

Application of the Limiting Surface Density Method to Arrays of 9975 Shipping Packages with Plutonium Oxide

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Introduction

 Reducing our computational burden by extending the Limiting Surface Density (LSD) Method to apply to arrays of 9975 shipping packages with Pu Oxide



• LA-14244-M (Hand Calculation Primer) has an overview and several example applications for the original method by Joe Thomas.

Background

• Previous work for Pu Metal contents is given in:

- 2017 NCSD Topical Meeting, Verification Suite for the Application of the Limiting Surface Density Method to Arrays of 9975 Shipping Packages, J. Baker, T. Stover, M. Ratliff and G. Mitschelen.
- Nuclear Science and Engineering, Volume 190, Issue 2, pp 176-194, Limiting Surface Density Method Adapted to Large Arrays of Heterogeneous Shipping Packages with Nonlinear Responses, T. Stover, J. Baker, M. Ratliff and G. Mitschelen, May 2018.

Full Derivation of LSD Relationships

• Start with basic reactor physics relationships:

$$B_g^2 = \frac{\pi^2}{(d_x + 2\lambda_x)^2} + \frac{\pi^2}{(d_y + 2\lambda_y)^2} + \frac{\pi^2}{(d_z + 2\lambda_z)^2}$$

• After 7 pages of algebra you have:

$$\frac{m_{c}n}{(2a_{n})^{2}}\left(1-\frac{c}{\sqrt{N}}\right)^{2} = c_{2}(m_{c}-m_{o})$$

• See derivation in *excruciating* detail in our Journal paper

• From Thomas' classic (original) method derivation:

$$\frac{m_c n}{(2a_n)^2} \left(1 - \frac{c}{\sqrt{N}}\right)^2 = c_2(m_c - m_o)$$

- where:

$$c = \sqrt{\frac{4\lambda_{array}^2 N B_N^2}{3\pi^2}}$$

-c and c_2 are empirically determined constants

• How to derive c??

- Clues given in Thomas' paper Y-CDC-10, Appendix B

Deriving the Constants (cont'd)

- KENO-VI calculations for critical mass of arrays across the parameter ranges of interest: array size & spacing (2a_n)
- Cubic arrays with number per side, n, from 4 to 10
 - $-N = n_x * n_y * n_z$ $64 \le N \le 1000$
 - Unit Spacing: $46.6 \text{ cm} \le 2a_n \le 150 \text{ cm}$
 - Reflected by 30 cm thick concrete on all 6 sides
 - Critical mass found for each combination of array size and spacing (49 different arrays)
- Pu oxide assumed to be theoretical density with 0.5 wt% moisture

Simplified 9975 KENO-VI Model



Vertical Slice of 4x4x4 Cubic Close-Packed Array



Horizontal Slice of Array Model



Critical Array Fissile Mass (kg) per Package



Non-Linear Response for Surface Density

$[=c_2(m_c-m_0)]$



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• From the derivation of array buckling (leakage):

$$NB_N^2 = \frac{3\pi^2}{m_c} c_3 e^{c_4 m_c}$$

where: $c_3 = 2.63031E+4$, and $c_4 = -0.694645$

• Extrapolation distance is calculated from:

$$\lambda_{array}^{2} = \frac{N3\pi^{2}}{4NB_{N}^{2}} \left(1 - \sqrt{\frac{4a_{n}^{2}NB_{N}^{2}}{n3\pi^{2}}}\right)^{2}$$

Computing the Geometric Constant, c

- Average value of $NB_N^2 \lambda_{array}^2 = 3.23$,
- Returning to the definition of c:

$$c = \sqrt{\frac{4\lambda_{array}^2 N B_N^2}{3\pi^2}}$$

- Yields c = 0.66
- Similar to Thomas' value of 0.55 +/- 0.18

Checking the Method—Using Cubic Arrays

• The relationship to estimate critical mass is:

$$m_c = \frac{(2a_n)^2 c_3 e^{c_4 m_c}}{n \left(1 - \frac{c}{\sqrt{N}}\right)^2}$$

• Using this to calculate m_c for the 49 cubic arrays

 $-4 \le n \le 10, 46.6 \text{ cm} \le \text{Pitch} \le 150 \text{ cm}$

- Comparing the LSD and KENO-VI critical mass values:
 - Average ∆% = **0.62**
 - Maximum Δ% = **1.7**

LSD vs. KENO-VI Critical Unit Mass for <u>Realistic</u> Arrays with Pu Oxide

- 42 non-cubic arrays were chosen: 2x20x1, 2x30x1, 2x20x2, 2x20x3, 4x20x3, 5x5x3. Also varying pitch: 46.6 cm ≤ 2a_n ≤ 150 cm
- Comparing the LSD and KENO-VI critical mass values:
 - Average ∆% = **2.5**
 - Maximum ∆% = **5.2**
 - LSD values generally under-predict the KENO-VI value
- Agreement not as good as for same arrays with Pu metal contents.
 For those:
 - Average Δ % = **0.6**
 - Maximum Δ% = **1.3**

Empirical Adjustment to Critical Mass Based on Array Shape and Pitch

$$m_{c,adj} = m_c \frac{(1+R^{2.5})}{100} * \left(1.01 - (14.3P^{-1.58})\right)$$

where: R = Shape Factor, given by: $R = \frac{\sqrt[3]{N}}{3} \left(\frac{1}{n_x} + \frac{1}{n_y} + \frac{1}{n_z} \right)$

P = Horizontal Pitch (unit cell dimension) in cm

With this adjustment, overall results improved:

- Average Δ% = 0.8
- Maximum Δ% = **2.2**

Application Considerations

- There is much more safety margin for Pu oxide contents that with Pu metal
- For quick reactivity effect estimates, great precision is not needed
- Empirical adjustment might not be needed
 - Does not improve results significantly for minimum pitch (no spacing), where the Δ is less than +/- 2.5%.

• Program relationships into a spreadsheet:

$$m_{c} = \frac{(2a_{n})^{2}c_{3}e^{c_{4}m_{c}}}{n\left(1 - \frac{c}{\sqrt{N}}\right)^{2}} \qquad k_{eff} = (\frac{m}{m_{c}})^{1/3}$$

- Examples for 10x14x3 array, with 5 kg PuO₂ in each package
 - What is safety margin for normal conditions?
 - What is the effect of:
 - Excess stacking (4 high)?
 - Double batching (10 kg)?
 - Changing the array pitch?

Example 1: 10x14x3 Array of 9975s, 5 kg PuO₂ Each, Units Touching

- What is the critical mass per unit?
- What is k_{eff}?

| 4 | | | | | | | | | | | | | | |
|----|----------------|----------------|----|------------|------------------|--------|-------|----------------|------------|----------|----------------|---------|-------|--|
| 5 | | | | c = | 0.6602 | | | | | | | | | |
| 6 | | | | c3 = | 26303 | | | | | | | | | |
| 7 | | | | c4 = | -0.6946 | | | | | | | | | |
| 8 | | | | | | | | | | | | | | |
| 9 | | | | | | | Array | | | | | | | |
| 10 | | | | | | Shape | Pitch | a _n | Constants | Solver | LSD | Loading | k-eff | |
| 11 | n _x | n _y | nz | N | N ^{1/3} | Factor | (cm) | (cm) | Multiplier | Equation | m _c | (kg) | | |
| 12 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 46.6 | 23.3 | 8143340 | -7.1E-13 | 18.7 | 5.000 | 0.644 | |
| 13 | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | |

- What is the critical mass per unit?
- What is k_{eff}?

| 8 | | | | | | | | | | | | | |
|----|----------------|----------------|----|-----|------------------|--------|-------|----------------|------------|----------|----------------|---------|-------|
| 9 | | | | | | | Array | | | | | | |
| 10 | | | | | | Shape | Pitch | a _n | Constants | Solver | LSD | Loading | k-eff |
| 11 | n _x | n _y | nz | N | N ^{1/3} | Factor | (cm) | (cm) | Multiplier | Equation | m _c | (kg) | |
| 12 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 46.6 | 23.3 | 8143340 | -7.1E-13 | 18.692 | 5.000 | 0.644 |
| 13 | 10 | 14 | 4 | 560 | 8.243 | 1.158 | 46.6 | 23.3 | 7333166 | -2.9E-07 | 18.552 | 5.000 | 0.646 |
| 14 | | | | | | | | | | | | | |

Example 3: Excess Mass, 10 kg PuO₂ Each

• What is Δk_{eff} ?

| 8 | | | | | | | | | | | | | | |
|----|----------------|----------------|----|-----|------------------|--------|-------|----------------|------------|----------|----------------|---------|-------|--|
| 9 | | | | | | | Array | | | | | | | |
| 10 | | | | | | Shape | Pitch | a _n | Constants | Solver | LSD | Loading | k-eff | |
| 11 | n _x | n _y | nz | N | N ^{1/3} | Factor | (cm) | (cm) | Multiplier | Equation | m _c | (kg) | | |
| 12 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 46.6 | 23.3 | 8143340 | -7.1E-13 | 18.692 | 5.0 | 0.644 | |
| 13 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 46.6 | 23.3 | 8143340 | -7.1E-13 | 18.692 | 10.0 | 0.812 | |
| 14 | | | | | | | | | | | | | | |

Example 4: Changing Spacing

- Assume decrease of 5 cm, or an increase of 10 cm
- What is new critical mass?
- What is Δk_{eff} ?

| 8 | | | | | | | | | | | | | | | |
|----|----------------|----------------|----|-----|------------------|--------|-------|------|------------|----------|----------------|---------|-------|--------|---|
| 9 | | | | | | | Array | | | | | | | | |
| 10 | | | | | | Shape | Pitch | an | Constants | Solver | LSD | Loading | k-eff | ∆k-eff | |
| 11 | n _x | n _y | nz | N | N ^{1/3} | Factor | (cm) | (cm) | Multiplier | Equation | m _c | (kg) | | | |
| 12 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 46.6 | 23.3 | 8143340 | -7.1E-13 | 18.692 | 5.0 | 0.644 | | |
| 13 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 41.6 | 20.8 | 6489592 | -3.2E-06 | 18.389 | 5.0 | 0.648 | 0.004 | |
| 14 | 10 | 14 | 3 | 420 | 7.489 | 1.260 | 56.6 | 28.3 | 12013335 | -8.8E-07 | 19.213 | 5.0 | 0.638 | -0.006 | |
| 15 | | | | | | | | | | | | | | | Γ |

Conclusions

- LSD Method provides good agreement with KENO-VI for arrays of 9975 shipping packages with Pu Oxide.
- Allows rapid estimates for safety margin for varying mass, spacing, and array sizes.
- Can be used to evaluate variety of normal and credible abnormal conditions.
- Helps develop understanding of the physics.

Questions?



Supplemental Discussion: Basic Concept

- Buckling relationships can be used to relate one critical array to another, using empirically derived constants.
- First, assume that one has a critical array of identical fissile items, specified by its isotopics, mass/unit, spacing, array shape, etc.
- Changes in one parameter (e.g., mass or spacing) may be compensated by changes in another parameter so that the resulting array is also critical.

Can LSD work for shipping package arrays?

- Thomas' LSD method is very good for air-spaced arrays of solid items (see Hand Calculation Primer Sec. 7)
 - Caveat 1: Derivation uses cubic arrays of cubic units
 - Caveat 2: Each unit may be surrounded by $\leq \frac{1}{2}$ inch of steel

• Problems and Challenges:

- 9975s are not cubic; nor are the arrays
- 9975s have several nested layers of packaging material (steel, lead, Celotex™)
- Some packaging varies among 9975s
- Unclear how to derive the necessary constants