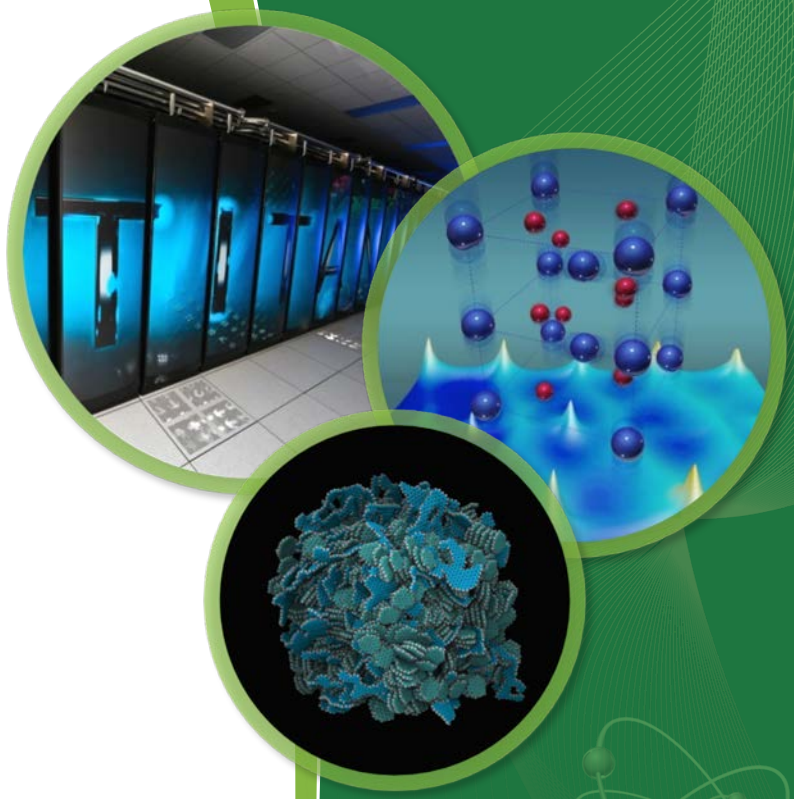
Determination of Critical Experiment Correlations for Experiments Involving Arrays of Low-Enriched Fuel Rods



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Outline

1. Experiments examined
2. Fuel rod placement uncertainty effects
3. Monte Carlo uncertainty effect
4. Conclusions

Experiments examined

- WPNCS/UACSA Phase IV benchmark
  - LCT-007 and LCT-039
  - LCT-007: variable square pitch arrays in light water
  - LCT-039: fixed square pitch arrays of different patterns in light water
  - Both used the same fuel with 4.74 wt% ^{235}U, performed at Valduc
- LCT-042
  - Three arrays of rods in light water
  - Different poison panels adjacent to central array
  - All used the same fuel with 2.35 wt% ^{235}U, performed at PNL
  - Fuel arrays identical in all seven cases

Fuel rod placement uncertainty effects

- Correlation is essentially a measure of shared uncertainty

Recall: $\rho = \frac{\mathrm{cov}(x, y)}{\sigma_x \sigma_y}$

- Total system uncertainty can depend very strongly on fuel rod position, as it controls moderation
- Fuel rod arrays in experiments present a problem:
  - Is pitch uncertain, or is the location of each rod uncertain?
  - Are rods in the same location in each experiment?
- These are the central questions in the WPNCS/UACSA benchmark

WPNCS/UACSA results

	7-1	7-2	7-3	39-1	39-2	39-3	39-4	39-5	39-6	39-7	39-8	39-9	39-10	39-11	39-12	39-13	39-14	39-15	39-16	39-17
7-1	1.000	0.933	0.391	0.978	0.975	0.974	0.974	0.956	0.957	0.974	0.971	0.978	0.969	0.977	0.972	0.980	0.979	0.973	0.977	0.978
7-2	0.933	1.000	0.557	0.923	0.920	0.925	0.930	0.925	0.929	0.933	0.920	0.936	0.940	0.925	0.924	0.928	0.933	0.928	0.937	0.931
7-3	0.391	0.557	1.000	0.405	0.390	0.409	0.417	0.459	0.463	0.415	0.389	0.434	0.451	0.384	0.406	0.405	0.382	0.399	0.418	0.420
39-1	0.978	0.923	0.405	1.000	0.978	0.970	0.973	0.957	0.958	0.976	0.972	0.970	0.973	0.979	0.976	0.976	0.981	0.977	0.972	0.977
39-2	0.975	0.920	0.390	0.978	1.000	0.972	0.970	0.953	0.954	0.975	0.967	0.968	0.963	0.974	0.974	0.976	0.977	0.970	0.972	0.977
39-3	0.974	0.925	0.409	0.970	0.972	1.000	0.967	0.945	0.954	0.971	0.963	0.971	0.966	0.974	0.969	0.974	0.971	0.967	0.972	0.970
39-4	0.974	0.930	0.417	0.973	0.970	0.967	1.000	0.956	0.954	0.971	0.965	0.968	0.965	0.972	0.971	0.973	0.974	0.968	0.973	0.978
39-5	0.956	0.925	0.459	0.957	0.953	0.945	0.956	1.000	0.946	0.958	0.944	0.955	0.955	0.953	0.949	0.952	0.952	0.954	0.951	0.958
39-6	0.957	0.929	0.463	0.958	0.954	0.954	0.954	0.946	1.000	0.956	0.953	0.960	0.961	0.955	0.954	0.963	0.955	0.957	0.958	0.960
39-7	0.974	0.933	0.415	0.976	0.975	0.971	0.971	0.958	0.956	1.000	0.973	0.974	0.970	0.979	0.978	0.974	0.979	0.974	0.979	0.978
39-8	0.971	0.920	0.389	0.972	0.967	0.963	0.965	0.944	0.953	0.973	1.000	0.964	0.970	0.973	0.966	0.970	0.973	0.964	0.964	0.966
39-9	0.978	0.936	0.434	0.970	0.968	0.971	0.968	0.955	0.960	0.974	0.964	1.000	0.967	0.976	0.968	0.976	0.976	0.969	0.975	0.974
39-10	0.969	0.940	0.451	0.973	0.963	0.966	0.965	0.955	0.961	0.970	0.970	0.967	1.000	0.964	0.968	0.969	0.966	0.964	0.968	0.970
39-11	0.977	0.925	0.384	0.979	0.974	0.974	0.972	0.953	0.955	0.979	0.973	0.976	0.964	1.000	0.973	0.980	0.979	0.977	0.977	0.977
39-12	0.972	0.924	0.406	0.976	0.974	0.969	0.971	0.949	0.954	0.978	0.966	0.968	0.968	0.973	1.000	0.978	0.976	0.968	0.972	0.976
39-13	0.980	0.928	0.405	0.976	0.976	0.974	0.973	0.952	0.963	0.974	0.970	0.976	0.969	0.980	0.978	1.000	0.977	0.979	0.976	0.976
39-14	0.979	0.933	0.382	0.981	0.977	0.971	0.974	0.952	0.955	0.979	0.973	0.976	0.966	0.979	0.976	0.977	1.000	0.976	0.977	0.979
39-15	0.973	0.928	0.399	0.977	0.970	0.967	0.968	0.954	0.957	0.974	0.964	0.969	0.964	0.977	0.968	0.979	0.976	1.000	0.970	0.973
39-16	0.977	0.937	0.418	0.972	0.972	0.972	0.973	0.951	0.958	0.979	0.964	0.975	0.968	0.977	0.972	0.976	0.977	0.970	1.000	0.976
39-17	0.978	0.931	0.420	0.977	0.977	0.970	0.978	0.958	0.960	0.978	0.966	0.974	0.970	0.977	0.976	0.976	0.979	0.973	0.976	1.000

Pitch sampled: all pitches are the same and are the same for all cases

Coefficients range from 0.96 to 0.98
(For cases with the same pitch)

	7-1	7-2	7-3	39-1	39-2	39-3	39-4	39-5	39-6	39-7	39-8	39-9	39-10	39-11	39-12	39-13	39-14	39-15	39-16	39-17
7-1	1.000	0.034	0.023	0.012	0.005	-0.040	0.069	-0.009	0.071	0.067	0.082	0.088	0.049	0.044	0.042	0.063	0.088	0.139	-0.021	0.082
7-2	0.034	1.000	0.074	-0.045	0.020	0.040	0.181	0.086	0.065	0.041	-0.028	-0.034	0.009	-0.030	0.018	0.047	-0.041	0.023	0.061	-0.028
7-3	0.023	0.074	1.000	0.118	0.063	0.094	0.061	0.086	0.201	0.079	0.100	0.134	0.047	0.091	0.012	0.125	0.050	0.117	0.172	0.055
39-1	0.012	-0.045	0.118	1.000	0.121	0.138	0.076	0.071	0.124	0.034	0.100	0.085	0.135	0.023	0.037	0.037	0.087	0.083	0.115	0.149
39-2	0.005	0.020	0.063	0.121	1.000	0.034	0.075	0.037	0.130	0.041	0.055	0.049	0.009	0.025	0.095	0.100	-0.050	0.124	-0.003	0.115
39-3	-0.040	0.040	0.094	0.138	0.034	1.000	0.079	0.077	0.044	0.007	0.048	-0.064	0.145	0.076	0.061	0.090	0.067	0.059	0.088	0.116
39-4	0.069	0.181	0.061	0.076	0.075	0.079	1.000	-0.051	0.090	-0.012	-0.017	0.036	0.026	-0.021	0.034	0.088	0.042	-0.004	0.025	-0.018
39-5	-0.009	0.086	0.086	0.071	0.037	0.077	-0.051	1.000	0.138	0.081	0.043	0.140	0.112	0.059	0.085	0.131	0.184	0.001	0.161	0.093
39-6	0.071	0.065	0.201	0.124	0.130	0.044	0.090	0.138	1.000	0.103	-0.014	0.035	0.149	0.051	0.062	0.116	0.013	0.074	0.153	0.127
39-7	0.067	0.041	0.079	0.034	0.041	0.007	-0.012	0.081	0.103	1.000	0.131	0.007	0.004	0.024	-0.003	0.111	0.053	0.081	0.173	0.035
39-8	0.082	-0.028	0.100	0.100	0.055	0.048	-0.017	0.043	-0.014	0.131	1.000	-0.067	0.047	-0.016	0.063	0.004	0.030	0.013	0.050	0.070
39-9	0.088	-0.034	0.134	0.085	0.049	-0.064	0.036	0.140	0.035	0.007	-0.067	1.000	0.082	0.041	0.070	0.000	0.046	-0.081	-0.009	0.077
39-10	0.049	0.009	0.047	0.135	0.009	0.145	0.026	0.112	0.149	0.004	0.047	0.082	1.000	0.080	0.069	-0.004	0.041	0.115	0.119	0.047
39-11	0.044	-0.030	0.091	0.023	0.025	0.076	-0.021	0.059	0.051	0.024	-0.016	0.041	0.080	1.000	0.115	0.022	-0.087	-0.048	0.112	0.046
39-12	0.042	0.018	0.012	0.037	0.095	0.061	0.034	0.085	0.062	-0.003	0.063	0.070	0.069	0.115	1.000	0.132	0.112	0.006	0.065	0.069
39-13	0.063	0.047	0.125	0.037	0.100	0.090	0.088	0.131	0.116	0.111	0.004	0.000	-0.004	0.022	0.132	1.000	0.184	0.206	0.232	0.138
39-14	0.088	-0.041	0.050	0.087	-0.050	0.067	0.042	0.184	0.013	0.053	0.030	0.046	0.041	-0.087	0.112	0.184	1.000	0.148	0.051	0.204
39-15	0.139	0.023	0.117	0.083	0.124	0.059	-0.004	0.001	0.074	0.081	0.013	-0.081	0.115	-0.048	0.006	0.206	0.148	1.000	0.090	0.037
39-16	-0.021	0.061	0.172	0.115	-0.003	0.088	0.025	0.161	0.153	0.173	0.050	-0.009	0.119	0.112	0.065	0.232	0.051	0.090	1.000	-0.023
39-17	0.082	-0.028	0.055	0.149	0.115	0.116	-0.018	0.093	0.127	0.035	0.070	0.077	0.047	0.046	0.069	0.138	0.204	0.037	-0.023	1.000

All fuel rod positions are sampled independently and differently in each case

Coefficients range from ~0 to ~0.23

Fuel rod position modeling makes a difference!

LCT-042 results

	42-1	42-2	42-3	42-4	42-5	42-6	42-7
42-1	1	0.97	0.99	0.97	0.99	0.99	0.98
42-2	0.97	1	0.97	0.96	0.97	0.98	0.96
42-3	0.99	0.97	1	0.97	0.99	0.99	0.97
42-4	0.97	0.96	0.97	1	0.98	0.98	0.96
42-5	0.99	0.97	0.99	0.98	1	0.99	0.98
42-6	0.99	0.98	0.99	0.98	0.99	1	0.98
42-7	0.98	0.96	0.97	0.96	0.98	0.98	1

Pitch sampled: all pitches are the same and are the same for all cases

Coefficients range from 0.96–0.99

	42-1	42-2	42-3	42-4	42-5	42-6	42-7
42-1	1	0.33	0.52	0.42	0.52	0.63	0.43
42-2	0.33	1	0.32	0.48	0.42	0.37	0.19
42-3	0.52	0.32	1	0.29	0.59	0.61	0.48
42-4	0.42	0.48	0.29	1	0.46	0.31	0.26
42-5	0.52	0.42	0.59	0.46	1	0.67	0.49
42-6	0.63	0.37	0.61	0.31	0.67	1	0.46
42-7	0.43	0.19	0.48	0.26	0.49	0.46	1

All fuel rod positions are sampled independently and differently in each case

Coefficients range from 0.19–0.67

Fuel rod position modeling makes a difference, but it is not the same in both cases

Monte Carlo uncertainty effects

- Stochastic uncertainty in each realization k_{eff} calculation contributes unique uncertainty to each case
- Higher uncertainties drive correlation coefficients lower
- How large is the effect, and how low must the uncertainty go?

WPNCS/UACSA results

	7-1	7-2	7-3	39-1	39-2	39-3	39-4	39-5	39-6	39-7	39-8	39-9	39-10	39-11	39-12	39-13	39-14	39-15	39-16	39-17
7-1	1.000	0.034	0.023	0.012	0.005	-0.040	0.069	-0.009	0.071	0.067	0.082	0.088	0.049	0.044	0.042	0.063	0.088	0.139	-0.021	0.082
7-2	0.034	1.000	0.074	-0.045	0.020	0.040	0.181	0.086	0.065	0.041	-0.028	-0.034	0.009	-0.030	0.018	0.047	-0.041	0.023	0.061	-0.028
7-3	0.023	0.074	1.000	0.118	0.063	0.094	0.061	0.086	0.201	0.079	0.100	0.134	0.047	0.091	0.012	0.125	0.050	0.117	0.172	0.055
39-1	0.012	-0.045	0.118	1.000	0.121	0.138	0.076	0.071	0.124	0.034	0.100	0.085	0.135	0.023	0.037	0.037	0.087	0.083	0.115	0.149
39-2	0.005	0.020	0.063	0.121	1.000	0.034	0.075	0.037	0.130	0.041	0.055	0.049	0.009	0.025	0.095	0.100	-0.050	0.124	-0.003	0.115
39-3	-0.040	0.040	0.094	0.138	0.034	1.000	0.079	0.077	0.044	0.007	0.048	-0.064	0.145	0.076	0.061	0.090	0.067	0.059	0.088	0.116
39-4	0.069	0.181	0.061	0.076	0.075	0.079	1.000	-0.051	0.090	-0.012	-0.017	0.036	0.026	-0.021	0.034	0.088	0.042	-0.004	0.025	-0.018
39-5	-0.009	0.086	0.086	0.071	0.037	0.077	-0.051	1.000	0.138	0.081	0.043	0.140	0.112	0.059	0.085	0.131	0.184	0.001	0.161	0.093
39-6	0.071	0.065	0.201	0.124	0.130	0.044	0.090	0.138	1.000	0.103	-0.014	0.035	0.149	0.051	0.062	0.116	0.013	0.074	0.153	0.127
39-7	0.067	0.041	0.079	0.034	0.041	0.007	-0.012	0.081	0.103	1.000	0.131	0.007	0.004	0.024	-0.003	0.111	0.053	0.081	0.173	0.035
39-8	0.082	-0.028	0.100	0.100	0.055	0.048	-0.017	0.043	-0.014	0.131	1.000	-0.067	0.047	-0.016	0.063	0.004	0.030	0.013	0.050	0.070
39-9	0.088	-0.034	0.134	0.085	0.049	-0.064	0.036	0.140	0.035	0.007	-0.067	1.000	0.082	0.041	0.070	0.000	0.046	-0.081	-0.009	0.077
39-10	0.049	0.009	0.047	0.135	0.009	0.145	0.026	0.112	0.149	0.004	0.047	0.082	1.000	0.080	0.069	-0.004	0.041	0.115	0.119	0.047
39-11	0.044	-0.030	0.091	0.023	0.025	0.076	-0.021	0.059	0.051	0.024	-0.016	0.041	0.080	1.000	0.115	0.022	-0.087	-0.048	0.112	0.046
39-12	0.042	0.018	0.012	0.037	0.095	0.061	0.034	0.085	0.062	-0.003	0.063	0.070	0.069	0.115	1.000	0.132	0.112	0.006	0.065	0.069
39-13	0.063	0.047	0.125	0.037	0.100	0.090	0.088	0.131	0.116	0.111	0.004	0.000	-0.004	0.022	0.132	1.000	0.184	0.206	0.232	0.138
39-14	0.088	-0.041	0.050	0.087	-0.050	0.067	0.042	0.184	0.013	0.053	0.030	0.046	0.041	-0.087	0.112	0.184	1.000	0.148	0.051	0.204
39-15	0.139	0.023	0.117	0.083	0.124	0.059	-0.004	0.001	0.074	0.081	0.013	-0.081	0.115	-0.048	0.006	0.206	0.148	1.000	0.090	0.037
39-16	-0.021	0.061	0.172	0.115	-0.003	0.088	0.025	0.161	0.153	0.173	0.050	-0.009	0.119	0.112	0.065	0.232	0.051	0.090	1.000	-0.023
39-17	0.082	-0.028	0.055	0.149	0.115	0.116	-0.018	0.093	0.127	0.035	0.070	0.077	0.047	0.046	0.069	0.138	0.204	0.037	-0.023	1.000

Stochastic uncertainty on the order of 0.06% Δk

Coefficients between ~0 and 0.23

	7-1	7-2	7-3	7-4	39-1	39-2	39-3	39-4	39-5	39-6	39-7	39-8	39-9	39-10	39-11	39-12	39-13	39-14	39-15	39-16	39-17
7-1	1.000	0.362	0.457	0.359	0.453	0.196	0.203	0.315	0.328	0.367	0.249	0.241	0.358	0.236	0.227	0.179	0.235	0.221	0.222	0.232	0.226
7-2	0.362	1.000	0.642	0.641	0.288	0.300	0.315	0.328	0.425	0.404	0.322	0.298	0.547	0.345	0.294	0.302	0.326	0.327	0.318	0.325	0.321
7-3	0.457	0.642	1.000	0.853	0.417	0.451	0.452	0.440	0.589	0.550	0.474	0.431	0.710	0.472	0.403	0.437	0.441	0.477	0.449	0.473	0.463
7-4	0.359	0.641	0.853	1.000	0.437	0.447	0.486	0.450	0.613	0.521	0.470	0.446	0.697	0.486	0.424	0.432	0.448	0.488	0.450	0.473	0.477
39-1	0.453	0.288	0.417	0.437	1.000	0.234	0.241	0.229	0.299	0.380	0.260	0.270	0.358	0.269	0.210	0.232	0.278	0.265	0.283	0.273	0.263
39-2	0.196	0.300	0.451	0.447	0.234	1.000	0.244	0.244	0.298	0.375	0.277	0.267	0.412	0.290	0.245	0.225	0.254	0.263	0.272	0.265	0.241
39-3	0.203	0.315	0.452	0.486	0.241	0.244	1.000	0.235	0.278	0.337	0.230	0.238	0.420	0.252	0.210	0.224	0.251	0.267	0.264	0.276	0.240
39-4	0.315	0.328	0.440	0.450	0.229	0.244	0.235	1.000	0.270	0.333	0.200	0.245	0.394	0.255	0.184	0.222	0.248	0.243	0.243	0.259	0.245
39-5	0.328	0.425	0.589	0.613	0.299	0.298	0.278	0.270	1.000	0.334	0.278	0.305	0.479	0.276	0.247	0.272	0.280	0.295	0.287	0.295	0.260
39-6	0.367	0.404	0.550	0.521	0.380	0.375	0.337	0.333	0.334	1.000	0.337	0.334	0.502	0.333	0.301	0.293	0.321	0.342	0.307	0.328	0.289
39-7	0.249	0.322	0.474	0.470	0.260	0.277	0.230	0.200	0.278	0.337	1.000	0.306	0.401	0.271	0.243	0.250	0.297	0.292	0.309	0.331	0.280
39-8	0.241	0.298	0.431	0.446	0.270	0.267	0.238	0.245	0.305	0.334	0.306	1.000	0.416	0.268	0.251	0.225	0.250	0.273	0.260	0.290	0.247
39-9	0.358	0.547	0.710	0.697	0.358	0.412	0.420	0.394	0.479	0.502	0.401	0.416	1.000	0.393	0.400	0.393	0.370	0.403	0.388	0.400	0.409
39-10	0.236	0.345	0.472	0.486	0.269	0.290	0.252	0.255	0.276	0.333	0.271	0.268	0.393	1.000	0.205	0.220	0.269	0.258	0.271	0.280	0.219
39-11	0.227	0.294	0.403	0.424	0.210	0.245	0.210	0.184	0.247	0.301	0.243	0.251	0.400	0.205	1.000	0.226	0.267	0.251	0.275	0.278	0.256
39-12	0.179	0.302	0.437	0.432	0.232	0.225	0.224	0.222	0.272	0.293	0.250	0.225	0.393	0.220	0.226	1.000	0.285	0.262	0.275	0.298	0.292
39-13	0.235	0.326	0.441	0.448	0.278	0.254	0.251	0.248	0.280	0.321	0.297	0.250	0.370	0.269	0.267	0.285	1.000	0.281	0.300	0.290	0.281
39-14	0.221	0.327	0.477	0.488	0.265	0.263	0.267	0.243	0.295	0.342	0.292	0.273	0.403	0.258	0.251	0.262	0.281	1.000	0.273	0.277	0.267
39-15	0.222	0.318	0.449	0.450	0.283	0.272	0.264	0.243	0.287	0.307	0.309	0.260	0.388	0.271	0.275	0.275	0.300	0.273	1.000	0.284	0.252
39-16	0.232	0.325	0.473	0.473	0.273	0.265	0.276	0.259	0.295	0.328	0.331	0.290	0.400	0.280	0.278	0.298	0.290	0.277	0.284	1.000	0.257
39-17	0.226	0.321	0.463	0.477	0.263	0.241	0.240	0.245	0.260	0.289	0.280	0.247	0.409	0.219	0.256	0.292	0.281	0.267	0.252	0.257	1.000

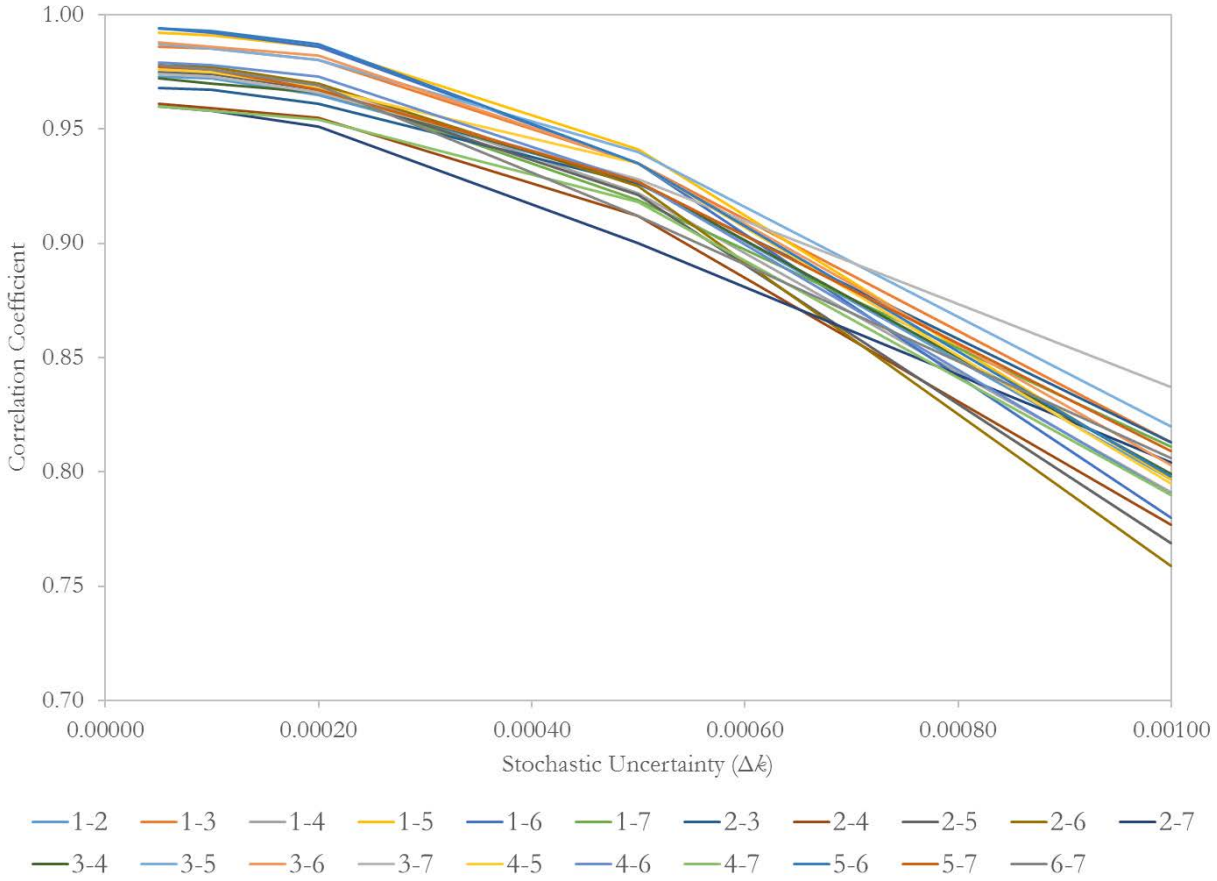
Stochastic uncertainty on the order of 0.01% Δk

Coefficients between 0.18 and 0.71

Stochastic uncertainty makes a difference!

LCT-042 results

- There are fewer cases in LCT-042, so more different stochastic uncertainties were examined
- Stochastic uncertainty effect was examined with the uniform pitch assumption
- There was a small change between 0.02% Δk and 0.01% Δk
- There was effectively no change between 0.01% Δk and 0.005% Δk



Stochastic uncertainty makes a difference, but it appears to saturate at low uncertainties

Conclusions

- Critical experiment correlations can be determined in SCALE 6.2 using the Sampler sequence; results are consistent with expectations and explicable with known inputs
- Treatment of fuel rod pitch is extremely important
- Stochastic uncertainty of individual realizations must be fairly low
- 100–300 realizations generally sufficient for convergence
- Plots can be generated to check convergence; see paper for examples

Questions?