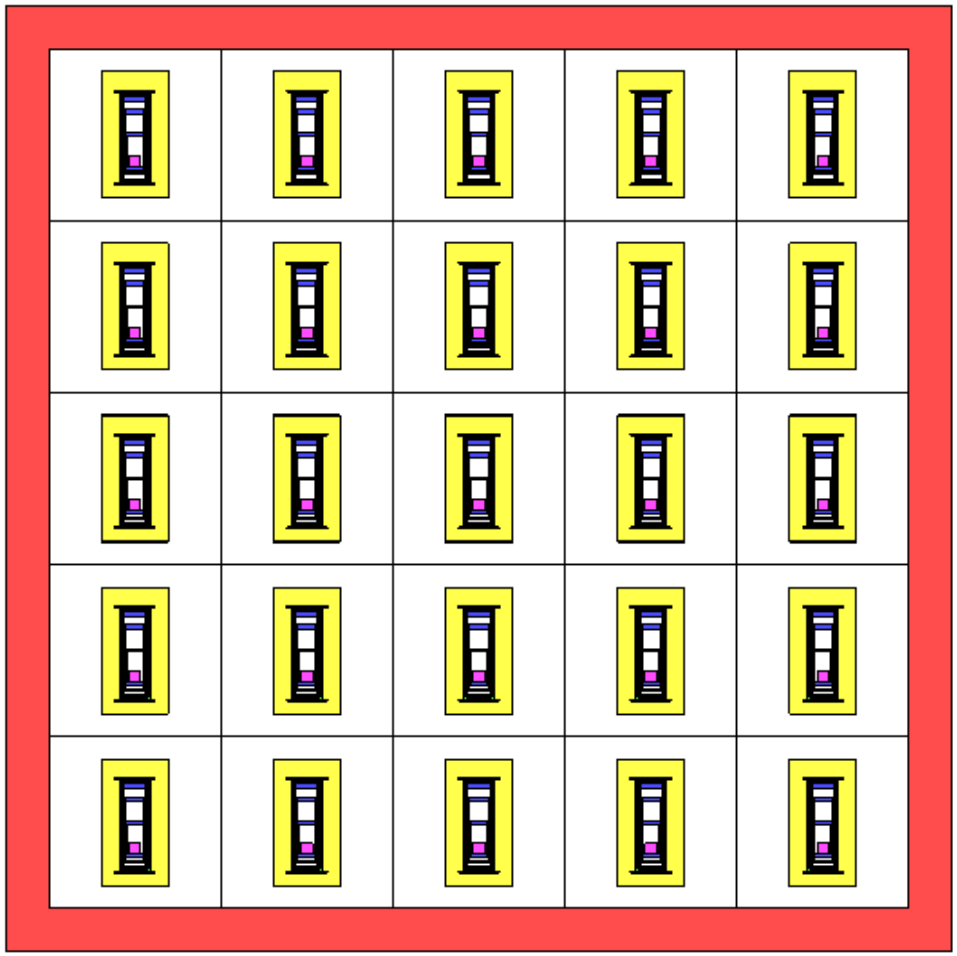
Deriving the Constants (cont'd)

- KENO-VI calculations for critical mass of arrays across the parameter ranges of interest: array size & spacing ($2a_n$)
- Cubic arrays with number per side, n, from 4 to 10
  - $N = n_x * n_y * n_z$ $\quad 64 \leq N \leq 1000$
  - Unit Spacing: $46.6 \text{ cm} \leq 2a_n \leq 150 \text{ cm}$
  - Reflected by 30 cm thick concrete on all 6 sides
  - Critical mass found for each combination of array size and spacing:

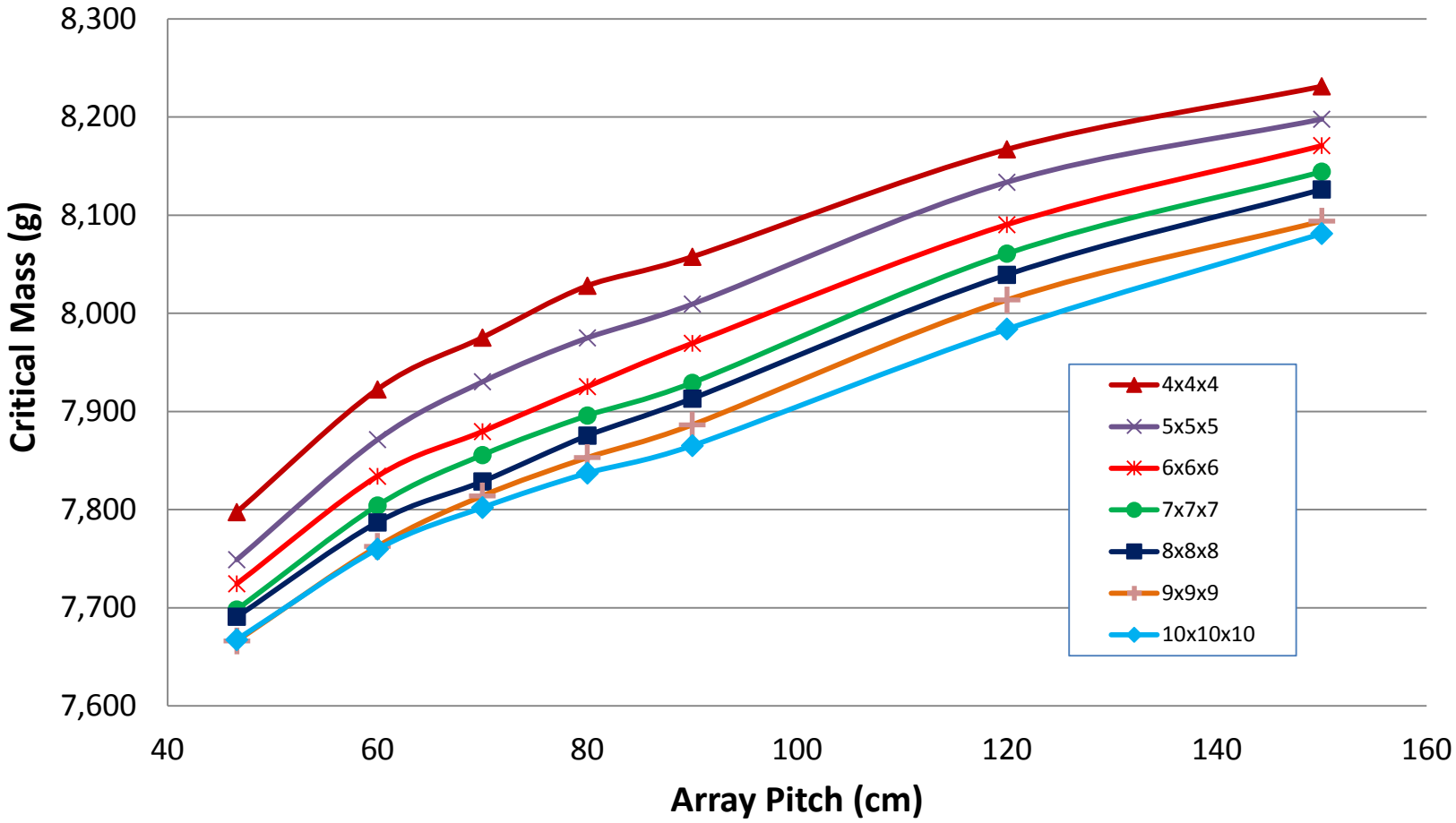
Vertical Slice of 5x5x5 Cubic Close-Packed Array



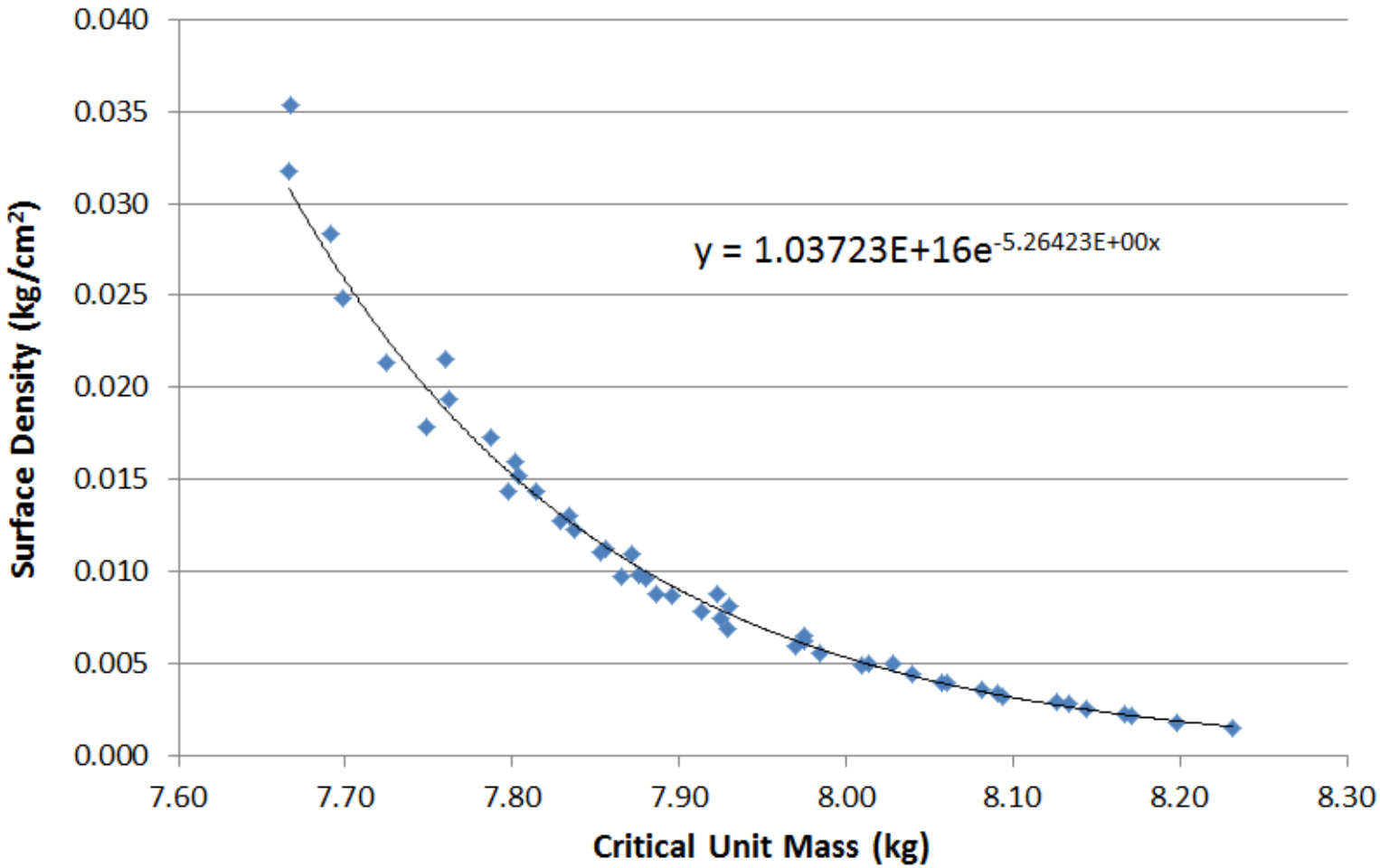
Vertical Slice of 5x5x5 Array, 120 cm Pitch



Fissile Mass (g) per Package for a Cubic Critical Array



Non-Linear Response for Surface Density [$=c_2(m_c-m_0)$]



Computing the Constants

- **From the revised derivation for array buckling (leakage):**

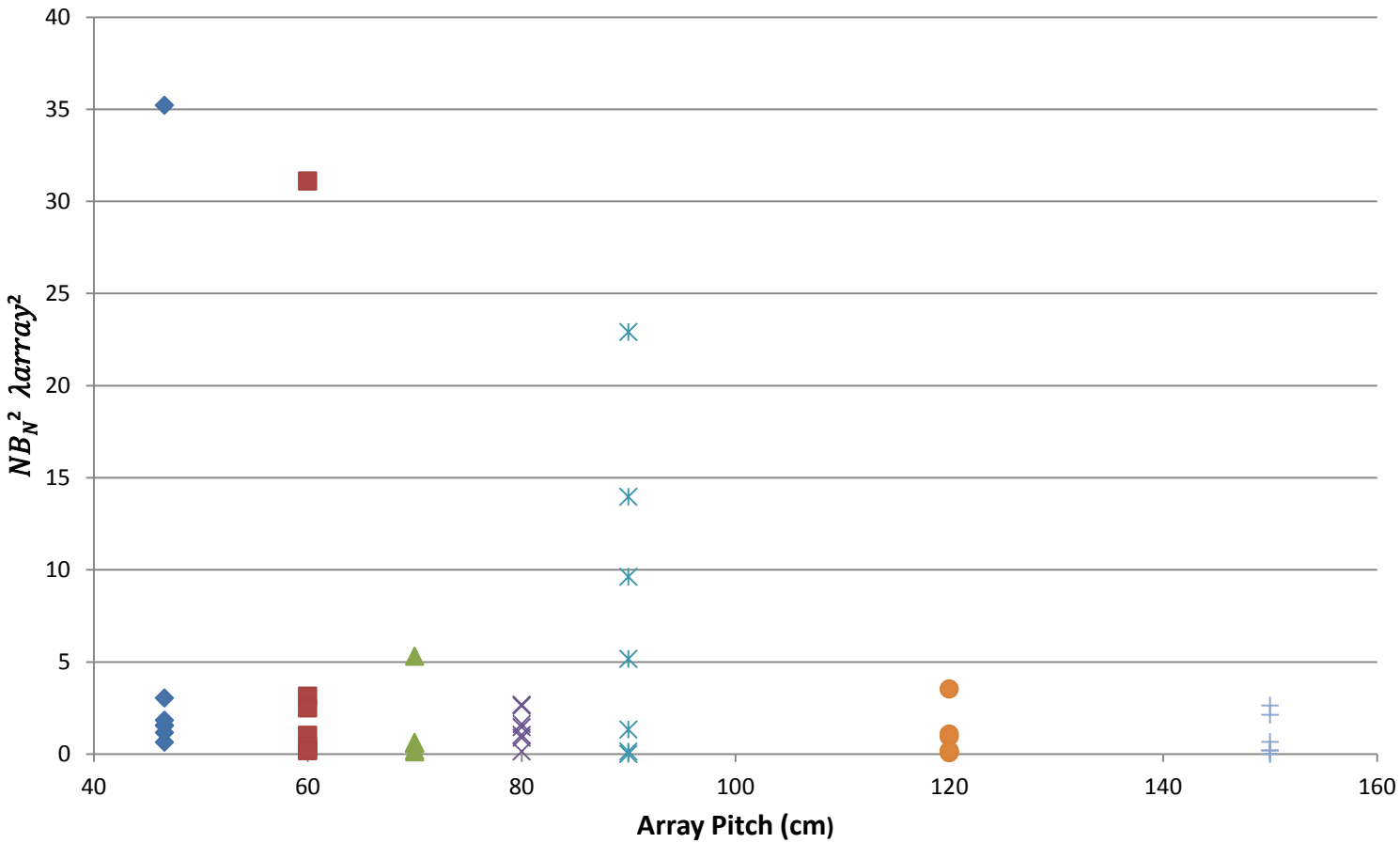
$$NB_N^2 = \frac{3\pi^2}{m_c} c_3 e^{c_4 m_c}$$

  – where: c_3 = 1.03723e+16 c_4 = -5.26423

- **Extrapolation distance is calculated from:**

$$\lambda_{array}{}^2 = \frac{N3\pi^2}{4NB_N^2}\left(1 - \sqrt{\frac{4a_n{}^2 NB_N^2}{n3\pi^2}}\right)^2$$

Computed Values of $NB_N^2\ \lambda_{array}{}^2$



Computing the Geometric Constant, c

- Average value of $NB_N^2\,\lambda_{array}{}^2$ = 1.43,
- Returning to the definition of c:

$$c = \sqrt{\frac{4\lambda_{array}{}^2 NB_N^2}{3\pi^2}}$$

- Yields c = 0.44
- Similar to Thomas' value of 0.55 +/- 0.18

Checking the Method—Using Cubic Arrays

- The relationship to estimate critical mass is:

$$m_c = \frac{(2a_n)^2 c_3 e^{c_4 m_c}}{n\left(1 - \frac{c}{\sqrt{N}}\right)^2}$$

- Using this to calculate m_c for the 49 cubic arrays
  - 4 ≤ n ≤ 10, 46.6 cm ≤ Pitch ≤ 150 cm
- **Average Δ% between LSD and KENO**-VI m_c = 0.16
- **Maximum Δ% = 0.46**

Checking with Realistic Arrays

- Selected arrays:
  - 2x20x1, 2x30x1, 2x20x2, 2x20x3, 4x20x3, 5x5x3
  - Critical unit masses computed for each array with KENO-VI
    - *46.6 cm ≤ Pitch ≤ 150 cm*

- But first, a word about Shape factor:

$$R = \frac{\sqrt[3]{N}}{3}\left(\frac{1}{n_x} + \frac{1}{n_y} + \frac{1}{n_z}\right)$$

- No helpful shape factor adjustment found for these arrays
- **Currently, we restrict use to arrays with R ≤ 2**
  - Not a significant limitation

LSD vs. KENO-VI Critical Unit Mass for Realistic Arrays

- For 42 non-cubic arrays:
  - Average Δ% = 0.6
  - Maximum Δ% = 1.3
  - LSD values slightly under-predict the KENO-VI value

Sample Application: 10x14x3 Array, 46.6 cm pitch

- Estimating multiplication:

$$k_{eff} = \left(\frac{m}{m_c}\right)^{1/3} = \left(\frac{4.4\ kg}{7.703\ kg}\right)^{1/3} = 0.830$$

Unit Mass (kg)	LSD k_{eff}	Δk_{eff} vs. LSD base case	KENO-VI k_{eff}	Δk_{eff} (LSD – KENO-VI)
3.4	0.761	-0.068	0.768	-0.007
3.9	0.797	-0.033	0.804	-0.007
4.4 (base case)	0.830	--	0.836	-0.006
4.9	0.860	0.030	0.866	-0.006
5.4	0.888	0.059	0.894	-0.005

Additional Examples

- The effect of stacking an extra layer on top of the array is evaluated by changing n_z from 3 to 4. Resulting $\mathbf{\Delta m_c}$ = 19 g. Multiplication change is insignificant.

- Changing array size from 10x14x3 to 6x6x3: **Δk** = -0.004

- Changing array size from 10x14x3 to 20x30x3: **Δk** = 0.003

Conclusions

- LSD Method provides very good agreement with KENO-VI for arrays of 9975 shipping packages.
- Allows rapid estimates for safety margin for varying mass, spacing, and array sizes.
- Can be used to evaluate variety of normal and credible abnormal conditions.
- Helps develop understanding of the physics.

Questions / Comments?