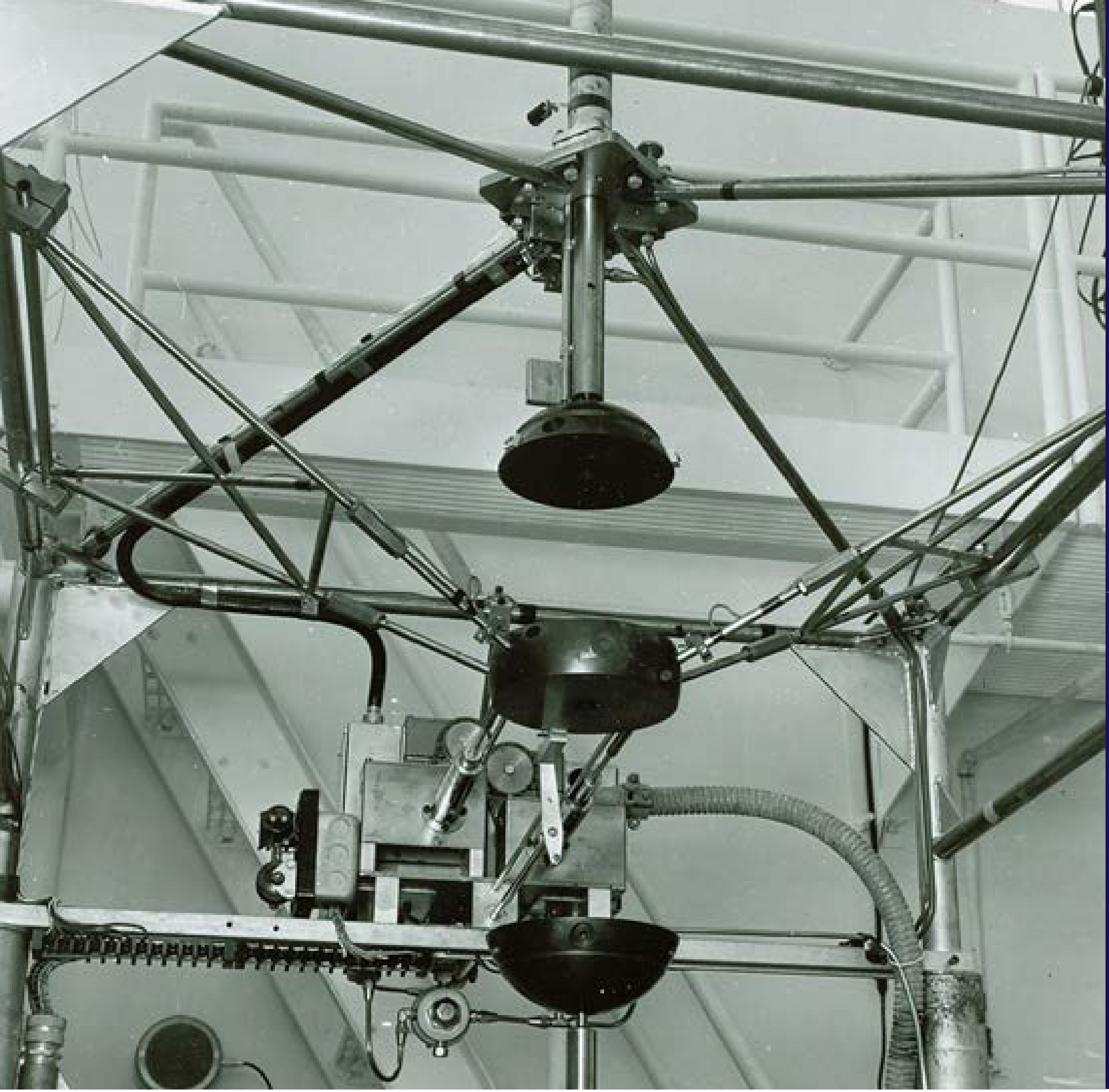

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Prompt Neutron Decay Constant Fitting Using the Rossi-alpha and Feynman Variance-to-Mean Methods

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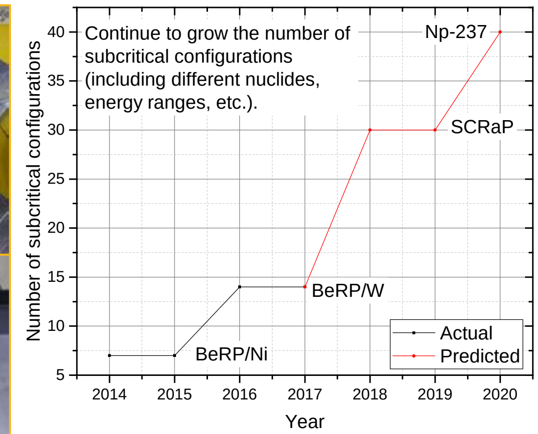
ANS Winter 2017

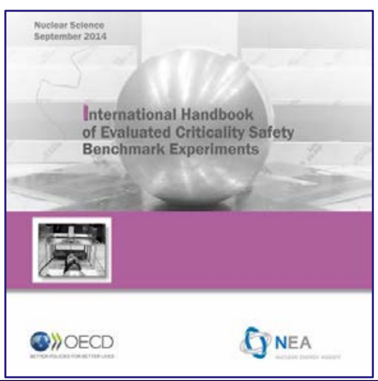
Overview

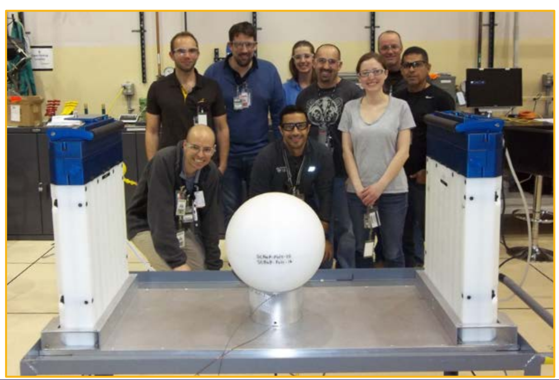
- Introduction
- Background
- Results
- Conclusions
- Future work

Introduction

- **Subcritical analysis methods are employed for many applications including nonproliferation, criticality safety, and Accelerator-Driven Systems.**
- **Many organizations (LANL, LLNL, SNL, IAEA, IRSN, CEA, AWE, universities, and others) have pursued sub-critical experiments and/or simulations in recent years.**
- **BeRP-Ni (published in 2014)**
    - Executed in 2012, ICSBEP evaluation published in 2014
- **BeRP-W (published in 2016)**
    - Executed in 2012, ICSBEP evaluation published in 2016
- **SCRαP (to be published in 2018?)**
    - Sub-critical copper/poly-reflected α-phase Pu
    - Executed in 2016, ICSBEP evaluation published in 2018

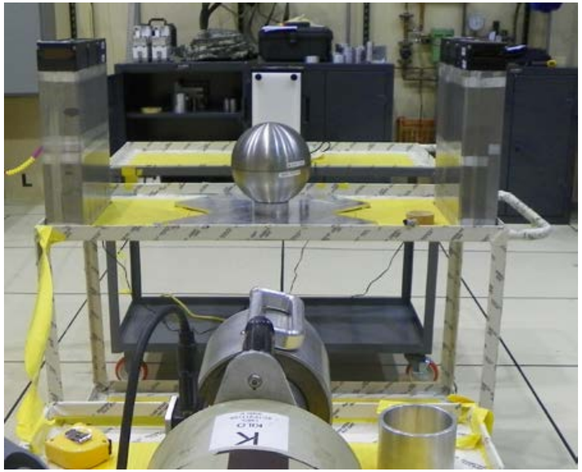






Introduction

- **One parameter of interest when performing subcritical measurements is the prompt neutron decay constant.**
  - This work details how to determine the prompt neutron decay constant using the Rossi-α and Feynman Variance-to-Mean methods.
  - It then looks at various ways to fit the data and makes recommendations on how best to fit the data for different configurations.
- **This work was applied to the BeRP/W data.**



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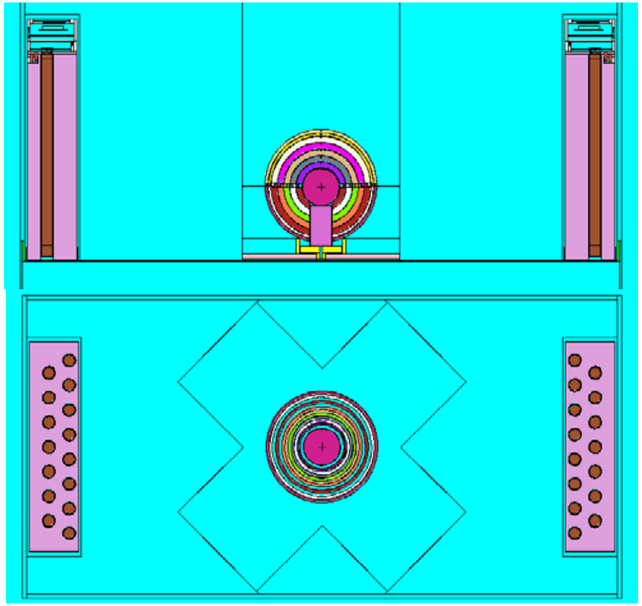
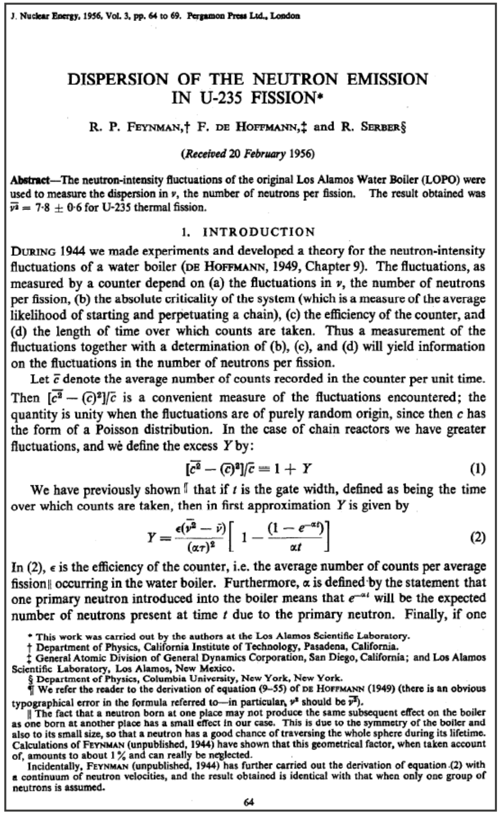


Background

- **Both the Rossi-α and Feynman Variance-to-Mean methods were introduced in this paper.**
- **Both methods are considered neutron noise methods and take advantage of the fact that prompt neutrons that are produced from fission are born immediately after the fission event.**
- **Recent works have described how one can perform a double integration of Rossi-α expressions in order to determine Feynman Variance-to-mean parameters.**

$$\frac{c_k(c_l - \delta_{k,l})}{2} = \int_0^{\Delta T} dt_c \int_0^{t_c} dt_g \, p_{rossi}(t_c - t_g) \; . \qquad (19)$$

R. SOULE, et. al., “Neutronic Studies in Support of Accelerator-Driven Systems: The MUSE Experiments in the MASURCA Facility,” Nuc. Sci. Eng., 148, 124-152 (2004).



J. Nuclear Energy, 1956, Vol. 3, pp. 64 to 69. Pergamon Press Ltd., London

DISPERSION OF THE NEUTRON EMISSION IN U-235 FISSION*

R. P. FEYNMAN,† F. DE HOFFMANN,‡ and R. SERBER§

(*Received* 20 *February* 1956)

Abstract—The neutron-intensity fluctuations of the original Los Alamos Water Boiler (LOPO) were used to measure the dispersion in ν, the number of neutrons per fission. The result obtained was $\overline{\nu^2} = 7{\cdot}8 \pm 0{\cdot}6$ for U-235 thermal fission.

1. INTRODUCTION

DURING 1944 we made experiments and developed a theory for the neutron-intensity fluctuations of a water boiler (DE HOFFMANN, 1949, Chapter 9). The fluctuations, as measured by a counter depend on (a) the fluctuations in ν, the number of neutrons per fission, (b) the absolute criticality of the system (which is a measure of the average likelihood of starting and perpetuating a chain), (c) the efficiency of the counter, and (d) the length of time over which counts are taken. Thus a measurement of the fluctuations together with a determination of (b), (c), and (d) will yield information on the fluctuations in the number of neutrons per fission.

Let $\bar{c}$ denote the average number of counts recorded in the counter per unit time. Then $[\overline{c^2} - (\bar{c})^2]/\bar{c}$ is a convenient measure of the fluctuations encountered; the quantity is unity when the fluctuations are of purely random origin, since then c has the form of a Poisson distribution. In the case of chain reactors we have greater fluctuations, and we define the excess Y by:

$$[\overline{c^2} - (\bar{c})^2]/\bar{c} = 1 + Y \qquad (1)$$

We have previously shown¶ that if t is the gate width, defined as being the time over which counts are taken, then in first approximation Y is given by

$$Y = \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\alpha\tau)^2}\left[1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right] \qquad (2)$$

In (2), ϵ is the efficiency of the counter, i.e. the average number of counts per average fission‖ occurring in the water boiler. Furthermore, α is defined by the statement that one primary neutron introduced into the boiler means that $e^{-\alpha t}$ will be the expected number of neutrons present at time t due to the primary neutron. Finally, if one

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§ Department of Physics, Columbia University, New York, New York.
¶ We refer the reader to the derivation of equation (9–55) of DE HOFFMANN (1949) (there is an obvious typographical error in the formula referred to—in particular, ν^2 should be $\overline{\nu^2}$).
‖ The fact that a neutron born at one place may not produce the same subsequent effect on the boiler as one born at another place has a small effect in our case. This is due to the symmetry of the boiler and also to its small size, so that a neutron has a good chance of traversing the whole sphere during its lifetime. Calculations of FEYNMAN (unpublished, 1944) have shown that this geometrical factor, when taken account of, amounts to about 1 % and can really be neglected.
Incidentally, FEYNMAN (unpublished, 1944) has further carried out the derivation of equation (2) with a continuum of neutron velocities, and the result obtained is identical with that when only one group of neutrons is assumed.

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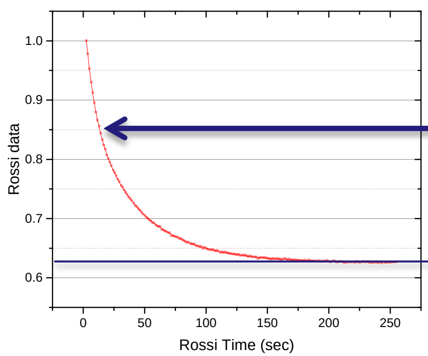
Background: Rossi-α

- **Rossi-α method involves creating a histogram of the time differences of observed neutron events.**

$$p(t)\Delta = A\Delta + Be^{-\alpha t}\Delta$$

- **p(t) is the probability of detecting a neutron in time interval Δ, A is the uncorrelated term, B is the magnitude of the correlated term, and α is the prompt neutron decay constant.**
- **Sometimes the inverse of the prompt neutron decay constant (1/α) is referred to as the system "lifetime".**
  - o Neutron lifetime, however, has a very specific definition: $\frac{1-k_p}{\alpha}$



Correlated ($Be^{-\alpha t}\Delta$ term)

Uncorrelated (AΔ term)

Background: Feynman Variance-to-Mean

$$[\overline{c^2} - (\bar{c})^2]/\bar{c} = 1 + Y$$

Excess variance

Variance

Mean

Detector efficiency (counts per fission)

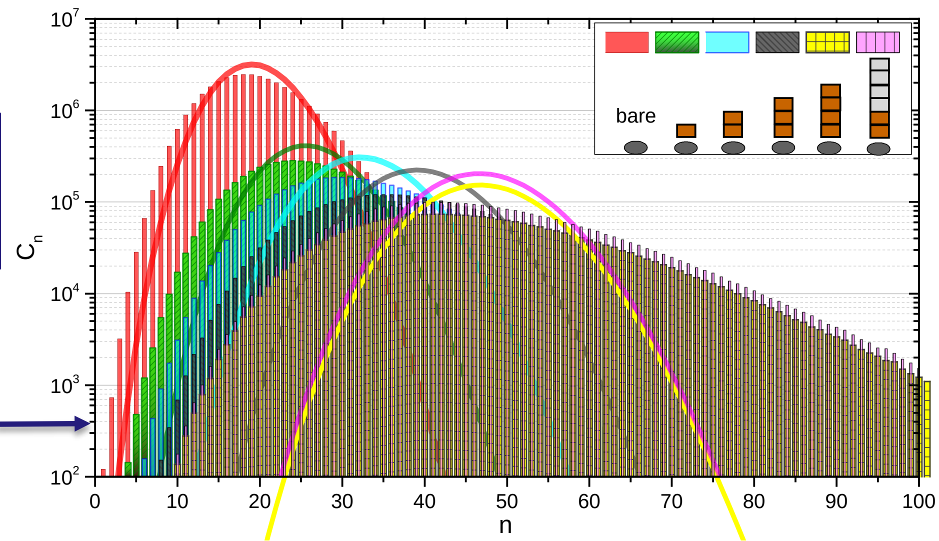
second moment (variance) of the distribution of the number of neutrons produced per induced fission

$$Y = \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\alpha\tau)^2}\left[1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right]$$

mean time between fissions

average number of neutrons produced per induced fission

If multiplication (M) = 1, then excess variance = 0.
As M increases, Y increases



Background: Feynman Variance-to-Mean

A parameter Y_2, which is proportional to excess variance is often used.

$$Y_2 = \frac{Y\bar{c}}{2} = \frac{\overline{c^2} - \bar{c}^2 - \bar{c}}{2}$$

Y_2 data are fit to the term in brackets.

$$Y = \frac{\epsilon(\overline{\nu^2} - \bar{\nu})}{(\alpha\tau)^2}\overbrace{\left[1 - \frac{(1 - e^{-\alpha t})}{\alpha t}\right]}$$

Background: Detector systems

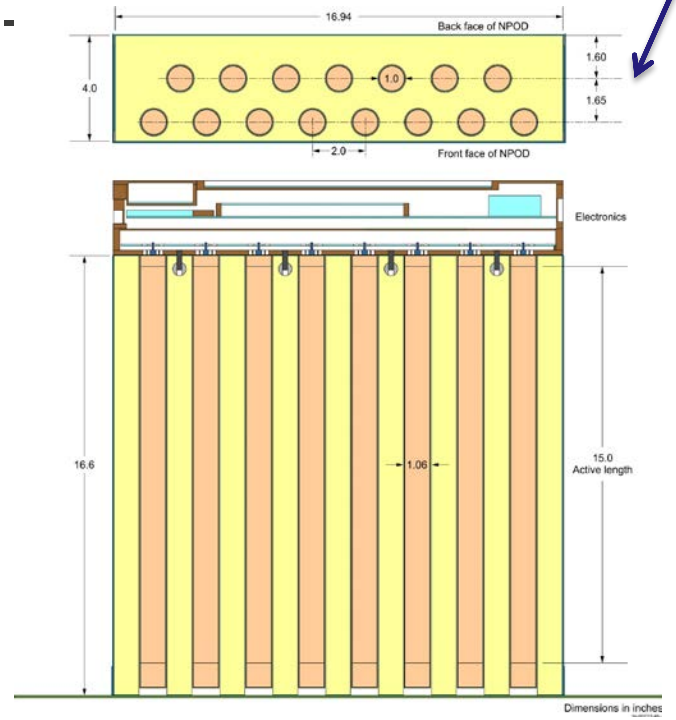
- **Some detector systems involve slowing down neutrons in order to increase the probability of detection (due to many materials having much larger cross-sections at lower energies).**
- **For many applications, He-3 detector systems are used and it is very common to use high-density polyethylene (HDPE) to slow down the neutron energy and increase detector efficiency.**
- **For such systems, the prompt-neutron decay constant includes the time in which neutrons scatter in the HPDE of the detector system.**

  λ is often used instead of **α** to denote inclusion of slowing-down time.

- **For this work, the term α is used. 1/α is called the lifetime/slowing-down time (it is know that this is not a true measure of neutron lifetime).**

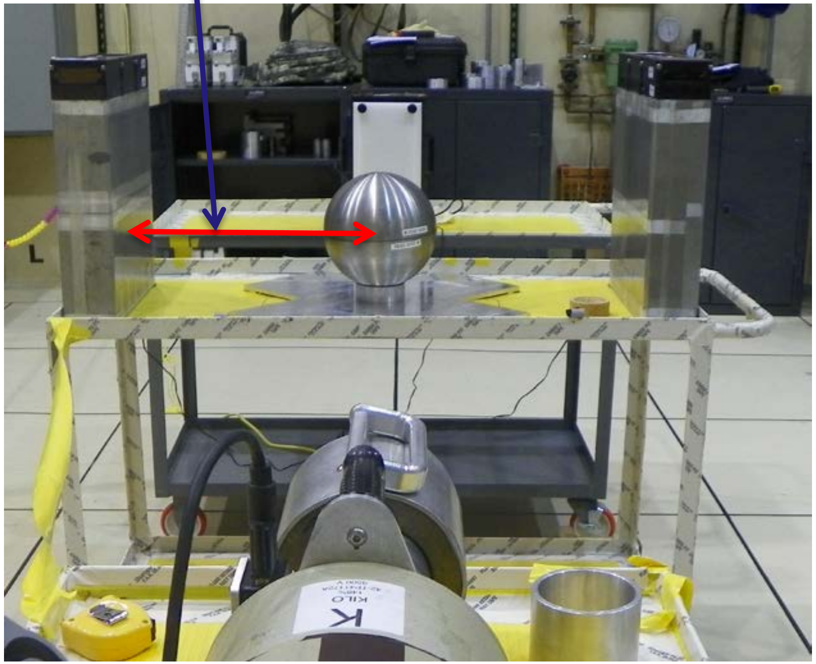
Background: Detector systems

- **NPOD detector system is used for multiplicity analysis.**
- **Has a known slowing-down time of 35-45 micro-**



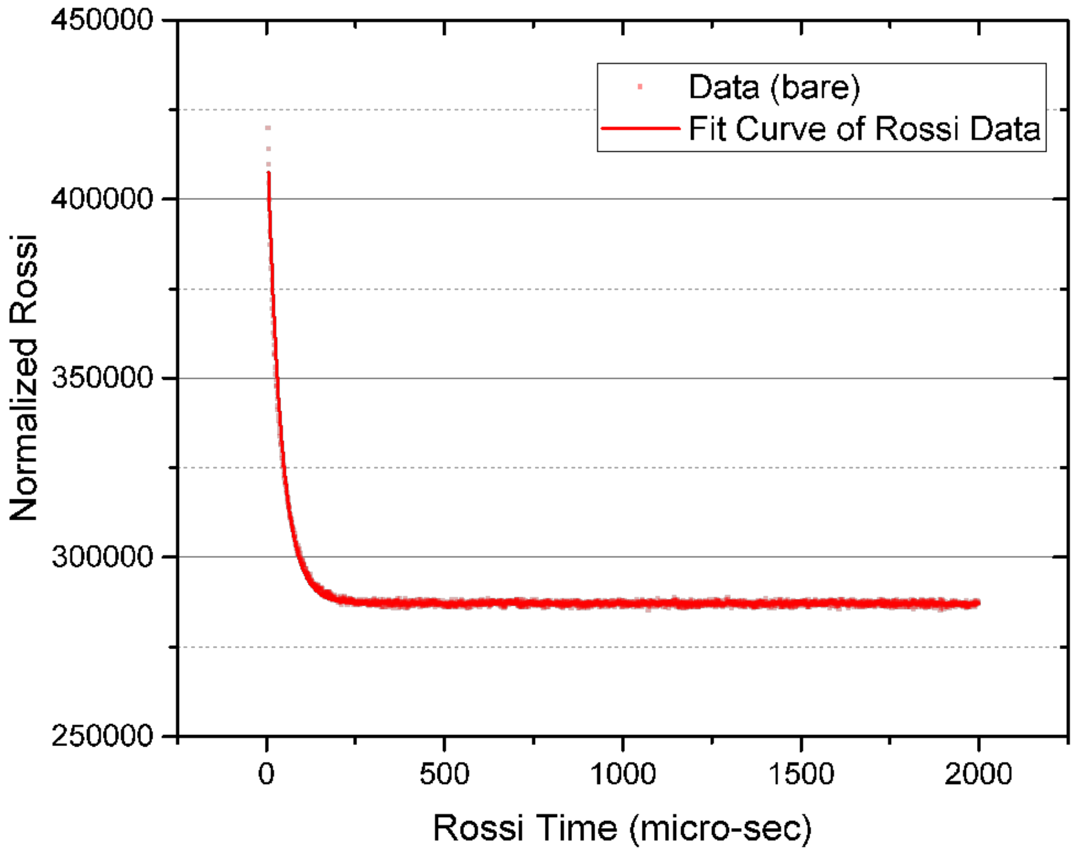
NPOD detectors: 15 He-3 tubes inside polyethylene and wrapped in Cd.

50.0 cm from center of BeRP to Cd face of NPOD.



Results: Rossi-α

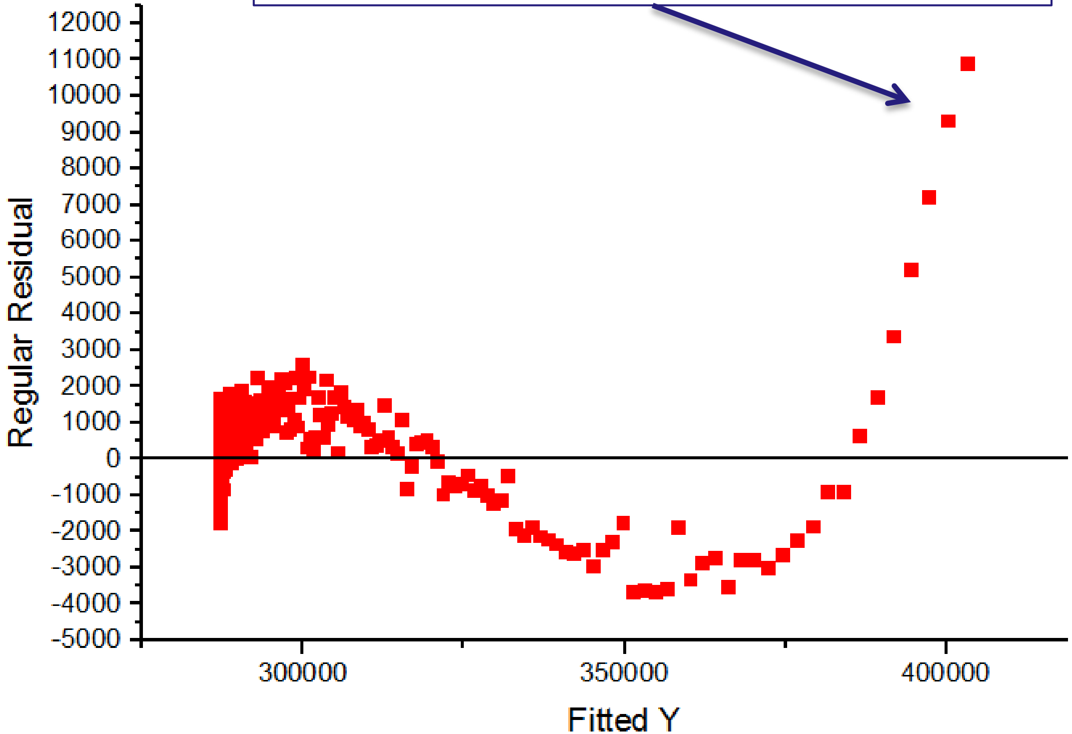
- **Bare BeRP configuration.**
- **Fit using the Levenberg Marquardt iteration algorithm.**
- **Visually this fit looks quite good and the R^2 for each curve is greater than 0.99.**



Results: Rossi-α

- **Residual plots, however, show that this is not the best possible fit.**
- **The fitted value is on the x-axis and the residual value is on the y-axis. For a good fit this plot would be centered around 0 with no increasing or decreasing trends.**
- **The residual plots for the reflected cases show even greater disparity.**

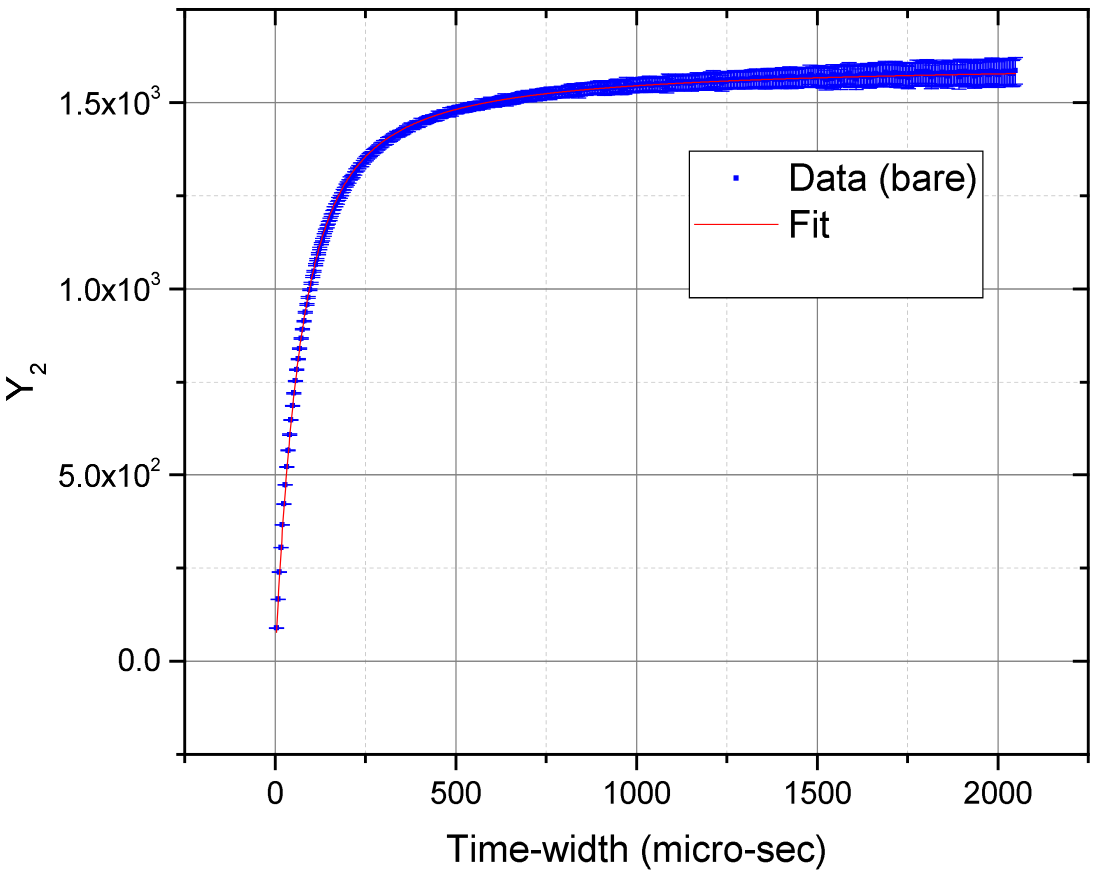
Data is much higher than fit at small times.



Rossi, single decay constant, bare BeRP.

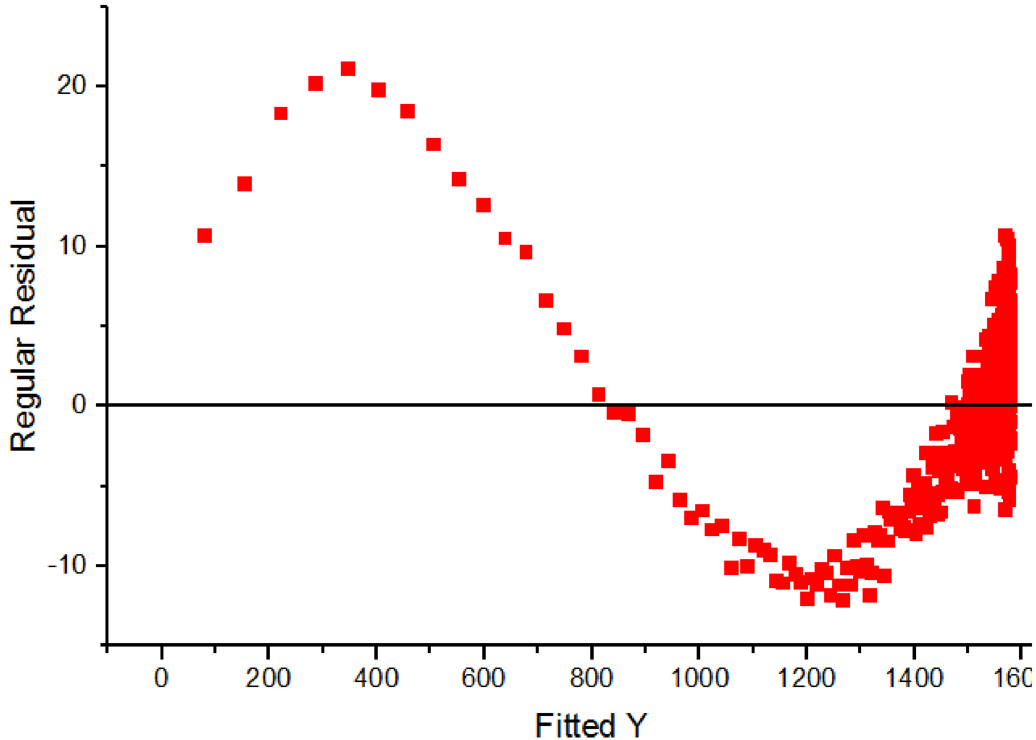
Results: Feynman V/M

- **Bare BeRP configuration.**
- **Fit using the Levenberg Marquardt iteration algorithm.**
- **Visually this fit looks quite good and the R^2 for each curve is greater than 0.999.**



Results: Feynman V/M

- **Similar to Rossi-α:**
  - o Residual plots show that this is not a great fit.
  - o The residual plots the reflected cases show even greater disparity.



Feynman, single decay constant, bare BeRP.

Results: two decay constant fitting

- **Since both methods produced poor fits, a second decay constant was added.**

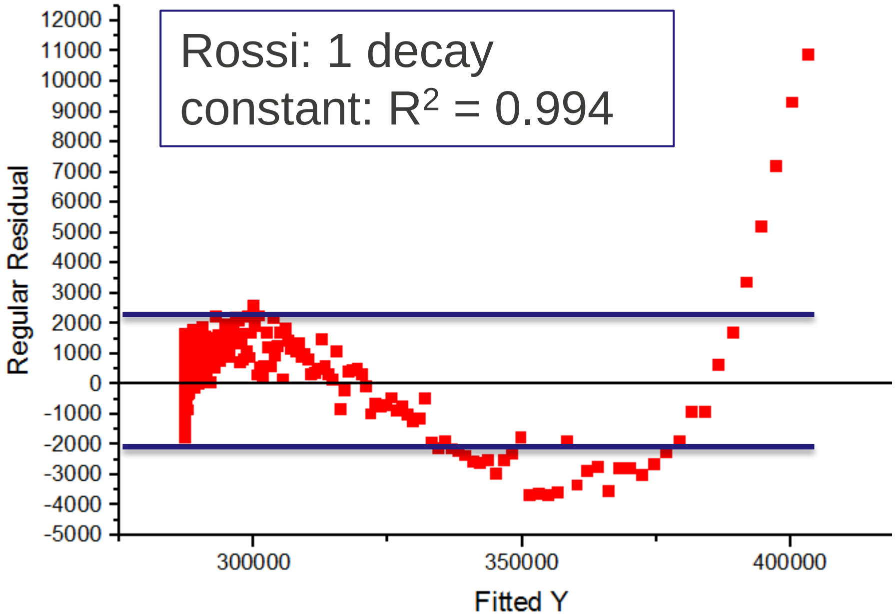
$$p(t)\Delta = A\Delta + Be^{-\alpha t}\Delta \Rightarrow p(t)\Delta = A\Delta + B_1 e^{-\alpha_1 t}\Delta + B_2 e^{-\alpha_2 t}\Delta$$

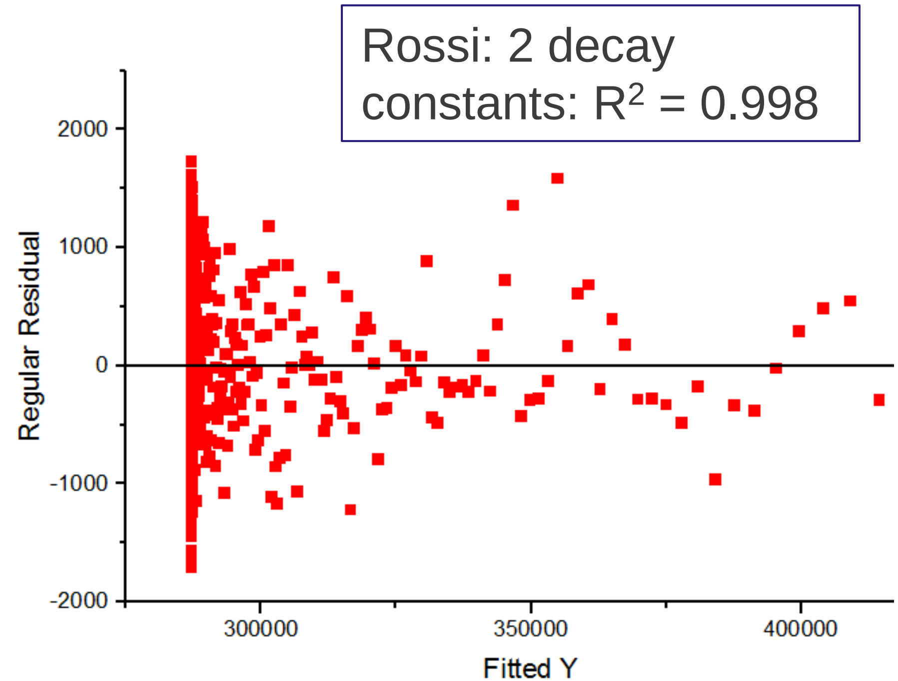
$$Y_2 = C\left(1 - \frac{1-e^{-\alpha t}}{\alpha t}\right) \Rightarrow Y_2 = C\left[D\left(1 - \frac{1-e^{-\alpha_1 t}}{\alpha_1 t}\right) + (1-D)\left(1 - \frac{1-e^{-\alpha_2 t}}{\alpha_2 t}\right)\right]$$

C and D are proportionality constants.

- **Fits with two decay constants have been used previously (both in general and for these equations specifically).**

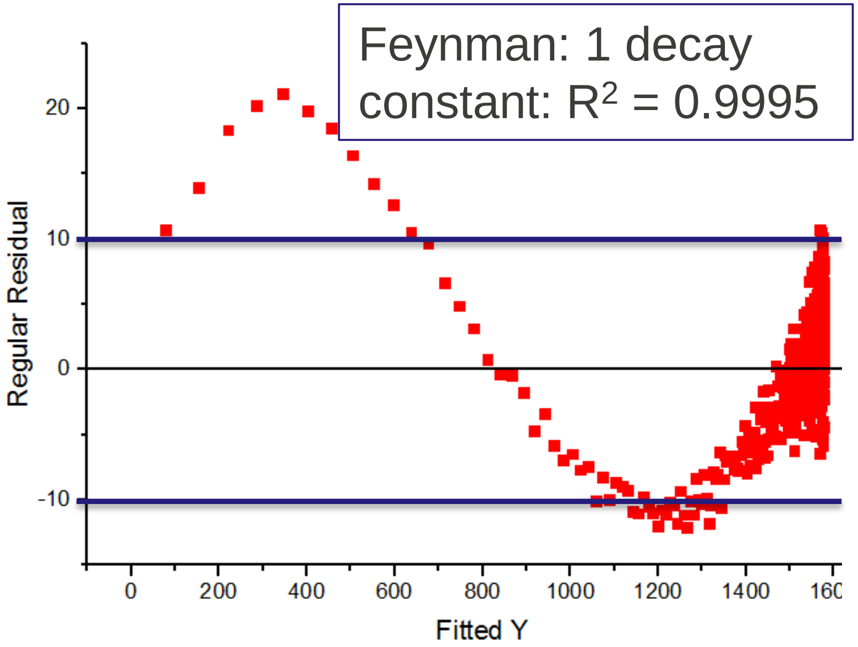
Results: two decay constant fitting

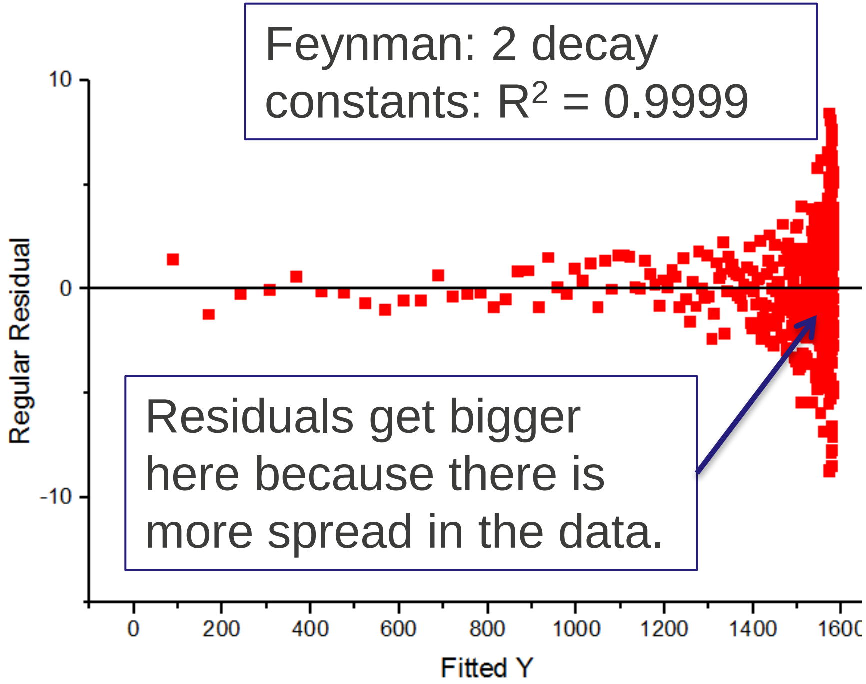




- **The fit using two decay constants is clearly much better.**
- **Not only did the clear trend go away, but the magnitude of the residuals is also much smaller.**

Results: two decay constant fitting

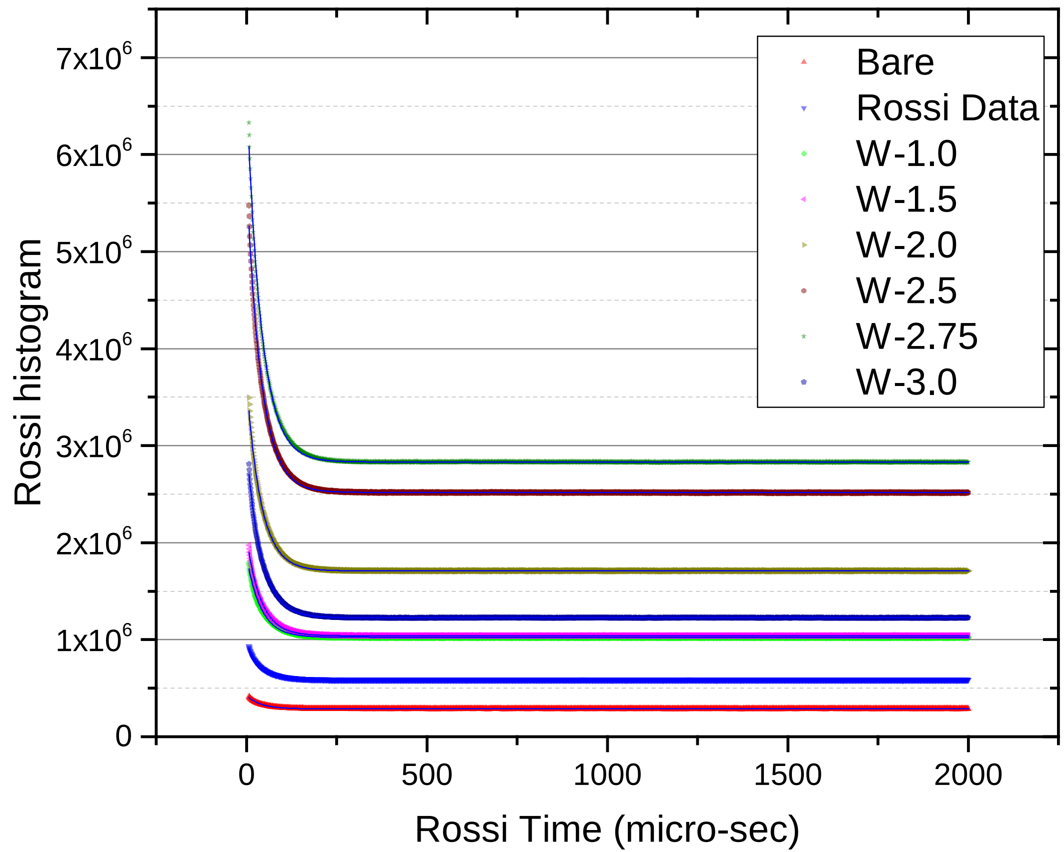




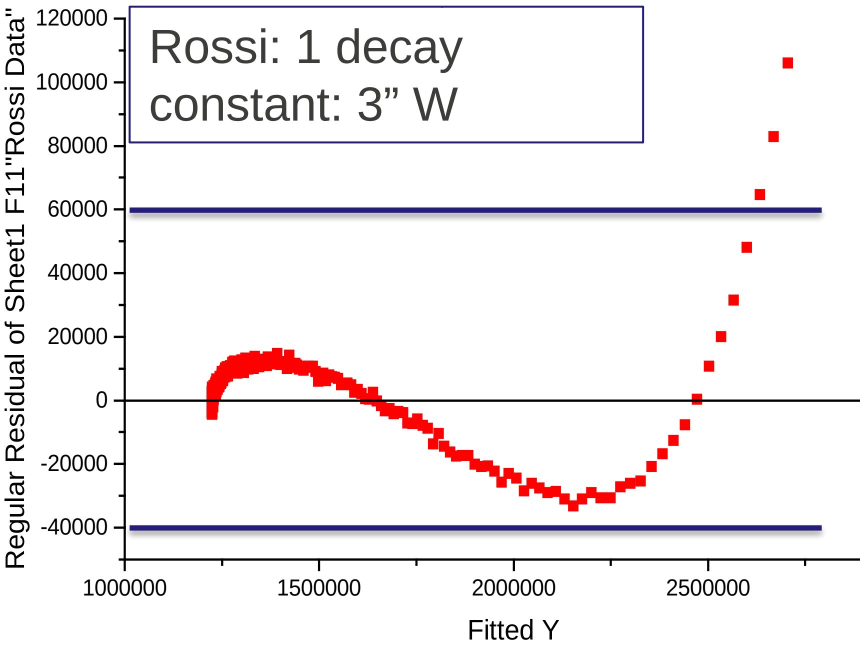
- **Same results as Rossi.**
- **Fit using two decay constants is clearly much better.**
- **Not only did the clear trend go away, but the magnitude of the residuals is also much smaller.**

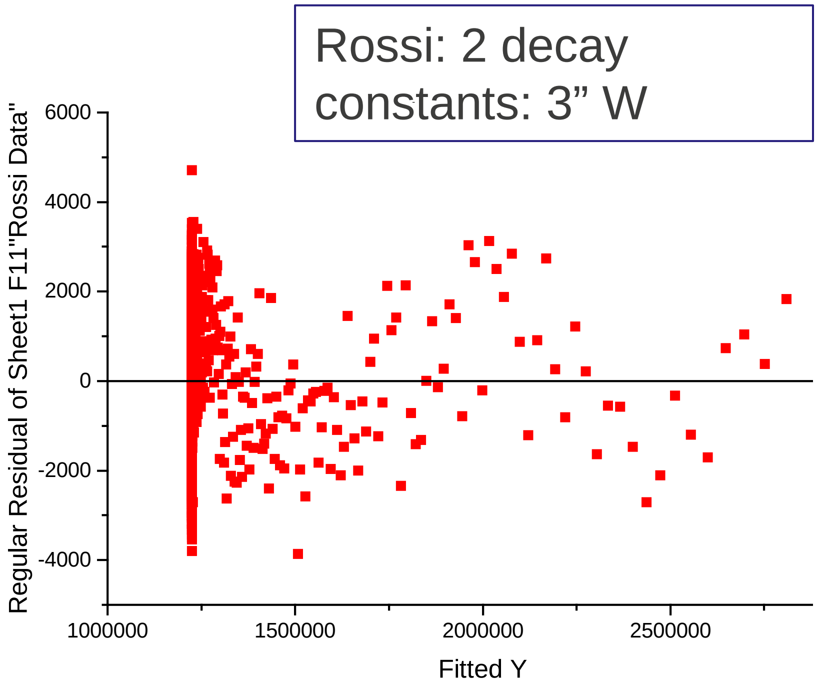
Results: reflected systems

- **8 configurations with varying W thickness.**



Results: reflected systems

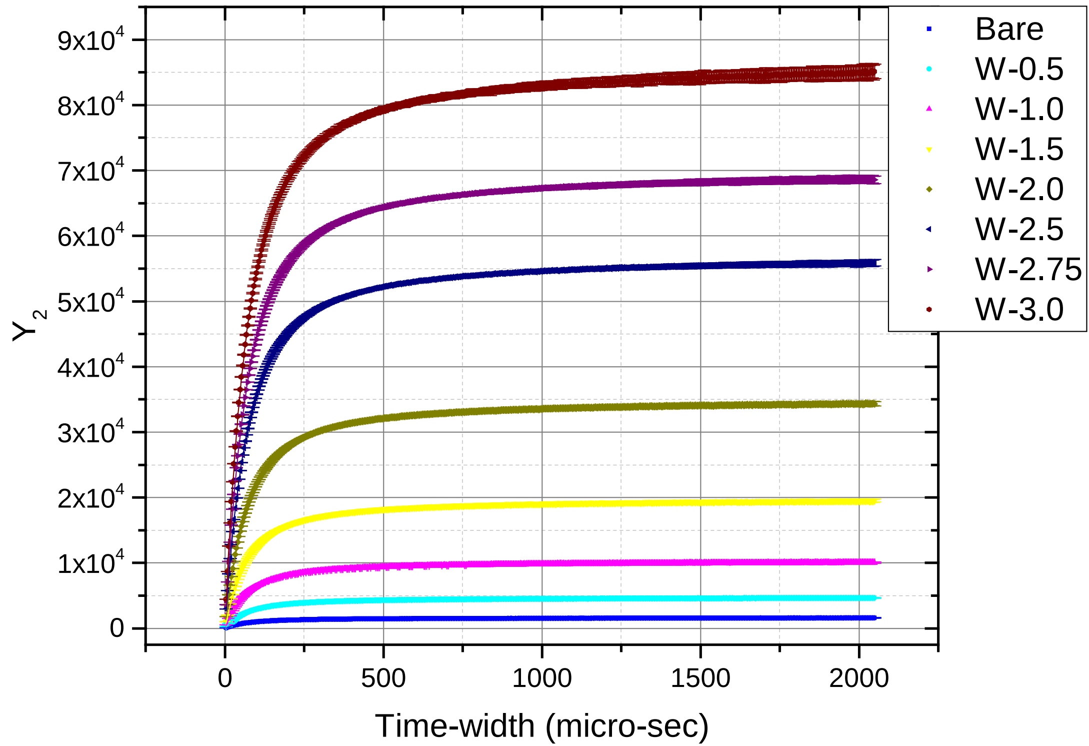




- **Like the bare results, the fit is clearly much better.**

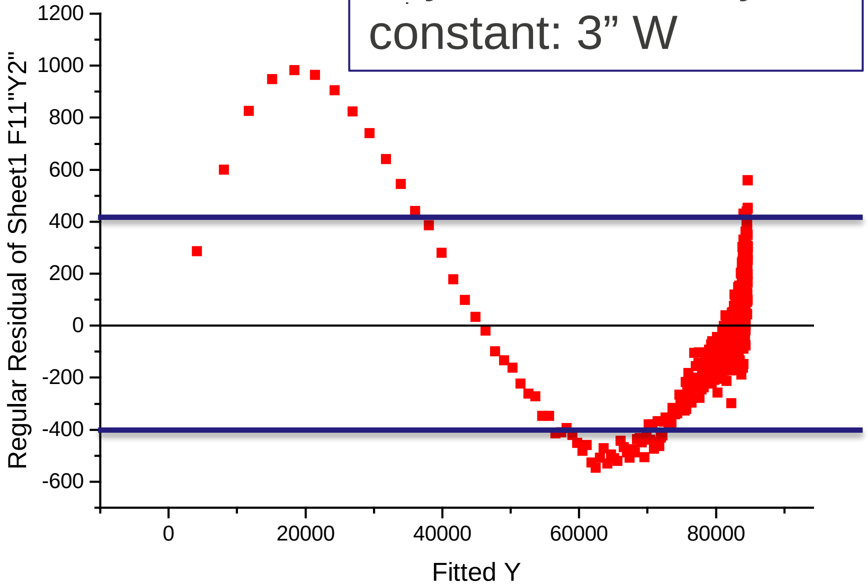
Results: reflected systems

- **8 configurations with varying W thickness.**

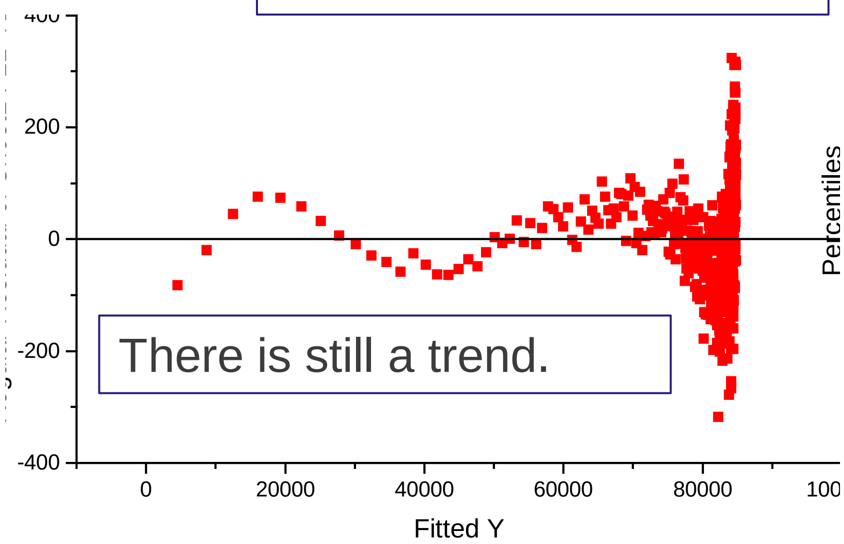


Results: two decay constant fitting

Feynman: 1 decay constant: 3” W



Feynman: 2 decay constants: 3” W

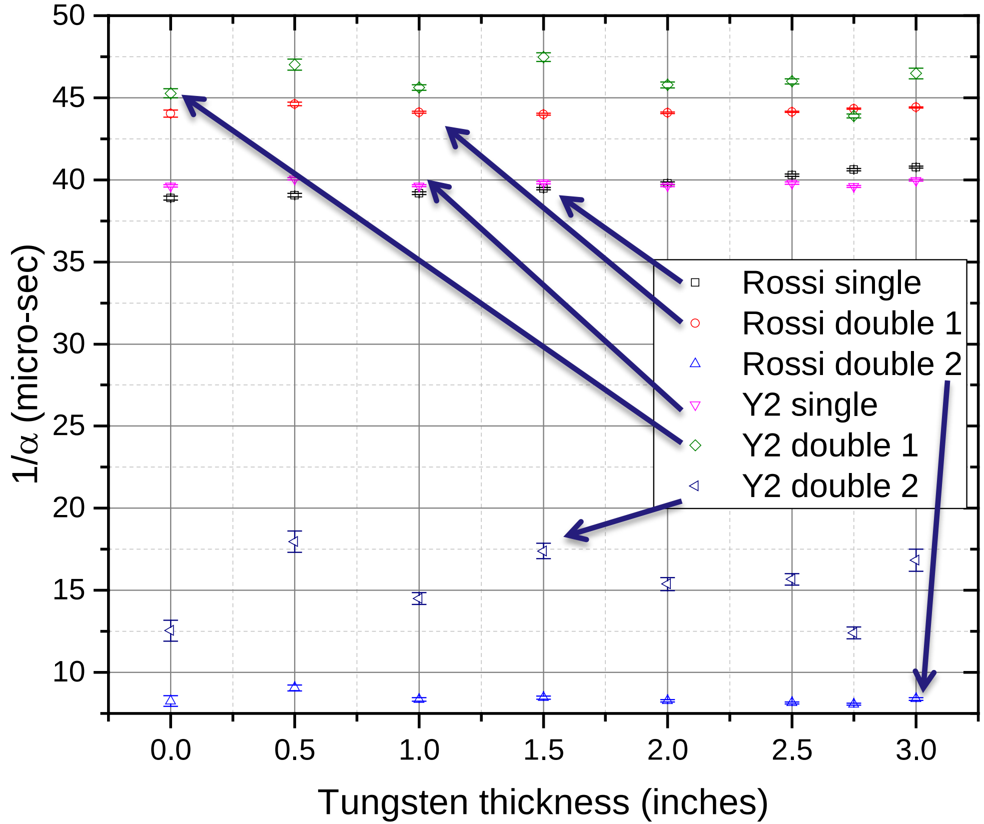


There is still a trend.

- **This was extended to a 3 decay constant fit, but that also had a trend in the residual plots.**
- **A different functional form is therefore required for fitting of reflected systems.**

Results: lifetime/slowing-down time

- **One of the times was 35-45 micro-sec, which is expected for this detector system.**
- **The second time constant was between 8-20 micro-sec. This will be investigated in future work.**



Conclusions/Future Work

- **The prompt neutron decay constant was determined using the Rossi-α and Feynman Variance-to-Mean methods.**
- **Results were shown for fits including both one and two decay constants.**
  - It was found that two decay constants were needed to get the best fit (even though the fit had an excellent R^2 with only one decay constant).
  - For reflected systems, a two decay constant fit worked well with for Rossi-α but not the Feynman Variance-to-Mean method.
- **In the future, further investigation on the faster decay constant will be performed. In addition, it is desired to have a new equational form for the Feynman Variance-to-Mean method for reflected systems.**

Thank you for your attention.

This work was supported by the DOE Nuclear Criticality Safety Program, funded and managed by the National Nuclear Security Administration for the Department of Energy.