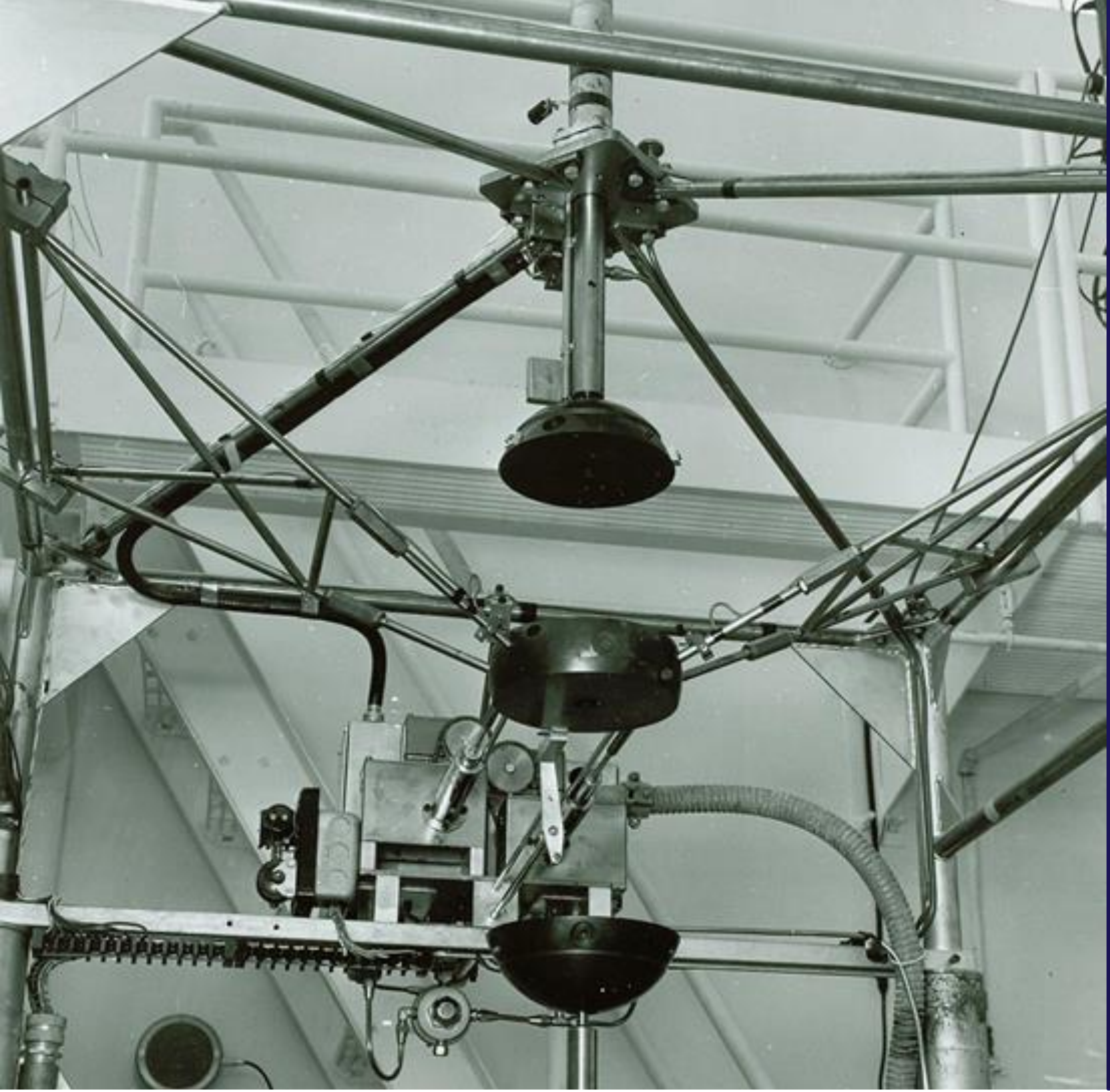

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Improved Figure of Merit for Feynman Histograms

J. Arthur[1,2], R. Bahran[1], J. Hutchinson[1], T. Grove[1], S. Pozzi[2]

[1]Los Alamos National Laboratory

[2]University of Michigan

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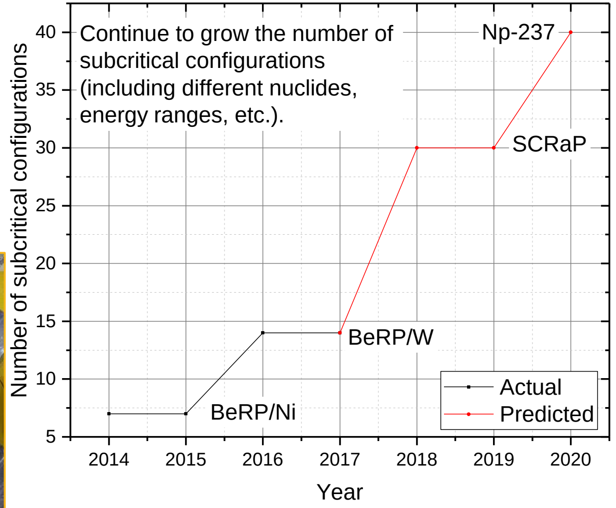
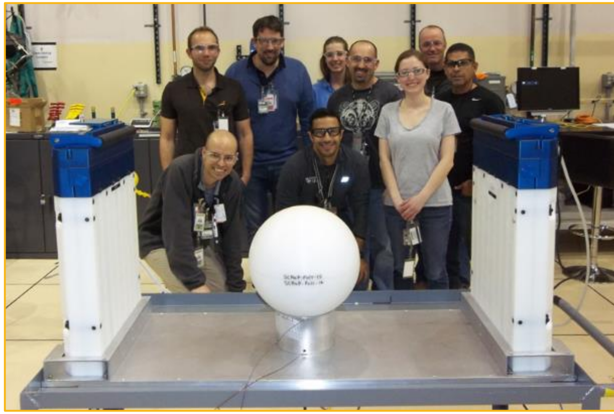
Motivation

- **Growing dataset of neutron multiplication benchmarks experiments/evaluations**
  - Culmination of several years of sub-critical experiment research
  - Goal is to validate nuclear data and computational methods

Chronology: 2012 - Present

- **BeRP-Ni (published in 2014)**
  - Executed in 2012, ICSBEP evaluation published in 2014
- **BeRP-W (published in 2016)**
  - Sub-critical tungsten-reflected α-phase Pu
  - Executed in 2012, ICSBEP evaluation published in 2016
- **SCRαP (to be published in 2018?)**
  - Sub-critical copper/poly-reflected α-phase Pu
  - Executed in 2016, ICSBEP evaluation published in 2018
- **Neptunium (to be published in 2020?)**
  - Sub-critical Neptunium w/various reflectors, in design phase



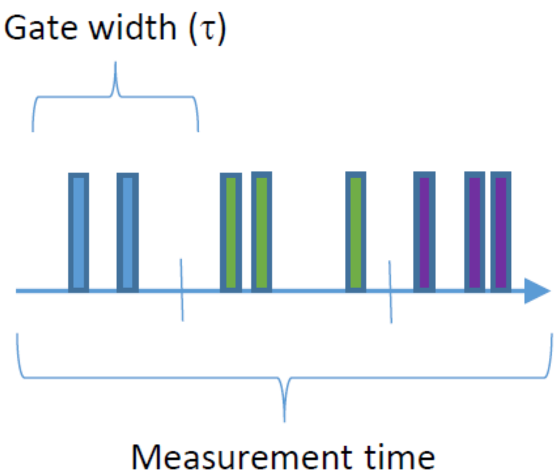


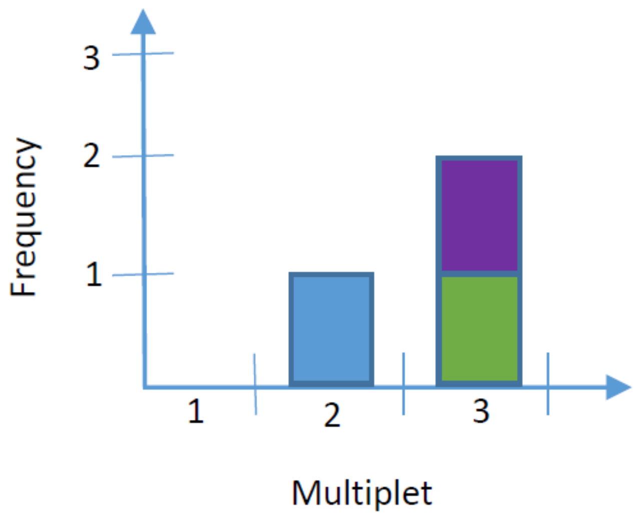
Motivation

- **Neutron multiplication inference measurements record list-mode data**
  - Includes only time and detector corresponding to each event
- **Observables of interest in advanced subcritical experiments include [1]:**
  - Feynman histogram (C_n): histogram showing the relative frequencies of various multiplets
  - Singles (R_1): rate of detection of single neutrons from a fission chain
  - Doubles (R_2): rate of detection of 2 neutrons from the same fission chain
  - Leakage multiplication (M_L): average number of neutrons escaping the system per neutron injected into the system
- **The goal is a quantitative comparison of measured and simulated histograms, for radiation transport code and nuclear data validation**

Feynman histograms

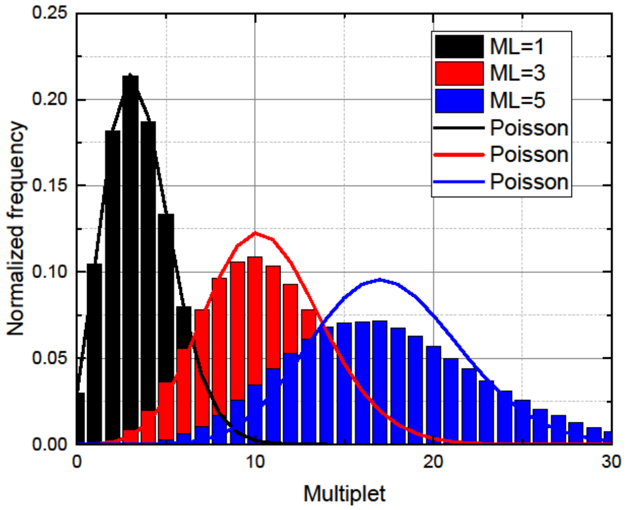
- **Method of creating a Feynman histogram from list-mode data:**
  - This is usually done for 1E5-1E6 time gates

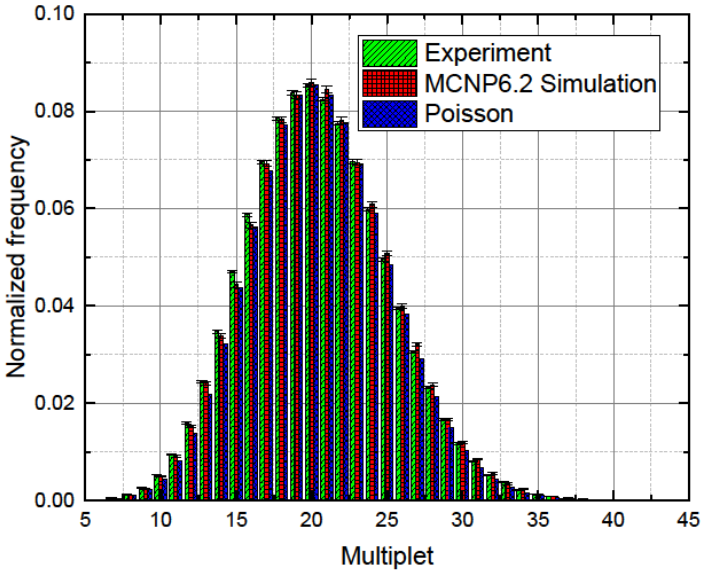




Feynman histograms

- Leakage multiplication is related to the deviation of the histogram from a Poisson distribution





- To date, no one has rigorously evaluated or established a suitable figure of merit (FOM) to quantify the degree of discrepancy between two Feynman histograms

Past FOM equation

- Typically a FOM that takes into account the differences between each bin of the histograms, as compared to the magnitude of the combination of the corresponding uncertainties, is used [2]

$$FOM = \frac{1}{N_{bins}} \sum_{i=1}^{N_{bins}} \frac{(M_i - S_i)^2}{\sigma_{M,i}^2 + \sigma_{S,i}^2}$$

- Because the uncertainties corresponding to the larger multiplet bins are inherently larger than those of the smaller multiplet bins, this type of FOM puts more weight on differences between smaller multiplet bins
- New proposed FOM also takes into account sensitivity of leakage multiplication (which is most sensitive to higher multiplet bins) to each bin in the histogram

Past FOM equation

$$FOM = \frac{1}{N_{bins}} \sum_{i=1}^{N_{bins}} \frac{(M_i - S_i)^2}{\sigma_{M,i}^2 + \sigma_{S,i}^2}$$

- **M_i and S_i represent the ith bins of the simulated and measured Feynman histograms**
- **$\sigma_{M,i}^2$ and $\sigma_{S,i}^2$ are the variances corresponding to these bins**
- **The ideal FOM value is 1**
  - The discrepancy between simulated and measured histograms is equal to the combined associated uncertainty.

Deriving an improved figure of merit

- **The sensitivities of leakage multiplication to each bin in the Feynman histogram were calculated**
  - Standard uncertainty propagation

Chain rule:

$$\frac{dy}{dx_n} = \frac{dy}{dx_1} * \frac{dx_1}{dx_2} * \frac{dx_2}{dx_3} * \cdots \frac{dx_{n-1}}{dx_n}$$

Chain rule
for partial derivatives:

$$\frac{dy}{dx_n} = \frac{\partial y}{\partial x_1} * \frac{dx_1}{dx_n} + \frac{\partial y}{\partial x_2} * \frac{dx_2}{dx_n} + \cdots \frac{\partial y}{\partial x_{n-1}} * \frac{dx_{n-1}}{dx_n}$$

- **The sensitivities are normalized and added as an additional factor to the past FOM**

Uncertainty propagation

- **Calculating leakage multiplication (M_L) from each bin in the Feynman histogram ($C_n(\tau)$) using the Hage-Cifarelli formalism based on the Feynman Variance-to-Mean method [1,3]**
- **Assumptions:**
  - Point source of spontaneous fission neutrons
  - No (α,n) contribution; spontaneous and induced fission only

$$p_n(\tau) = \frac{C_n(\tau)}{\sum_{n=0}^{\infty} C_n(\tau)}$$

$$m_r(\tau) = \frac{\sum_{n=0}^{\infty} n(n-1)\ldots(n-r+1)\, p_n(\tau)}{r!}$$

$$R_1(\tau) = \frac{m_1(\tau)}{\tau}$$

$$Y_2(\tau) = \frac{m_2(\tau) - \frac{1}{2}[m_1(\tau)]^2}{\tau}$$

$$\omega_2(\lambda,\tau) = 1 - \frac{1 - e^{-\lambda\tau}}{\lambda\tau}$$

$$R_2(\tau) = \frac{Y_2(\tau)}{\omega_2(\lambda,\tau)}$$

$$M_L = \frac{-C_2 + \sqrt{C_2^2 - 4C_1C_3}}{2C_1}$$

$$C_1 = \frac{\upsilon_{s1}\upsilon_{I2}}{\upsilon_{I1}-1},\ C_2 = \upsilon_{s2} - \frac{\upsilon_{s1}\upsilon_{I2}}{\upsilon_{I1}-1},\ C_3 = -\frac{R_2(\tau)\upsilon_{s1}}{R_1(\tau)\epsilon}$$

Figure of merit equation

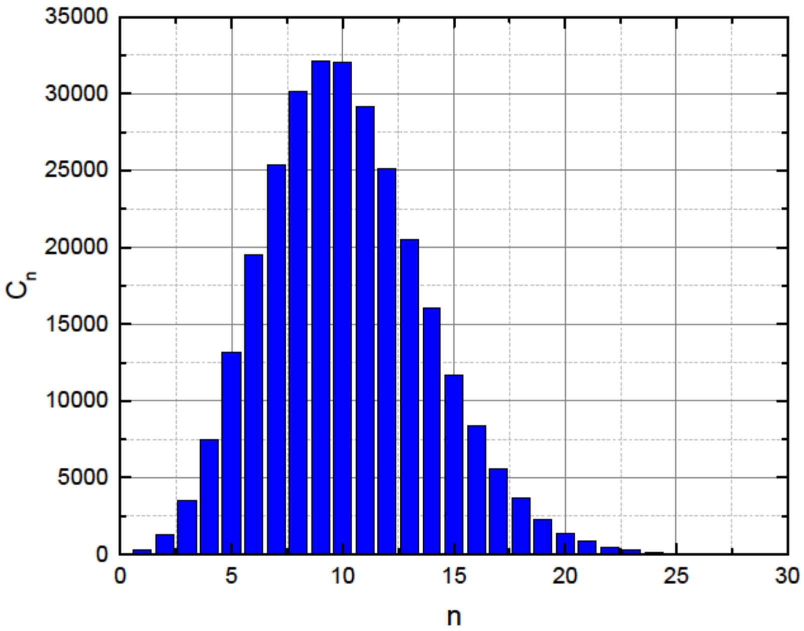
- **Ideal value is still unity**
  - Sensitivities are normalized and sum to unity

$$\frac{dM_L}{dC_n} = \frac{\left(C_n(\tau) - \sum_{n=1}^{N_{bins}} C_n(\tau)\right)\upsilon_{s1}n}{\left(\sum_{n=1}^{N_{bins}} C_n(\tau)\right)^2 \sqrt{C_2^2 - 4C_1C_3\tau\epsilon R_1}} \left[-\frac{R_2}{R_1} - \frac{m_1}{\omega_2} + \frac{n-1}{2\omega_2}\right]$$

$$FOM = \sum_{i=1}^{N_{bins}} \frac{(M_i - S_i)^2}{\sigma_{M,i}^2 + \sigma_{S,i}^2} \left|\frac{dM_L}{dC_i}\right|_{norm}$$

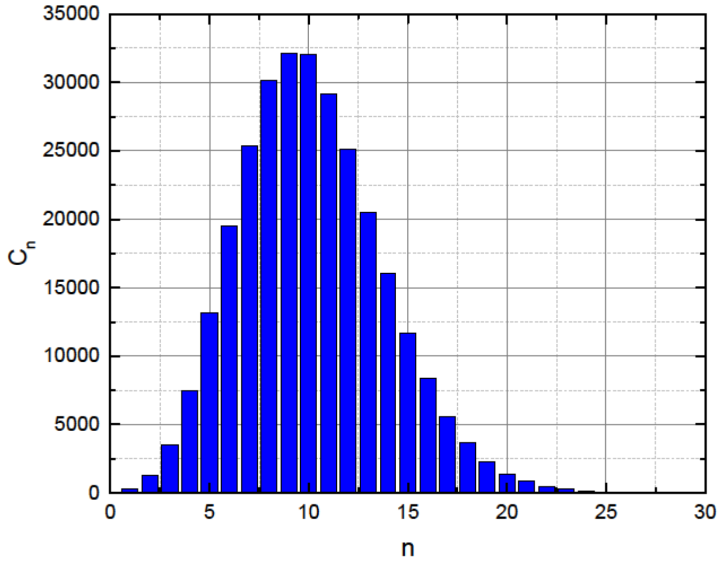
Test case

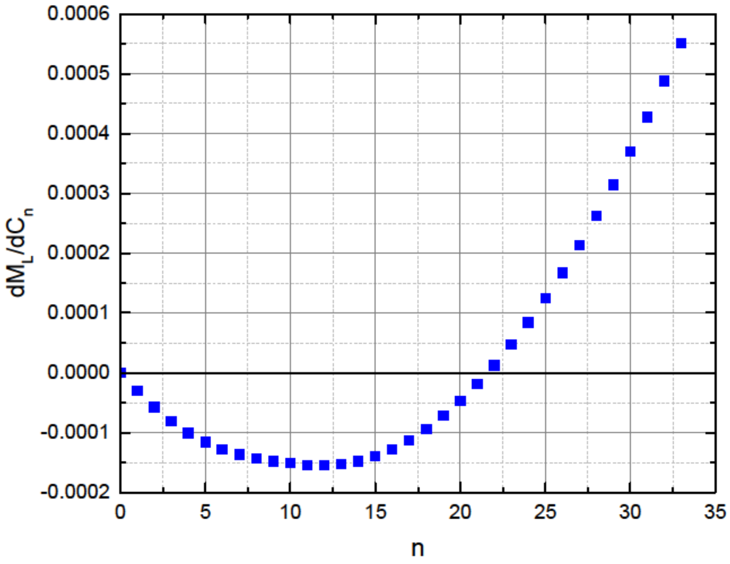
- Sphere of pure Pu-240 and Pu-239
- Fission rate = 130423 s^{-1}
- Detector efficiency = 0.012
- Tube dead time = 4.0 μs
- Neutron lifetime = 40.0 μs
- Count time = 300 s
- M_L = 3.0



Preliminary results

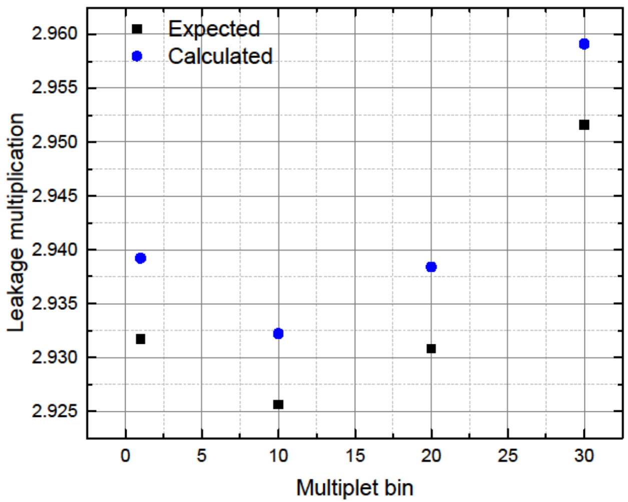
- **Important to keep in mind: leakage multiplication is related to the deviation of the Feynman histogram from a Poisson distribution**
- **Calculated M_L=2.9332**





Preliminary results

- **Unperturbed calculated M_L=2.9332**
- **Each multiplet bin perturbed by 50 counts**
  - Consistent downward bias of expected leakage multiplication



Multiplet bin	1	10	20	30
Sensitivity	-3.0211E-5	-1.5158E-4	-4.7141E-5	3.6906E-4
Expected M_L	2.9317	2.9256	2.9308	2.9516
Calculated M_L	2.9392	2.9322	2.9384	2.9591

Preliminary results

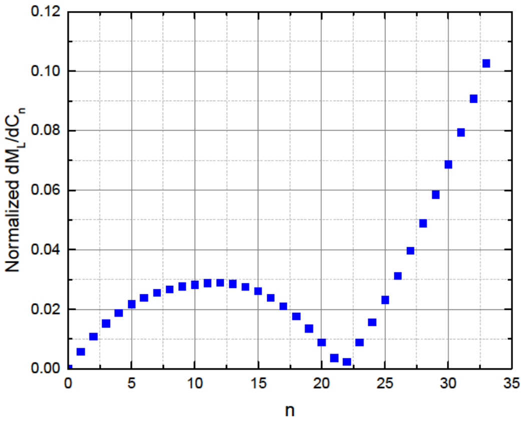
- **Each multiplet bin perturbed such that:** $(M_{\boldsymbol{n}} - S_{\boldsymbol{n}})^2 = \sigma_{M,\boldsymbol{n}}^2 + \sigma_{S,\boldsymbol{n}}^2$
- **Each bin of interest perturbed such that:** $(M_{\boldsymbol{n}} - S_{\boldsymbol{n}})^2 = \mathbf{10}(\sigma_{M,\boldsymbol{n}}^2 + \sigma_{S,\boldsymbol{n}}^2)$

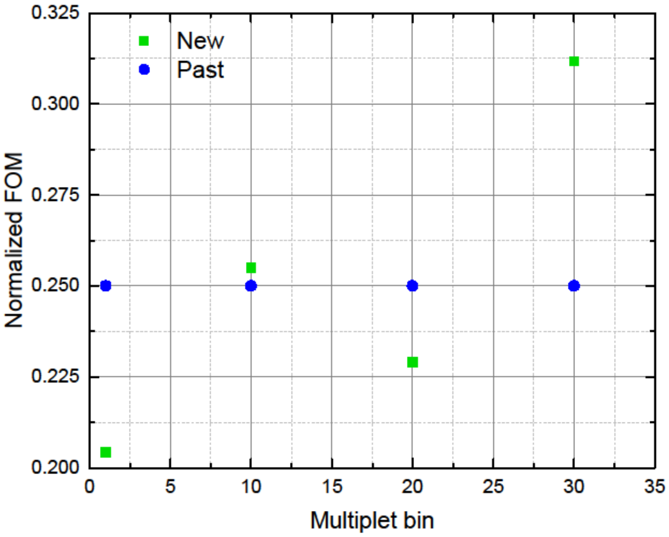
$$Past\ FOM = \frac{1}{N_{bins}} \sum_{i=1}^{N_{bins}} \frac{(M_i - S_i)^2}{\sigma_{M,i}^2 + \sigma_{S,i}^2} \qquad New\ FOM = \sum_{i=1}^{N_{bins}} \frac{(M_i - S_i)^2}{\sigma_{M,i}^2 + \sigma_{S,i}^2} \left|\frac{dM_L}{dC_i}\right|_{norm}$$

Multiplet bin	1	10	20	30
Past FOM	1.2647	1.2647	1.2647	1.2647
New FOM	1.0000	1.2480	1.1210	1.5261

Preliminary results

- Past FOM isn't affected by which multiplet bin is perturbed, but only by how much it is perturbed
- New FOM follows the trend of sensitivity of M_L to the perturbed multiplet bins





Conclusions and future work

- **The FOM that has typically been used to quantitatively compare Feynman histograms puts more weight on discrepancies at multiplet bins with lower uncertainties**
  - Leakage multiplication, the final observable of interest, is most sensitive to multiplet bins that increase or decrease the amount of deviation of the histogram from a Poisson distribution
- **New FOM puts more weight on discrepancies at bins that affect the amount of deviation of the histogram from a Poisson distribution**
- **This improved FOM will provide a better quantitative comparison between measured and simulated Feynman histograms for radiation transport code and nuclear data validation applications**
  - The new FOM will be applied to comparisons of Feynman histograms between measured data and simulated data produced by various codes that take into account the correlated physics of fission neutrons [4]

References

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