Sensitivity analysis and uncertainty quantification applied to the Feynman Y and Sm_2

Alexander R. Clark,[1,2] Jeffrey A. Favorite,[2] Alex McSpaden,[3] and Mark Nelson[3]
[1]Department of Nuclear Engineering, NCSU
[2]Monte Carlo Methods, Code, and Applications Group (XCP-3), LANL
[3]Advanced Nuclear Technology Group, (NEN-2), LANL

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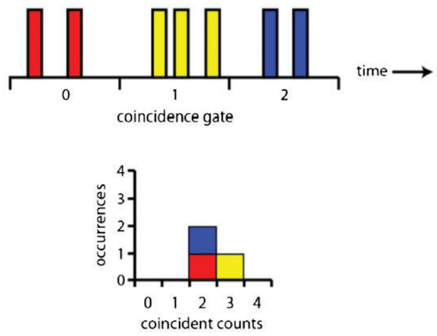
Introduction

Motivation

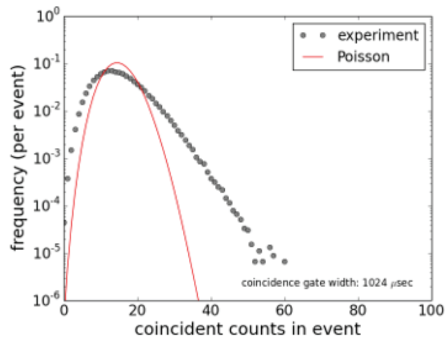
- Cross section evaluation using critical experiments and reaction rate measurements has led to over-calibration in nuclear cross sections
- Neutron multiplicity counting (NMC) is a method of non-destructive assay of SNM assemblies
  - Each NMC distribution moment is a function of the cross sections raised to the power of the moment’s order
  - Higher-order NMC distribution moments are more sensitive to the cross sections than the mean (first moment)
- Y and Sm_2 are ratios of NMC distribution moments
- Reliable characterization of SNM requires Sensitivity analysis (SA) and uncertainty quantification (UQ) applied to Y and Sm_2

Neutron multiplicity counting

- NMC accumulates distribution of coincident neutron counts
- Independent neutron emissions e.g. (α, n) reactions characterized by Poisson distribution
- Fission-chain reactions are described by generalized Poisson distribution



Accumulation of counting distribution
Miller, Nuc. Sci. and Eng., 2014



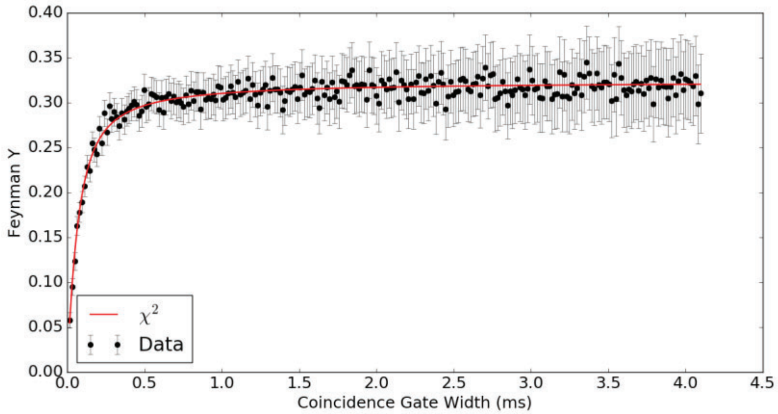
Counting distribution and Poisson distribution with the same mean
Mattingly, IEEE Trans. Nuc. Sci., 2012

Feynman Y definition and use

- Ratio of variance-to-mean of NMC distribution

$$Y = \frac{\sigma_n^2}{\overline{n}} - 1$$

- Measure of deviation of NMC distribution from Poisson statistics
- Can infer integral properties of SNM, such as neutron lifetime and neutron multiplication



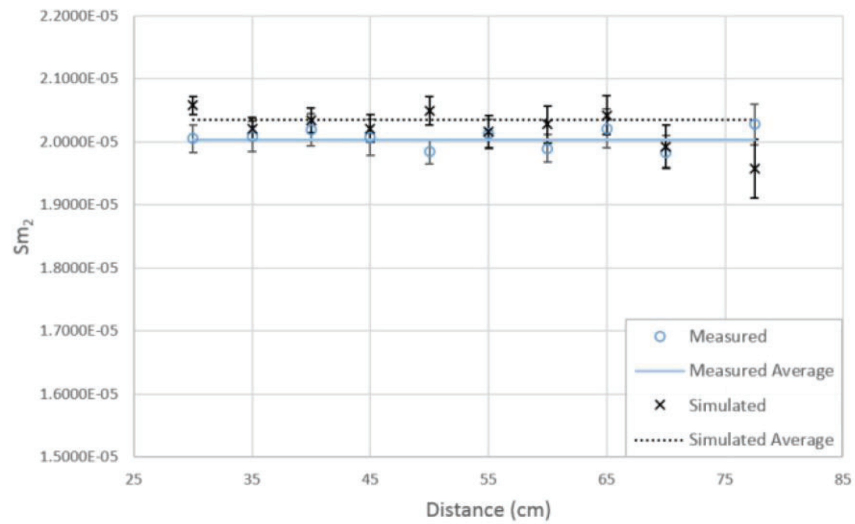
Y vs coincidence gate

Sm_2 definition and use

- Ratio of doubles-to-singles-squared

$$\mathrm{Sm}_2 = \frac{D}{S^2}$$

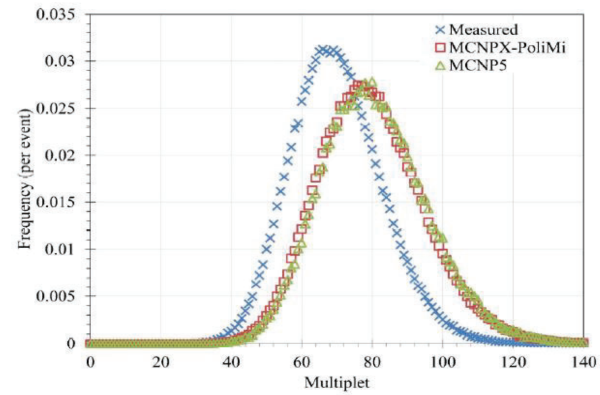
- Independent of detector response function
- Observe same Sm_2 for SNM assembly counted at different source-detector distances

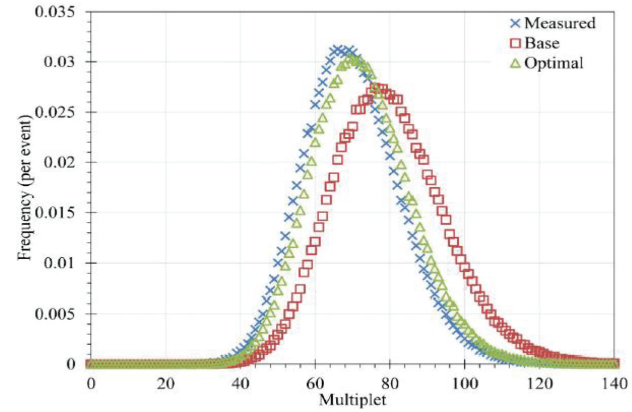


Measured and simulated Sm_2 for MC-15 counting plutonium sphere from 30 cm to 80 cm
McSpaden, Trans. ANS Winter, 2017

Over-calibration of Pu-239 $\bar{\nu}$

- Simulations of NMC of a 4.5-kg sphere of weapons grade plutonium metal (BeRP ball) overpredicted NMC distribution moments
- 1% reduction in Pu-239 $\bar{\nu}$ improved accuracy of simulated NMC distribution moments
- Over-calibration in $\bar{\nu}$ had greater effect on the variance than the mean
- $\bar{\nu}$ artificially adjusted to match JEZEBEL critical experiments





Simulated NMC distribution before (top) and after (bottom) Pu-239 $\bar{\nu}$ reduction
Miller, Nuc. Sci. and Eng., 2014

Sensitivity analysis

Forward NTE

- Describes a balance of production and loss terms for expected number of neutrons

$$L\psi = \overbrace{Q}^{\text{intrinsic source}}$$

$$L = \underbrace{\hat{\Omega}\cdot\nabla}_{\text{streaming loss}} + \overbrace{\sigma_t}^{\substack{\text{total} \\ \text{interaction loss}}} - \underbrace{\int d\Omega \int dE \sigma_s}_{\text{scatter source}} - \overbrace{\frac{\chi}{4\pi}\int d\Omega \int dE \bar{\nu}\sigma_f}^{\text{fission source}}$$

Adjoint NTE

- Counterpart to forward NTE

$$L^{+}\psi_1^{+} = Q_1^{+}$$

$$L^{+} = -\hat{\Omega} \cdot \nabla + \sigma_t - \int d\Omega' \int dE' \sigma_s - \\ \bar{\nu}\sigma_f \int d\Omega' \int dE' \frac{\chi}{4\pi}$$

- Adjoint flux is “importance” of source neutrons to the mean count rate

NMC distribution moments

- First moment of NMC distribution (mean count rate)

$$R_1 = \langle \psi, Q_1^+ \rangle = \langle \psi, \sigma_d \rangle$$

- Higher-order adjoint NTE have the same form as usual adjoint NTE with special fixed source terms

$$L^+ \psi_k^+ = Q_k^+, k = 1,2, \ldots$$

- Higher-order detector responses are computed the same way as R_1

$$R_k = \langle \psi, Q_k^+ \rangle$$

Second NMC distribution moment

$$R_2 = \langle \psi, Q_2^+ \rangle$$

$$Q_2^+ = \overline{\nu(\nu - 1)}\sigma_f I_1^2$$

$$I_1 = \int d\Omega \int dE \, \frac{\chi}{4\pi} \psi_1^+$$

- ψ_1^+ is a linear function of the cross sections
- Q_2^+ is proportional to the square of the cross sections

Sensitivity of mean count rate

- Perturbation theory allows the sensitivity to be computed with few transport solves

$$\frac{\partial R_1}{\partial \alpha_{m,g}} = \left\langle \frac{\partial Q_1^+}{\partial \alpha_{m,g}}, \psi \right\rangle + \left\langle \psi_1^+, \frac{\partial Q}{\partial \alpha_{m,g}} - \frac{\partial L}{\partial \alpha_{m,g}} \psi \right\rangle$$

- Sensitivity of higher-order detector response moments have a similar form

Relative sensitivity definition

- Absolute sensitivity is scaled by cross section and detector response values

$$\frac{\delta R_k}{R_k} = S_{R_k,\alpha_{m,g}} \frac{\delta \alpha_{m,g}}{\alpha_{m,g}}$$

- Represents a linear relationship between change in cross section value and corresponding change in detector response

Sensitivity of Y and Sm_2

$$Y = \frac{\sigma_n^2 - \bar{n}}{\bar{n}} = \frac{R_2}{R_1}$$

$$S_{Y,\alpha_{m,g}} = S_{R_2,\alpha_{m,g}} - S_{R_1,\alpha_{m,g}}$$

$$\mathrm{Sm}_2 = \frac{D}{S^2} = \frac{1}{2}\frac{R_2}{R_1^2}$$

$$S_{\mathrm{Sm}_2,\alpha_{m,g}} = S_{R_2,\alpha_{m,g}} - 2S_{R_1,\alpha_{m,g}}$$

Uncertainty quantification

Linear propagation of uncertainty

- Variance in Y and Sm_2 may be estimated using linear propagation of uncertainty

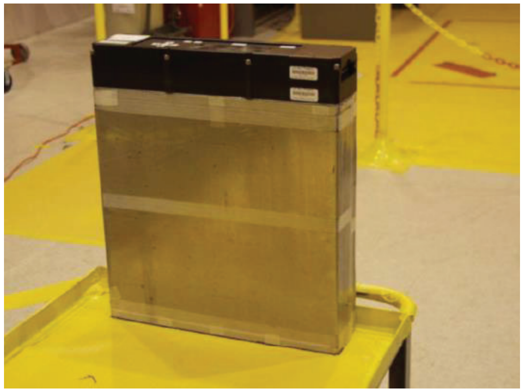
$$\mathrm{var}(f) = \boldsymbol{S}_{f,\boldsymbol{\alpha}}^{\mathrm{T}} \mathbf{C}_{\boldsymbol{\alpha}} \boldsymbol{S}_{f,\boldsymbol{\alpha}}\,, f = Y, \mathrm{Sm}_2$$

- Y and Sm_2 variance due to random nature of counting may be reduced by longer counting
- Y and Sm_2 variance due to cross sections does not change

Current results

Response and sensitivity calculations

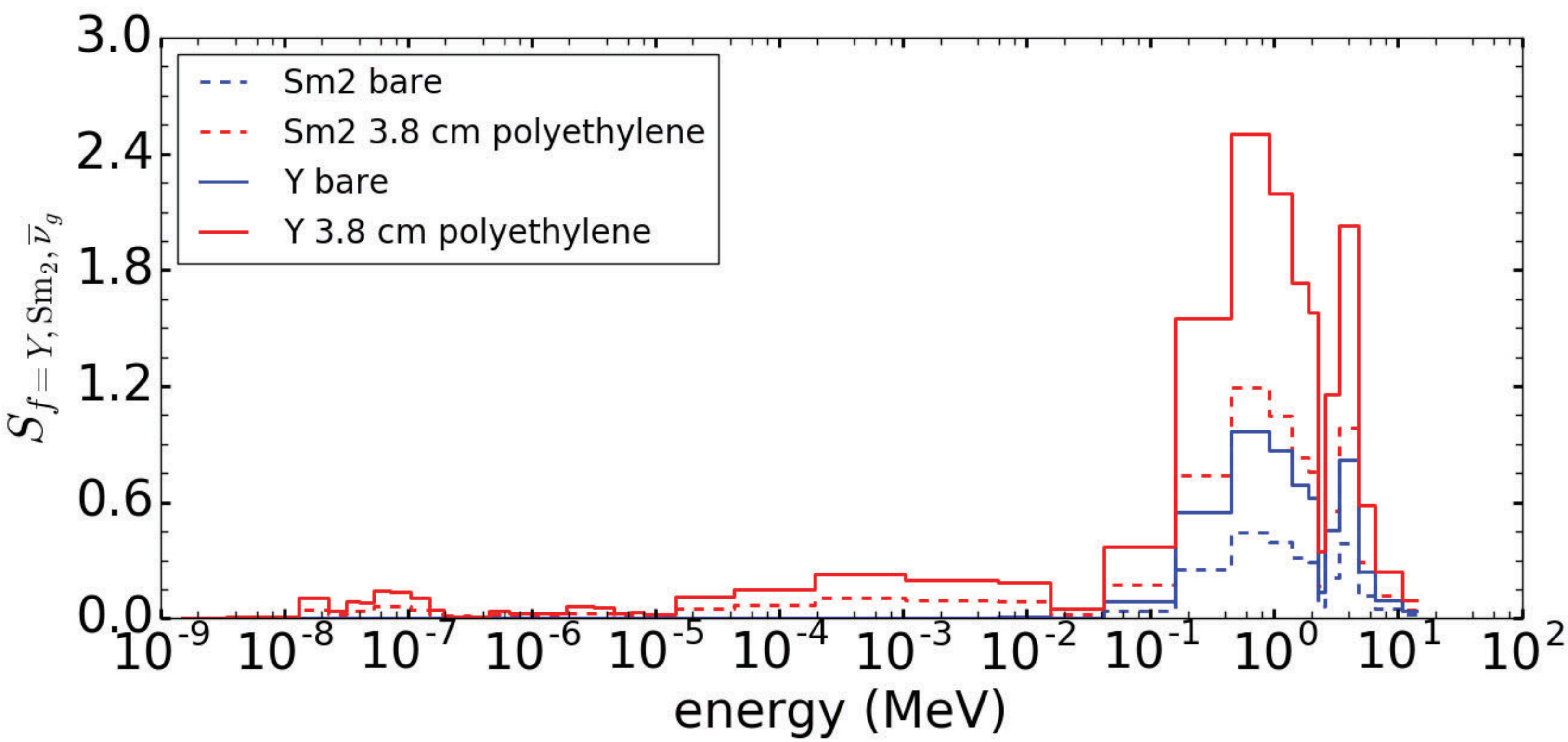
- 44-group, 1D PARTISN simulations of NMC of BeRP ball simulated with:
  - No reflector
  - 3.8 cm of polyethylene
- Cross sections present in the SA/UQ process:
  - $\boldsymbol{\alpha}_g = \left\{\chi_g,\ \bar{\nu}_g,\ \overline{\nu(\nu-1)}_g,\ \sigma_{fg},\ \sigma_{el,g},\ \sigma_{in,g},\ \sigma_{cg}\right\}$
  - Only Pu-239 and H-1 isotopes



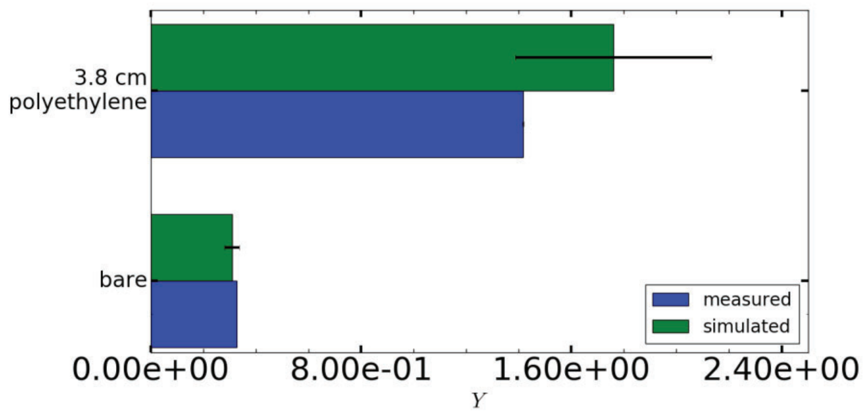
nPod neutron multiplicity counter
Miller, Nuc. Sci. and Eng., 2014

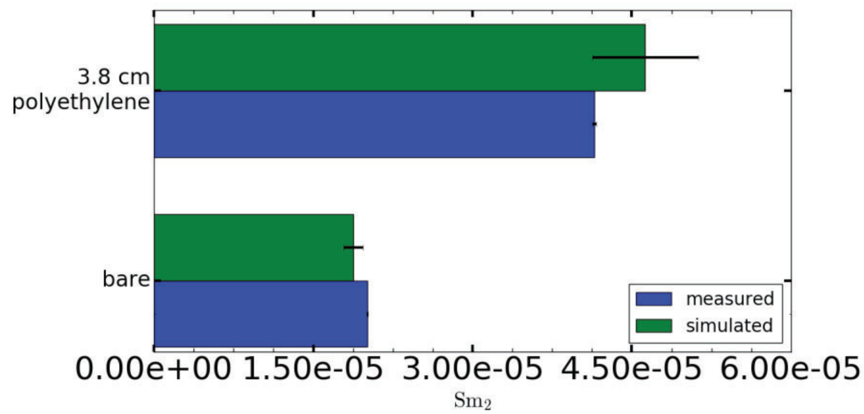
BeRP ball nested in polyethylene reflectors
Miller, Nuc. Sci. and Eng., 2014

Relative sensitivity of Y and Sm_2 to Pu-239 $\overline{\nu}$

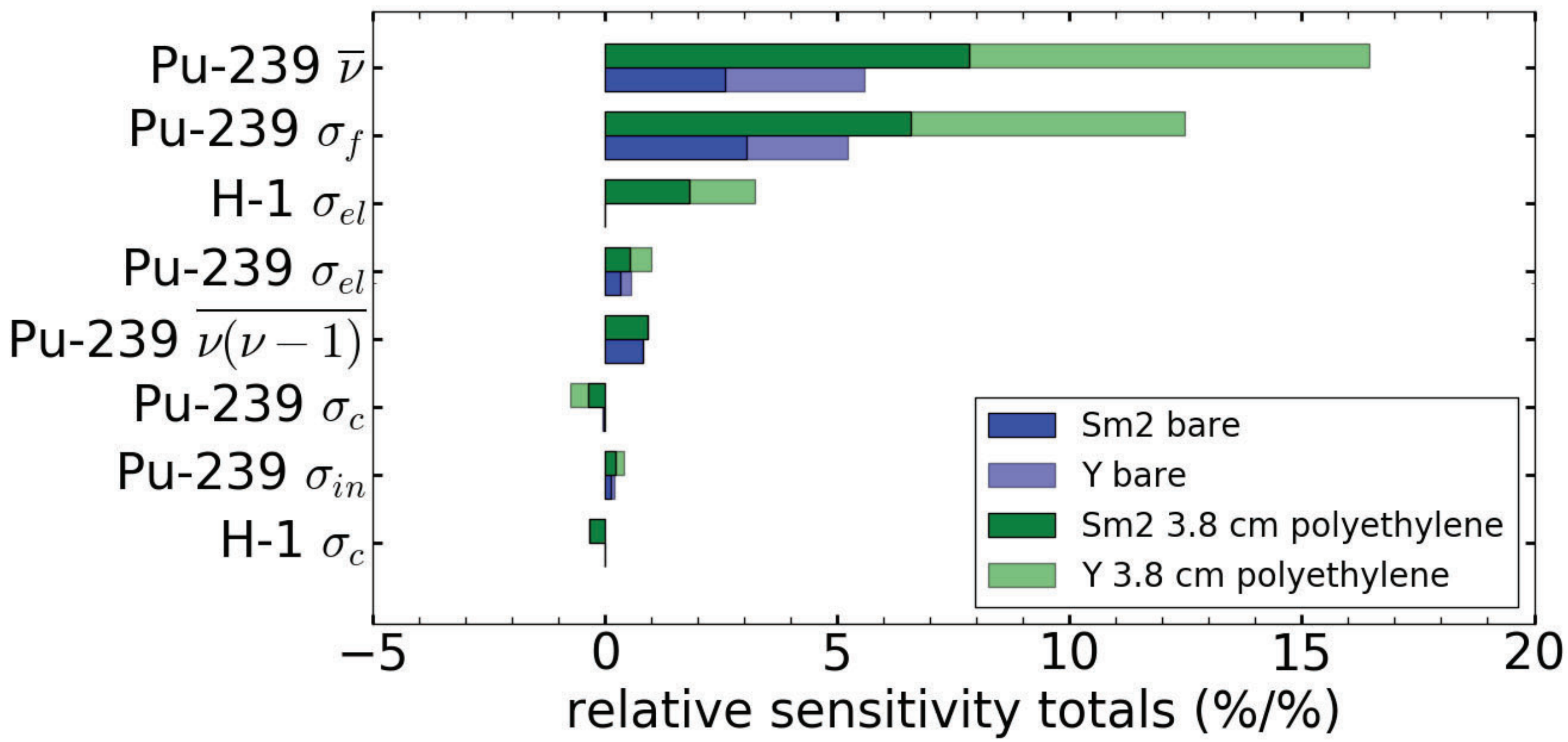


Y and Sm_2 and their uncertainty





Relative sensitivity totals of Y and Sm_2



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Summary

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- Performed first calculation of variance in Y and Sm_2 due to covariance in the cross sections
- Assay of SNM utilizing Y may be preferred for its historical use in inferring integral properties of the material
- Assay of SNM utilizing Sm_2 may be preferred because it
  - Is independent of detector response function
  - Has less variance due to the cross sections than Y to the cross sections

Future work

- Concurrent dissertation work is optimization of cross sections utilizing NMC experiments
  - SA/UQ applied to NMC distribution moments informs calibration of cross sections via parameter estimation (PE)
  - Estimate cross sections such that NMC experiments are more accurately simulated
- PE that includes SA/UQ applied to NMC distribution moments as well as Y and Sm_2 may further improve accuracy of simulation of NMC experiments