NCSD Hand Calculations Workshop

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Overview

- Why Hand Calculations
- Choosing the Right Method
- Single Unit Methods
  - Diffusion Theory
  - Buckling
  - Core Density Conversions
- Array Methods
  - Surface Density
  - Density Analog
  - Limiting Surface Density
  - Solid Angle

Why Hand Calculations

- Develop Intuition in New Analysts
- Used to Support Evaluations
- Useful for Scoping Calculations & Critical Estimates
- Must be Verified and Validated

Choosing the Right Single Unit Method

- Diffusion
  - Large, Homogeneous Systems
  - Problems with Boundaries, Small Systems, & Absorption
- Buckling Conversions
  - Convert Between Simple Geometric Shapes
  - Requires Similar Leakage Characteristics
- Core-Density Conversions
  - Homogeneous, Critical Systems With Uniform Volume/Density Changes
  - Bare or Reflected Systems

Diffusion Theory

- Steady State
- B_m^2 – Material Buckling
  - Function of Absorption and Fission Cross Sections
- B_g^2 – Geometric Buckling
  - Leakage of the System

$$-(-D\nabla^2\phi) - \Sigma_a\phi + \upsilon\Sigma_f\phi = \frac{1}{\upsilon}\frac{d\phi}{dt}$$

$$B_m^2 = \left(\frac{\upsilon\Sigma_f - \Sigma_a}{D}\right) \text{ and } B_g^2 = \left(\frac{\pi}{2X'}\right)^2$$

$$B_m^2 = B_g^2 \text{ then } k = 1 \quad \text{(Critical)},$$

$$B_m^2 > B_g^2 \text{ then } k > 1 \quad \text{(Supercritical)},$$

$$B_m^2 < B_g^2 \text{ then } k < 1 \quad \text{(Subcritical)}.$$

Buckling Conversions

- Derived From Diffusion Equation
- Used to Equate One Critical Shape to Another

Sphere: Radius-r $$\left(\frac{\pi}{r+d}\right)^2$$

Cylinder: Radius-r, Height-h $$\left(\frac{2.405}{r+d}\right)^2 + \left(\frac{\pi}{h+2d}\right)^2$$

Parallelepiped: Height, Width, Length $$\left(\frac{\pi}{a+2d}\right)^2 + \left(\frac{\pi}{b+2d}\right)^2 + \left(\frac{\pi}{c+2d}\right)^2$$

Infinite Cylinder: Radius-r $$\left(\frac{2.405}{r+d}\right)^2$$

Infinite Slab: Thickness-h $$\left(\frac{\pi}{h+2d}\right)^2$$

Hemi-Sphere: Radius-r $$\left(\frac{4.49}{r+d}\right)^2$$

Core Density Conversions

- Homogeneous, Uniform Compositions
- If Reflectors Are Present: Reflector Density Changes at Same Ratio
- Relates a Critical Core With a Known Density to an **Equivalent** Critical Core With **Another** Density

Sphere

$$\frac{r_s}{r_{s0}} = \left(\frac{\rho}{\rho_0}\right)^{-1}$$

Infinite Cylinder

$$\frac{r_c}{r_{c0}} = \left(\frac{\rho}{\rho_0}\right)^{-1}$$

Infinite Slab

$$\frac{t}{t_0} = \left(\frac{\rho}{\rho_0}\right)^{-1}$$

Choosing the Right Array Method

- Surface Density
  - Subcritical Center-Center Spacing in Finite Planar Array
  - Useful for Irregular Shapes
- Density Analog
  - Subcritical Center-Center Spacing in Any Array Configurations
- Limiting Surface Density (NB_N^2)
  - Center-Center Spacing in Any Array Configuration >64 Units
  - Calculate Trends Due to Change in Unit Shape/Density
- Solid Angle
  - Small Numbers of Moderated Units
  - Restrictions on Single Unit Characteristics

Surface Density

- Derived from Limiting Surface Density
- Known Information:
  - Fissile Unit Height
  - Mass of Fissile Material in Each Array Unit
  - Critical Dimensions of Infinite Water-Reflected Slab
  - Critical Mass of Unreflected Sphere of Fissile Material in Array
- Array No More Than 2 Units High
- Gives Conservative Estimate

Surface Density Cont.

- d – Center to Center Spacing
- n – Number of Fissile Units
  Projected Onto a Plane
- m – Fissile Material Mass (g)
- σ_0 – Surface Density Water
  Reflected Infinite Slab (g/cm^2)
- f – Ration of Mass of Unit to
  Unreflected Critical Sphere
  - **MUST BE ≤ 0.73**

$$f = \frac{m}{m_0}$$

$$k_{eff} = \left(\frac{m}{m_0}\right)^{\frac{1}{3}} = \left(f\right)^{\frac{1}{3}}$$

$$d = \left[\frac{nm}{0.54\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}} \cong 1.37\left[\frac{nm}{\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}}$$

Density Analog

- Similar to Surface Density Method
- Used for Cubic Arrays
- Known Information:
  - Fissile Unit Height
  - Mass of Fissile Material in Each Array Unit
  - Critical Dimensions of Infinite Water-Reflected Slab
  - Critical Mass of Unreflected Sphere of Fissile Material in Array
- Array Stored in Unlimited Fashion
- Gives Conservative Estimate

Density Analog Cont.

- d – Center to Center Spacing
- n – Number of Fissile Units in One Dimension (Total Units ^1/3)
- m – Fissile Material Mass (g)
- σ_0 – Surface Density Water Reflected Infinite Slab (g/cm^2)
- f – Ration of Mass of Unit to Unreflected Critical Sphere
  - **SHOULD BE ≤ 0.73**

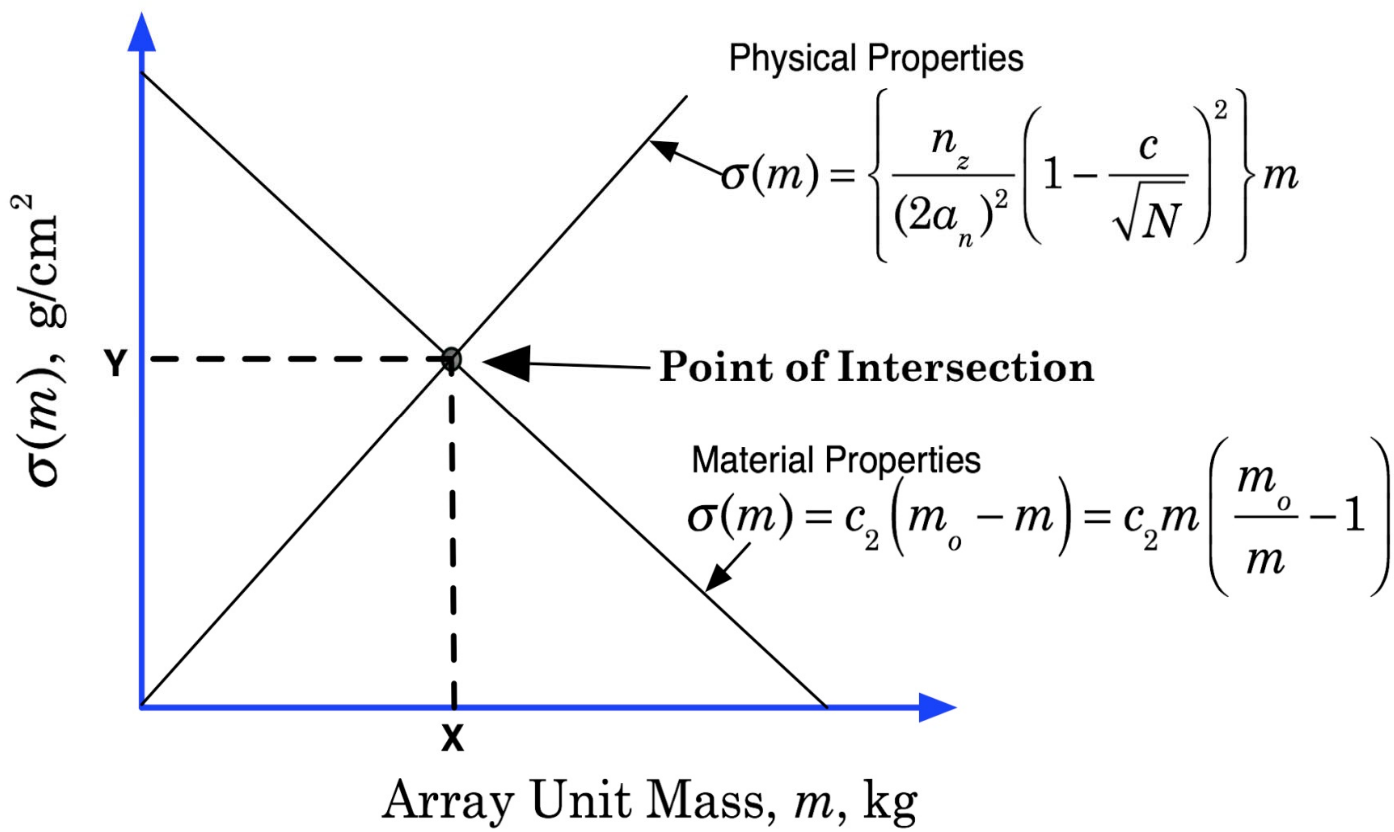
$$d = \left[\frac{nm}{2.1\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}} \cong 0.69\left[\frac{nm}{\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}}$$

$$f = \frac{m}{m_0}$$

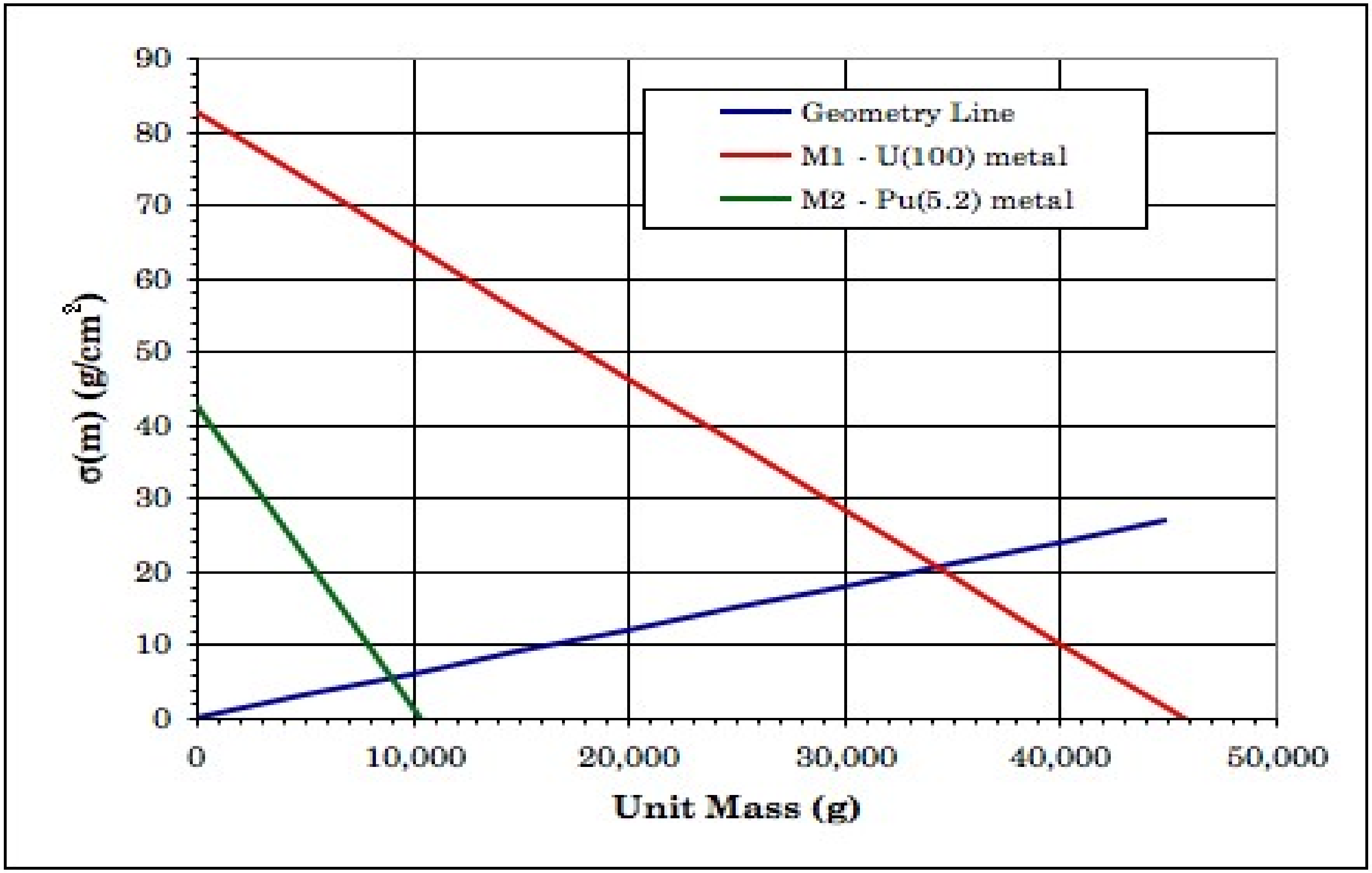
Limiting Surface Density (NB_N^2)

- Combination of Density Analog & Diffusion Theory
  - Expands Geometric Buckling Expanded for a Cubic Array
- Known Information:
  - Fissile Unit Height
  - Mass of Fissile Material in Each Array Unit
  - Critical Mass of Unreflected Sphere
  - Characteristic Constant as Function of Material Properties (c_2)
- Can Substitute Other Fissile & Reflector Materials
- $H/X \leq 20$ & $0.3 \leq H/D \leq 3$
- Calculates Critical Array

Limiting Surface Density (NB_N^2) Cont.



Limiting Surface Density (NB_N^2) Cont.

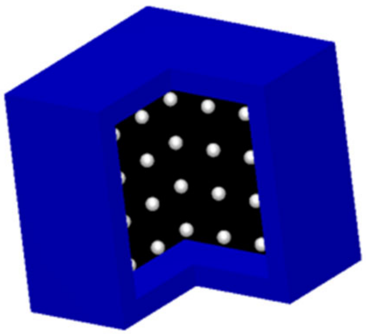


Limiting Surface Density (NB_N^2) Cont.

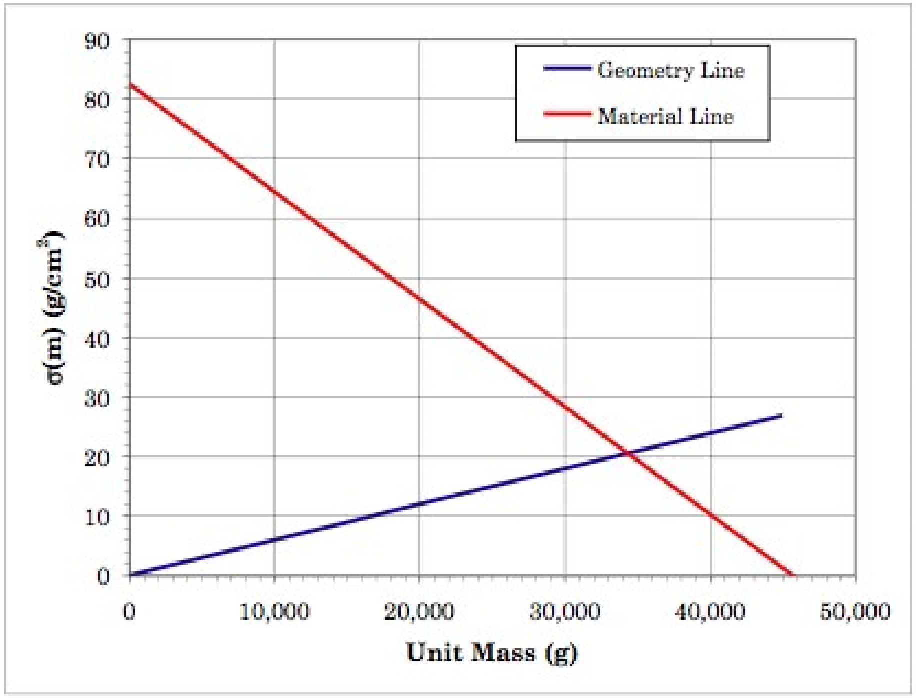
- c_2 – a constant that depends on all of the material properties of the arrays except for the mass, m, and is also equal to the slope of the "material-line" (cm^{-2})
  - the Primer provides Thomas' values for this constant for various fissile systems,
- c – an empirically determined constant equal to 0.55 ± 0.18 for the range of fissile materials and arrays studied by Thomas.
  - tends toward zero in a thermal system (i.e., moderated units in the array)

Limiting Surface Density (NB_N^2) Cont.

Graphical Limiting Surface Density Example:



- 4x4x4 Array
- 76.2 cm (30" c-to-c spacing between units)
- U (100) metal units
- Determine unit mass for array criticality



Geometry Line: $\sigma(m) = \frac{n_z m}{(2a_n)^2}\left(1 - \frac{c}{\sqrt{N}}\right)^2 = \frac{4m}{(2 \times 38.1\ cm)^2}\left(1 - \frac{0.55}{\sqrt{64}}\right)^2$

$\sigma(m) = 5.974 \times 10^{-4} m$ or $\frac{\sigma(m)}{m} = 5.974 \times 10^{-4}$

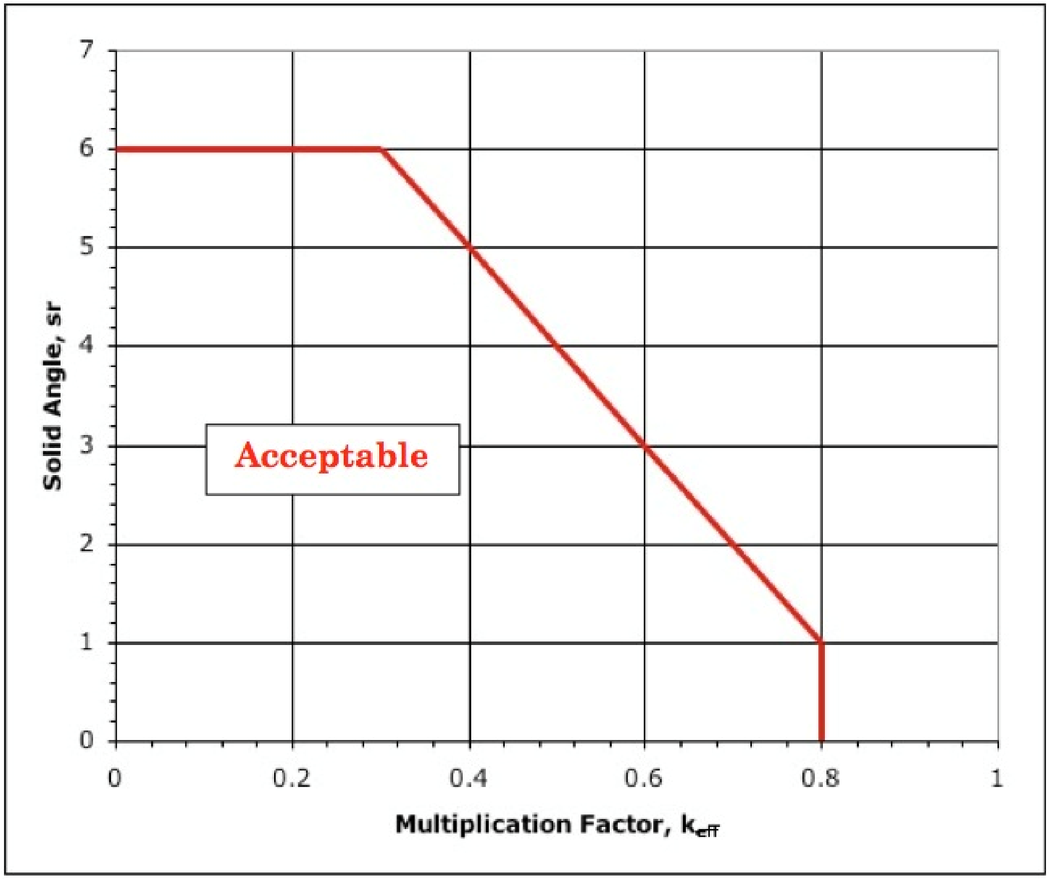
Material Line: $\sigma(m) = c_2(m_o - m)$

$\sigma(m) = 1.806 \times 10^{-3}(45.68 - m)$ or $\frac{\sigma(m)}{m} = 1.806 \times 10^{-3}\left(\frac{45.68}{m} - 1\right)$

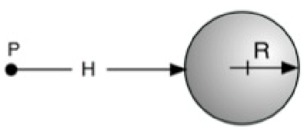
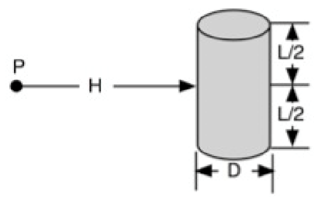
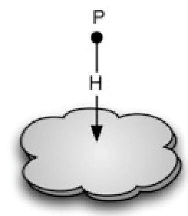
Solid Angle

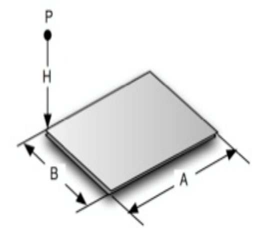
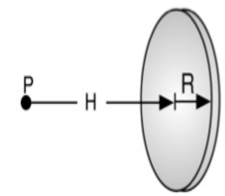
- Small Numbers of Moderated Fissile Units
- Can Be Non-Conservative for Fast Neutron Systems
- Restrictions:
  - Single Unit $k_{eff} \leq 0.80$
  - Each Unit Subcritical with Thick Water Reflection
  - Units Must be ≥ 0.3 m
  - Solid Angle ≤ 6 Steradians
  - Reflection Must be Bound by Water

Solid Angle Cont.



Solid Angle Cont.

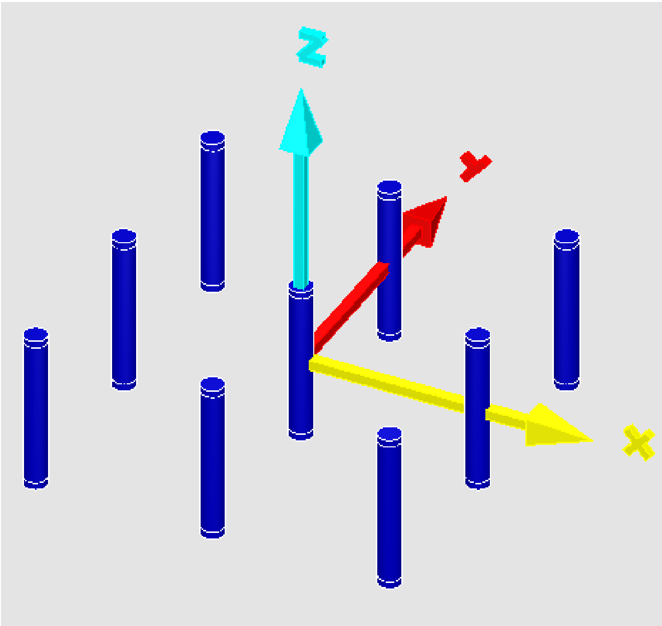
Point-to-Sphere	Point-to-Cylinder	Point-to-Arbitrary Shape
 $\Omega = 2\pi\left(1 - \frac{1}{\sqrt{1+\left(R/H\right)^2}}\right)$ where R = Radius of the sphere. H=Distance from the point to the surface of the sphere.	 $\Omega = \frac{LD}{H\sqrt{\left(L/2\right)^2 + H^2}}$ where L=Length of the cylinder D=Diameter of the cylinder H=Distance from the point to the surface of the cylinder.	 $\Omega = \frac{\text{Cross Sectional Area}}{\left(\mathrm{H}\right)^2}$

Point-to-Plane	Point-to-Disk
 $\Omega = \sin^{-1}\left(\frac{AB}{\sqrt{A^2+H^2}\sqrt{B^2+H^2}}\right)$ where A=Length of one side of the plane B=Length of the other side of the plane H=Perpendicular distance from the point to the plane. If the point, *P*, is directly above the center of the plane (not directly over a corner as shown in the figure) with dimensions 2A × 2B, multiply Ω by 4 to obtain the solid angle.	 $\Omega = 2\pi\left(1 - \frac{1}{\sqrt{1+\left(R/H\right)^2}}\right) \le \frac{\pi R^2}{H^2}$ where R=Radius of the disk H=Distance from the point P to the surface of the disk.

Solid Angle Cont.

- Problem: We want to store nine "safe"-bottles of fissile solution in a 3-by-3 array with 2-foot edge-to-edge spacing
- Question: What is the highest k_{eff} that each bottle can have as a single unit and still be subcritical in the array?

  - Each bottle is 4-feet tall
  - Each bottle diameter ½-foot



Solid Angle Cont.

- Question: What unit is the most reactive unit? Why?

Units are 2 feet edge to edge in X and Y

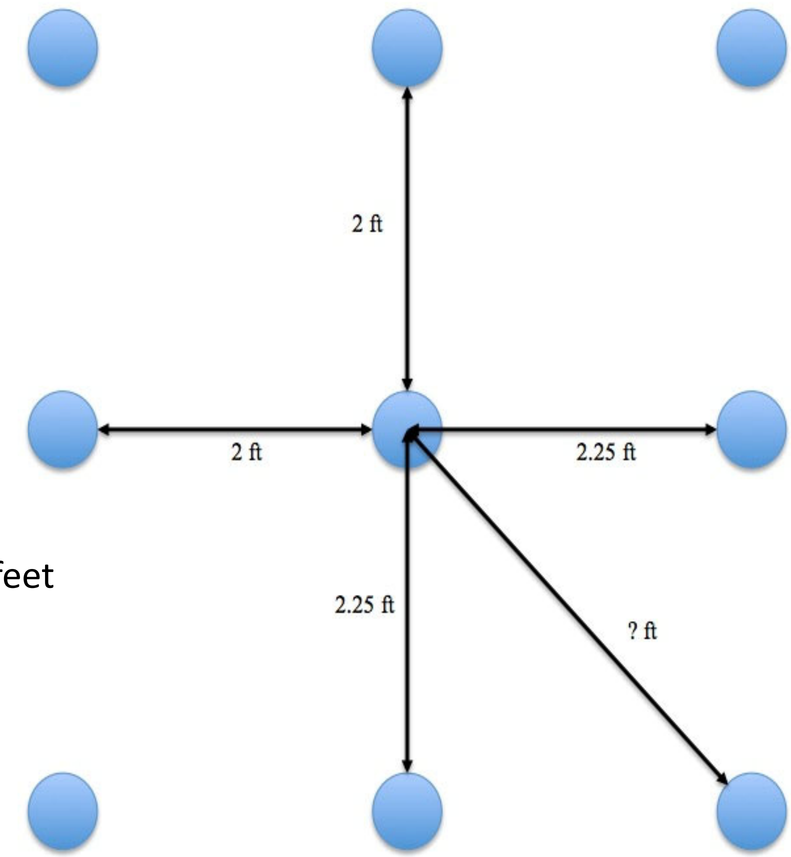
Center of central unit to edge of outer unit is ½ of ½ foot, or ¼ foot.

Point to cylinder distance is 2 feet + 0.25 feet, or 2.25 feet

The center to center distance in X and Y is 2.5 feet.

The center to center distance on the diagonal is sqrt(2.5^2), or 3.54 feet

Point to cylinder distance on the diagonals is 3.54 – 0.25 = 3.29 feet.

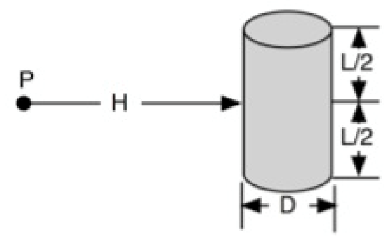


Solid Angle Cont.

- $$\Omega_n = \frac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{2.25\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (2.25\ ft)^2\right]^{\frac{1}{2}}} = 0.30\ SR$$
- $$\Omega_f = \frac{(4\ ft)\left(\frac{1}{2}\ ft\right)}{3.29\ ft\left[\left(\frac{4\ ft}{2}\right)^2 + (3.29\ ft)^2\right]^{\frac{1}{2}}} = 0.16\ SR$$

$$\Omega_{total} = 4\Omega_n + 4\Omega_f = 1.84\ SR$$

Point-to-Cylinder



$$\Omega = \frac{LD}{H\sqrt{(L/2)^2 + H^2}}$$

where

L=Length of the cylinder

D=Diameter of the cylinder

H=Distance from the point to the surface of the cylinder.

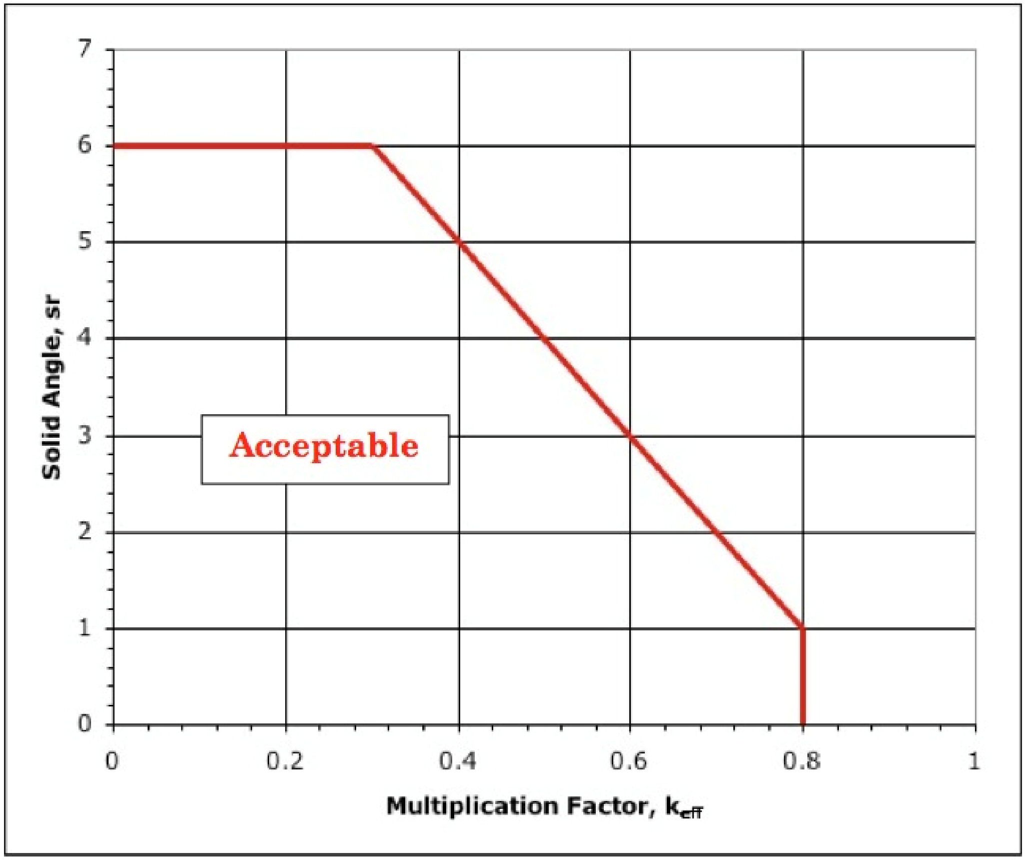
- Question: What is the highest allowed k_{eff}?

Solid Angle Cont.

$$\Omega_{total} = 1.84\ SR$$

$$1.84 = 9 - 10k_{eff}$$

$$k_{eff} = \left\lfloor \frac{9 - 1.84}{10} \right\rfloor = 0.72$$



What else is required?

What happens if we move the bottles closer together?

Potential Solutions to Competition

ANSI/ANS 8.7

- Cubic Array
- $k_{eff} < 0.95$
- 20 cm Water Reflection
- All Units are Spheres
- *DOUBLE BATCHING SHALL BE CONSIDERED*

Table 5.7
Unit Mass Limit in Kilograms of Plutonium per Cell in Water-Reflected Storage Arrays: Metal, 100 wt-% ^{239}Pu

Number of Units in Cubic Storage Arrays	*Minimum Dimension of Cubic Storage Cell (mm)*					
	254	305	381	457	508	610
($H/Pu \leq 0.01$; Pu density $\leq 19.7\ g/cm^3$)						
64	3.4	4.1	4.9	5.5[a]	5.8[a]	6.3[a]
125	2.9	3.6	4.4	5.1[a]	5.4[a]	6.0[a]
216	2.6	3.2	4.1	4.7	5.1[a]	5.7[a]
343	2.3	2.9	3.8	4.4	4.8[a]	5.4[a]
512	2.1	2.7	3.5	4.2	4.6	5.2[a]
729	1.9	2.5	3.3	3.9	4.3	5.0[a]
1000	1.7	2.3	3.1	3.7	4.1	4.8

[a] Values are greater than 90 % of critical spherical mass, water reflected.

ANSI/ANS 8.7 Cont.

- 125 Units, 4.4kg/unit = 550 kg total
  - 381 mm Cells (15 in)
  - 5x5x5 Array
  - **6.9 m^3**
- 216 Units, 2.6kg/unit = 561.6 kg total
  - 254 mm Cells (10 in)
  - 6x6x6 Array
  - **3.5m^3**

Surface Density

$$d = \left[\frac{nm}{0.54\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}} \cong 1.37\left[\frac{nm}{\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}}$$

- Choose Mass and Unit Depth (m&n)
  - f is determined from m
- σ_0 = Critical, Infinite Slab Height * Density ($h_{slab,crit}$ * ρ)
  - $h_{slab,crit}$ is looked up or calculated
    - Look up: ANSI/ANS 8.1
    - Calculated with Buckling Calculations $\left(\frac{\pi}{r+d}\right)^2 = \left(\frac{\pi}{h+2d}\right)^2$
      - d – Extrapolation Distance (3.28cm)
      - r – Critical Radius Sphere (Given or looked up)

Surface Density Cont.

- Masses: 2 kg, 4.5 kg, 7 kg & Unit Depth: 1 Unit, 2 Units
  - 2.0 kg, 1 Unit: d = 17.6 cm
  - 2.0 kg, 2 Unit: d = 25.0 cm

    250 Total Units
  - 4.5 kg, 1 Unit: d = 35.3 cm
  - 4.5 kg, 2 Unit: d = 49.9 cm

    112 Total Units
  - 7.0 kg, 1 Unit: d = 91.9 cm
  - 7.0 kg, 2 Unit: d = 130.0 cm

    72 Total Units

Surface Density Final

- Distance between units isn't the final step
- Choose an array size:
  - 2.0 kg, 1 Unit: 1x8x32 V=0.82 m^3
  - 2.0 kg, 2 Unit: 2x8x16 V=2.12 m^3
  - 4.5 kg, 1 Unit: 1x4x28 V=1.62 m^3
  - 4.5 kg, 2 Unit: 2x4x14 V=5.97 m^3
  - 7.0 kg, 1 Unit: 1x2x36 V=5.55 m^3
  - 7.0 kg, 2 Unit: 2x2x18 V=42.7 m^3

$$\text{Volume} = (2r+d(x-1)) * (2r+d(y-1)) * (2r+d(z-1))$$

Density Analog

$$d=\left[\frac{nm}{2.1\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}}\cong 0.69\left[\frac{nm}{\sigma_0\left(1-1.37f\right)}\right]^{\frac{1}{2}}$$

- Again, choose n&m and σ_0 consistent
- Same masses as Surface Density
- Cubic Arrays:
  - 2.0 kg ~ 6x6x6 array (6.3)
  - 4.5 kg ~ 5x5x5 array (4.8)
  - 7.0 kg ~ 4x4x4 array (4.2)

Density Analog Cont.

- 2.0 kg: d=22.3 cm   V=1.6
- 4.5 kg: d=39.6 cm   V=4.6
- 7.0 kg: d=94.4 cm   V=24.9

$$\text{Volume} = (2r+d(x-1))^3$$

- Problems getting the right number of units

Conclusion

- Hand calculations have a place
- Fast, relatively easy to do
- Very conservative calculations
- Struggle with abnormal conditions
- Great for scoping and to support further calculations

Questions?